Back-calculation of the Saint-Alban A test embankment with a new modelling approach in LEM

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ABSTRACT
To facilitate the continued use of limit equilibrium method (LEM) in stability design of embankments on soft clays, the new calculation method "Hybrid s_u" (HSU) has been developed. It is used to derive undrained shear strength from effective strength parameters, or to predict the excess pore pressure at failure. The HSU method uses an anisotropic effective stress soil model with volumetric hardening, from which a closed form solution for the effective mean stress at failure p'_f is derived. This in turn is used to derive the anisotropic undrained shear strength (for use in total stress analyses), or excess pore pressure (for use in undrained effective stress analyses). The model accounts for factors such as anisotropy, consolidation state, volumetric hardening and to some extent, rate effects. An advantage of the model over traditional undrained effective stress calculations is that the overestimation of shear strength at $F > 1$ is avoided.

To illustrate the use and results of the model, a back-calculation of the classic Saint-Alban A test embankment failure is presented. In the test, an embankment was built on a soft Champlain clay deposit. The height of the embankment was gradually increased until failure. The HSU model parameters are fitted with data available from literature. The model is then applied to both total stress and undrained effective stress analyses of the case using LEM. It is shown that the relatively simple HSU model gives realistic results with both analysis types, comparable with the results of an advanced FEM analysis and literature data.

Keywords: Slope stability, Modelling, LEM, Undrained shear strength, Pore pressure

1 INTRODUCTION

The stability of existing railway embankments in the Finnish Railway network has been studied extensively in the recent years. In studies commissioned by the Finnish Traffic Agency (FTA), it has been found that the calculated factor of safety (FOS) is very low for many railway embankments on soft subsoils. In some cases the calculated FOS can be as low as $F < 1$, without any partial factors applied. As the embankments are still standing without apparent issues, it seems that there is room for improvement both in the methods used to determine the shear strength of soil, and stability calculation methods.

The traditional stability design method in Finnish engineering practice has been to use a field vane to determine the undrained shear strength of soft soils. The stability calculations are done with the Limit Equilibrium Method (LEM) as total stress ($\varphi = 0$) analyses. In the recent years, better field vane systems and CPTU equipment have been adopted in use, which somewhat improves the situation of strength determination.

An alternative for $\varphi = 0$ analyses is of course the undrained effective stress analysis (φ'-c'). Often such analyses are employed to obtain an alternative result when total stress analyses are considered unreliable (e.g. due to unreliable strength data) or when data on undrained shear strength is missing altogether. Here, the uncertainty lies in determining the excess pore pressure, which is quite difficult especially in the context of LEM.

While FEM analyses with advanced effective stress models are slowly gaining use in large projects, LEM is still the primary stability calculation tool. One aim of the Finnish Transport Agency has been to improve the accuracy and usability of LEM in undrained *φ'-c'* analyses.

Länsivaara (2010) has proposed the parameter r_u', which is used to model the shear-induced excess pore pressure in normally consolidated clays. With a set of assumptions (triaxial compression, no hardening, normal consolidation), the parameter r_u' is a function of critical state friction angle φ'_c. In conjunction with a separate parameter that models loading-induced excess pore pressure, undrained effective stress analyses are possible.

To further increase the usability of undrained *φ'-c'* stability analyses with LEM, a more complex calculation method Hybrid s_u was developed by Lehtonen (2015). In its current developed form, its main purpose is to predict undrained shear strength based on a derivative of an effective stress soil model.

2 HYBRID SU METHOD

2.1 General

The Hybrid s_u (HSU) method (Lehtonen 2015) can be used for two purposes:

a) Calculation of undrained shear strength (φ = 0 analyses)
b) Calculation of excess pore pressure at failure (undrained *φ'-c'* analyses)

HSU is intended to be used for soft, normally consolidated or lightly overconsolidated inorganic clays. Its main purpose is to provide calculation parameters to LEM stability analyses. HSU takes into account physical factors such as anisotropy, consolidation state and hardening properties.

The anisotropic effective stress soil model S-CLAY1 (Wheeler et al 2003) is used as a starting point. The original S-CLAY1 model is a Modified Cam Clay derivative with an anisotropic initial yield surface, coupled with volumetric and rotational hardening of the yield surface (Figure 1).

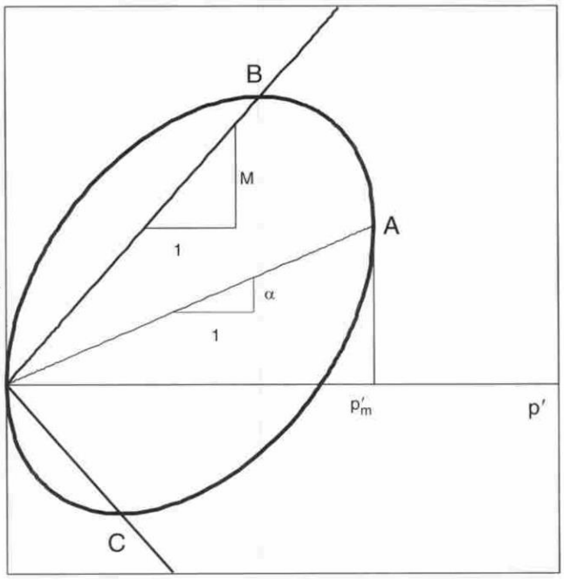

Figure 1 .S-CLAY1 initial yield surface (Wheeler et al 2003)

In HSU, a closed form solution can be derived for effective mean stress at failure p'_f with a set of suitable assumptions. The most notable assumptions are the use of the Drucker-Prager failure surface fitted for triaxial compression, and the omission for rotational hardening from the original S-CLAY1 model. The value of p'_f can then be used to derive either the undrained shear strength s_u or excess pore pressure at failure Δu_f.

The solution for p'_f needs to be omitted here for brevity. For a detailed derivation, see Lehtonen (2015). In essence, p'_f is a function of strength, stress, hardening and anisotropy parameters:

$$p'_f = f(\varphi', c', \sigma'_{v0}, OCR, \lambda/\kappa, C, D, b, \theta) \quad (1)$$

The model parameters of the HSU method are:

φ'	critical state friction angle (°)
c'	effective cohesion at critical state (kPa)
σ'_{vo}	initial vertical effective stress
OCR	overconsolidation ratio
λ/κ	hardening control parameter
C	coefficient for K_{0NC}
D	coefficient for initial K_0 (overconsolidated)
b	intermediate principal stress parameter (assumed b = 0.3)
θ	principal stress rotation angle (°)

Effective cohesion at critical state is included in HSU because some clays do exhibit apparent cohesion at large strains (e.g. Karlsrud & Hernandez-Martinez 2013). While the original S-CLAY1 is a "classical" critical state model with zero cohesion, HSU can model cohesion through a fairly simple coordinate conversion. In the conversion, the "correct" stress state p' is replaced with a calculation value p'_{calc}:

$$p'_{calc} = p' + p'_{att} \tag{2}$$

where p'_{att} is the value of attraction in the p'-q coordinate system (Figure 2).

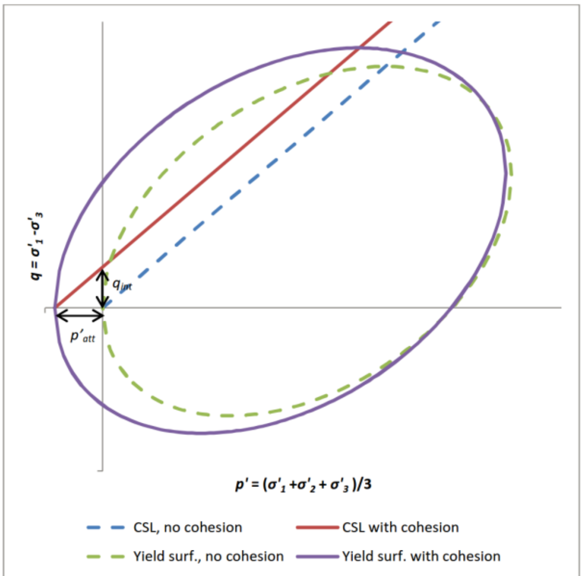

Figure 2. Concept of applying cohesion in HSU via a coordinate shift (Lehtonen 2015)

The hardening control parameter λ/κ is (by its textbook definition) the ratio between the slopes of the normal compression line and unloading-reloading line in the (ln p'-e) coordinate space. Essentially λ/κ is a parameter that controls volumetric hardening, and therefore the direction of the effective stress path (Figure 3).

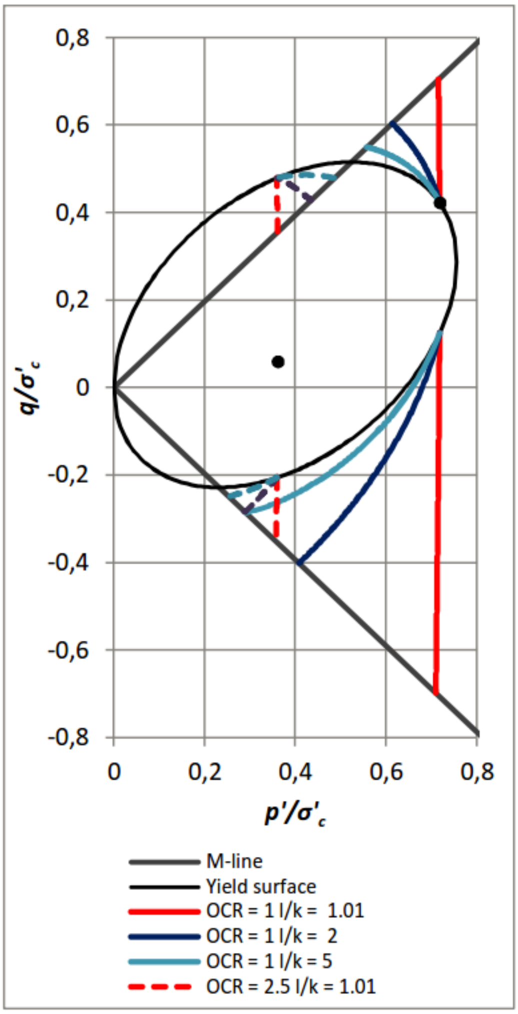

Figure 3. An example of NC and OC effective stress paths predicted by the HSU method. (Lehtonen 2015)

When λ/κ is set to 1, elastoplastic strains are equal to elastic strains, i.e. the behaviour is elastic and the stress path is vertical. When the ratio is increased, plastic strains begin to dominate and the effective stress path becomes more horizontal. This behaviour is completely analogous with any Modified Cam Clay-derivative soil model.

In the intended usage (LEM) strains are not considered, but the predicted strength is important. As such, in HSU the parameter λ/κ

can be decoupled from its physical meaning. It is simply used as a control parameter that allows the user to select a desired effective stress path that is likely for the given type of loading. Even rate dependency of strength and pore pressure response can be modelled with HSU to a certain degree: Faster loading rates result in less excess pore pressure, and consequently higher strength and more vertical effective stress paths - this can be achieved with a low λ/κ value.

Based on model fitting to laboratory data examples and failure back-calculations (Lehtonen 2015), the relevant range of λ/κ would be about λ/κ = 2.5…5.0. As a cautious engineering default value, $\lambda/\kappa = 5$ would be applicable. For very fast loading, λ/κ close to 1 can be possible.

The parameters C and D control the assumed K_0 values as follows:

$$K_{0NC} = C(1-\sin\varphi') \tag{3}$$

$$K_0 = K_{0NC} \cdot OCR^{D\cdot\sin\varphi'} \tag{4}$$

Essentially, C controls the inclination of the initial yield surface through K_0, while D only has a slight effect on the assumed initial stress state. The value of C has a considerable effect on strength anisotropy, especially on extension strength.

The intermediate principal stress parameter b (Habib 1953) controls the relative value of σ'_2.

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \tag{5}$$

For simplicity, it is assumed that $b = 0.3$. Principal stress rotation θ can be solved for each slice bottom:

$$\theta = 45° + \varphi'/2 - \alpha \tag{6}$$

where α is the inclination angle of the slice bottom.

2.2 *Use in $\varphi = 0$ analyses*

For $\varphi = 0$ analyses, HSU is simply used to predict the anisotropic undrained shear strength along the slip surface. The strength is defined (using the Cambridge notation) simply as:

$$s_u = \frac{q_f}{2} = \frac{p'_f \cdot M}{2} \tag{7}$$

As p'_f and s_u are functions of principal stress rotation, the s_u needs to be calculated separately for each slice bottom along each different slip surface. The limit equilibrium analysis for a given slip surface is then conducted as usual, with only the strength input calculated with the HSU method.

2.3 *Use in undrained φ'-c' analyses*

Lehtonen (2015) presents two different approaches for calculating Δu_f with the HSU method.

The main idea is that in stability calculations, the most important soil variable is the shear strength τ_f that can be attained at failure. In traditional undrained effective stress analyses the calculated shear strength does not correspond to failure, but to the mobilised state. This causes an inherent overestimation of τ_f and F, when $F > 1$, as the pore pressure increase between the mobilised state and failure is disregarded (Figure 4).

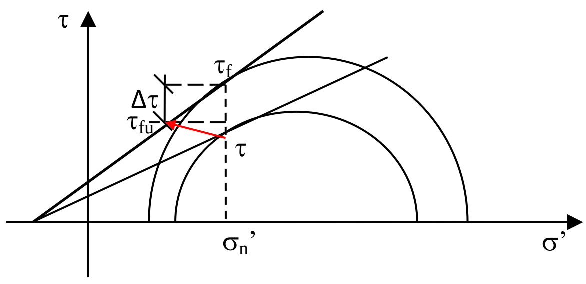

Figure 4. Overestimation of shear strength $\Delta\tau$ in traditional undrained effective stress analyses. τ_f is the assumed shear strength, while τ_{fu} is the actual strength at failure in undrained conditions.

In HSU, priority is given to attaining a "correct" shear strength that corresponds to the failure state. This means that the calculated excess pore pressure does not correspond to the mobilised state, but to failure. The overestimation of shear strength when $F >> 1$ is therefore eliminated.

In the first approach, the excess pore pressure is "forced" so that the resulting shear strength τ_f is equal to the undrained shear strength s_u predicted by HSU:

$$\begin{aligned} s_u &= \tau_f = c' + \sigma'_n \cdot \tan\varphi' \\ &= c' + (\sigma_n - u_0 - \Delta u_f) \cdot \tan\varphi' \end{aligned} \tag{8}$$

$$\Delta u_f = \sigma_n - u_0 - \frac{s_u - c'}{\tan\varphi_c'} \tag{9}$$

Equation 9 contains the total normal stress acting on the slice bottom. To obtain this, an iteration is necessary. First an assumption of Δu_f is made, the limit equilibrium is calculated and a value for σ_n is obtained from the results. Then Δu_f is calculated again, and the process is repeated until Δu_f converges.

As it is "decided" that $s_u = \tau_f$, the calculated factor of safety of the undrained effective stress analysis is completely identical to the corresponding total stress analysis.

A second, more "traditional" approach with HSU is to calculate Δu_f based on stress changes. From the principle of effective stresses it can be written that:

$$\Delta p' = \Delta p - \Delta u \tag{10}$$

$$\Delta u_f = \Delta p_f - \Delta p_f' \tag{11}$$

The change of effective mean stress $\Delta p'$ can be calculated with the HSU model (the initial stress state is known, and the failure state is calculated). The change in total stresses can be calculated with basic continuum mechanics, based on changes in vertical stresses and an assumption of the pore pressure at failure. Again, a first assumption of Δu_f is made, Δp_f is calculated, a new value for Δu_f is calculated and the iteration is continued until the calculation converges.

Even when Δu_f is calculated based on stress changes, these stress changes correspond to the assumed failure state. Therefore, the pore pressure and the resulting shear strength should also correspond to the actual failure.

3 SAINT-ALBAN TEST FILL A

3.1 Site description

In 1972, a test embankment in Saint-Alban, Quebec, Canada was brought to failure in a testing program by Laval University. The embankment was built on a sensitive Champlain Sea clay deposit. Its height was gradually increased until it failed at a height of 3.9 m. (La Rochelle et al 1974)

The subsoil consisted of a 1.5-2 m thick stiff crust, followed by a slightly overconsolidated clay layer. Figure 5 presents the range of field vane tests and some CIU tests (after Trak et al 1980), as well as model predictions made with the MIT-E3 effective stress soil model.

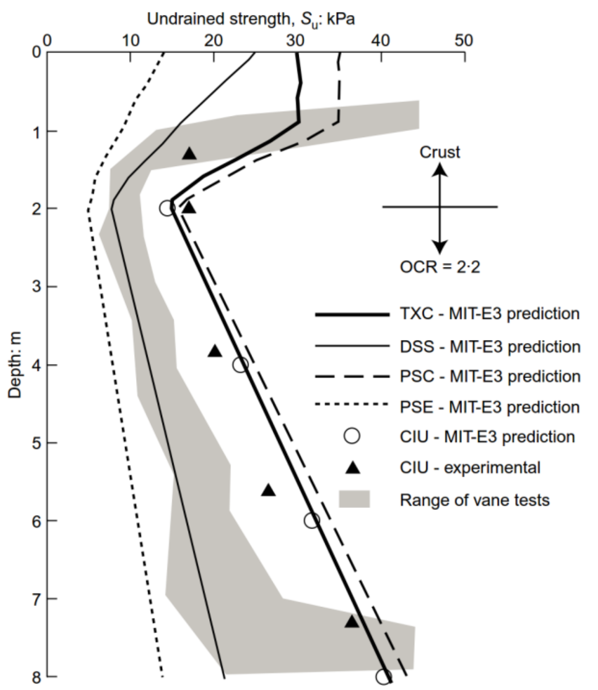

Figure 5. Measured strengths and predicted strength profiles (Zdravkovic et al 2002)

The friction angle in triaxial compression is given as $\varphi' = 27°$, and the lateral earth pressure in triaxial compression is K_{0NC} = 0.49 (Zdravkovic et al 2002).

3.2 HSU parameters and calculation results

To study the validity of the HSU method a back-calculation of the Saint-Alban Test Fill A is conducted. HSU is applied to the soft clay layer below the depth of 2.0 m.

The parameter selection for HSU is fairly straightforward. The given friction angle φ' = 27° is utilized as it is. As the K_{0NC} value is given, the parameter C can be calculated as:

$$C = \frac{K_{0NC}}{1 - \sin\varphi'} = \frac{0.49}{1 - \sin 27°} \approx 0.90 \qquad (12)$$

The parameter D is left at its default value D = 1, as there is no data available about the in situ lateral stresses. The OCR value for the clay is set to OCR = 2.2 (Zdravkovic et al 2002). Unit weight of γ'_{sat} = 16 kN/m3 is used for the clay (Trak et al 1980).

The embankment and crust properties are given in Table 1. The dry crust is modelled with given s_u values, but different strength profiles are used for the active and passive parts. The values are consistent with data used by Zdravkovic et al (2002).

Table 1. Soil parameters for the embankment and dry crust

	φ' [°]	s_u (kPa)	ds_u (kPa/m)	γ (kN/m^3)
Embankment	44	0	0	19
Crust (z =0-1 m, active side)	0	30	0	19
Crust (z = 1-2 m, active side)	0	30 (top of layer)	-20	19
Crust (z =0-2 m, passive side)	0	13 (top of layer)	-4	19

The only unknown HSU model parameter is λ/κ. Its "proper" value can be determined by a back-calculation setup where λ/κ is varied.

Figure 6 illustrates the initial yield surface obtained with the given HSU parameters as well as effective stress paths in compression and extension for different λ/κ values. Note that compression and extension stress states are not considered in the stability calculation itself, but the intermediate principal stress parameter is set as b = 0.3, as assumed in the derivation of the HSU method (see Section 2.1).

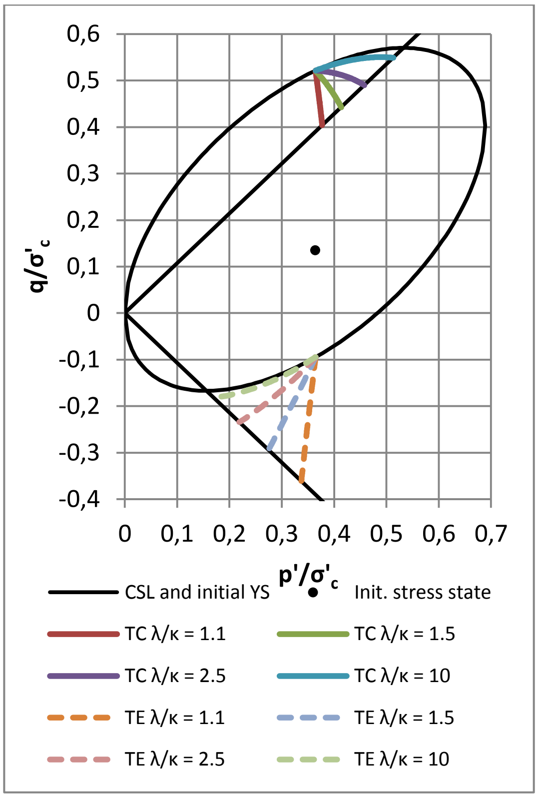

Figure 6. HSU compression and extension stress paths with different λ/κ values. Other parameters are defined in the text.

A circular slip surface with 20 slices, consistent with the observed mechanism (La Rochelle et al 1974), is created (Figure 7). The stress state and other relevant parameters are determined for each slice bottom. It is assumed that the initial state includes only the crust and clay layer. In the failure state the embankment with a height of h = 3.9 m (the stated failure height) is applied.

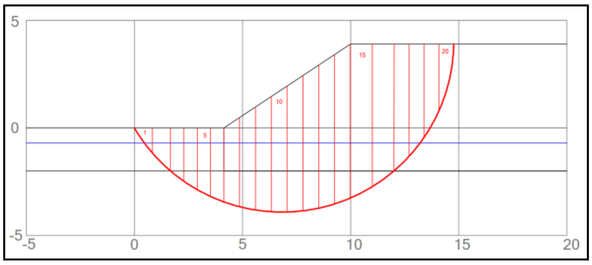

Figure 7. The slip surface used in the analyses

The limit equilibrium calculation was carried out in a spreadsheet using Spencer's method (Spencer 1967). Practically any other limit equilibrium method could have been used as well, but Spencer's method can be considered a good compromise between accuracy and simplicity in spreadsheet applications.

By varying the λ/κ value, the factor of safety can be calculated as a function of λ/κ, with φ = 0 limit equilibrium analyses. The calculated factors of safety versus λ/κ is illustrated in Figure 8.

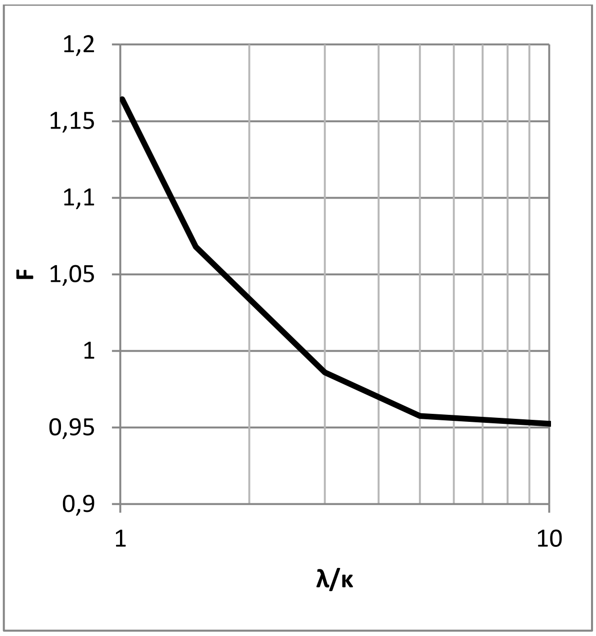

Figure 8. Calculated F versus λ/κ, total stress analyses

From the total stress calculations where HSU is used to calculate s_u in the clay layer, it becomes apparent that F = 1 when λ/κ = 2.53. The resulting strength profile along the slip surface is shown in Figure 9a. For comparison, the strength profile predicted by Zdravkovic et al (2002) with the MIT-E3 soil model is shown in Figure 9b. Note that as the failure surfaces are not exactly the same, the coordinates along the failure surface length differ slightly. Additionally, the HSU strength profile as shown in Figure 9a shows the average strengths calculated for each slice bottom, and includes the embankment in the far right of the graph.

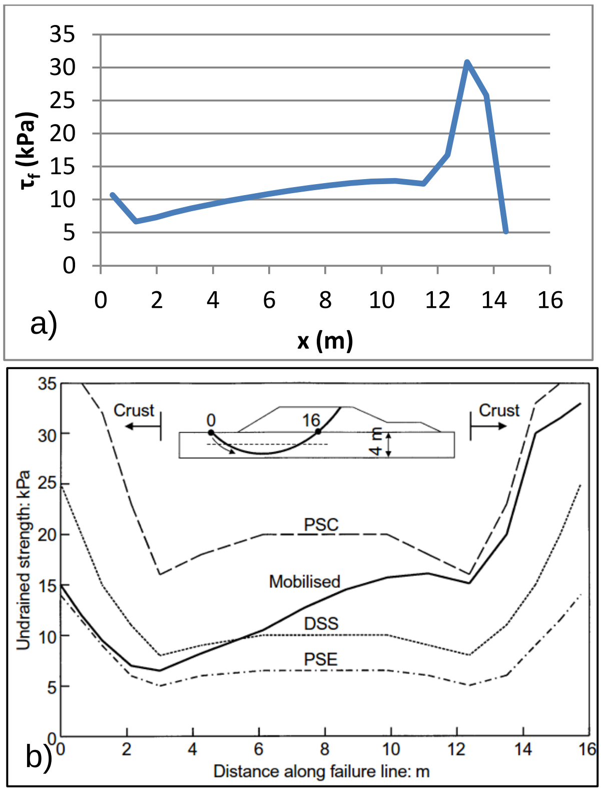

Figure 9. a) Shear strength profile along the slip surface at λ/κ = 2.53, HSU total stress analysis. b) Mobilised shear stress and predicted strength envelope, MIT-E3 (Zdravkovic et al 2002).

The predicted HSU strength profiles (λ/κ = 2.53) against depth are shown in Figure 10. These can be compared to the predicted strength profiles from Zdravkovic et al (2002), see Figure 5.

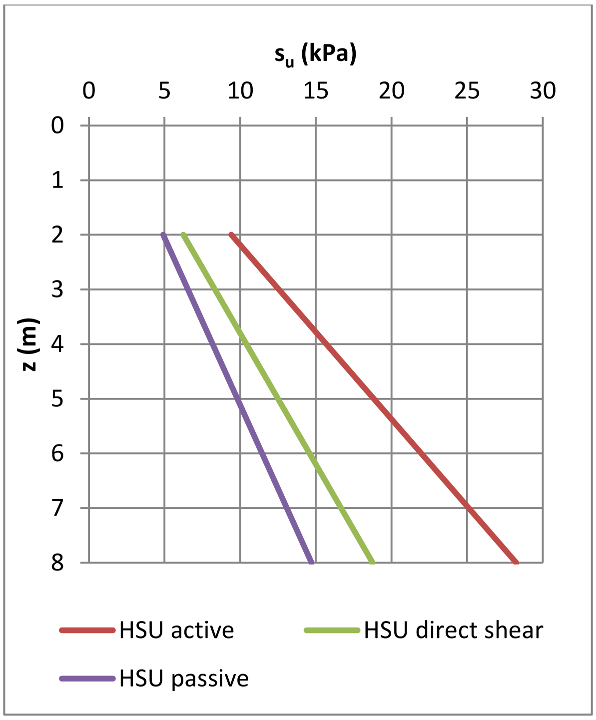

Figure 10. HSU strength profiles at λ/κ = 2.53.

The stability calculation was also conducted as an undrained φ'-c' analysis where the HSU method was used to predict the excess pore pressure at failure Δu_f. Figure 11 shows the excess pore pressure distribution at failure (λ/κ = 2.53) for the two different approaches (forcing to a given s_u value, and Δu_f calculation based on stress changes).

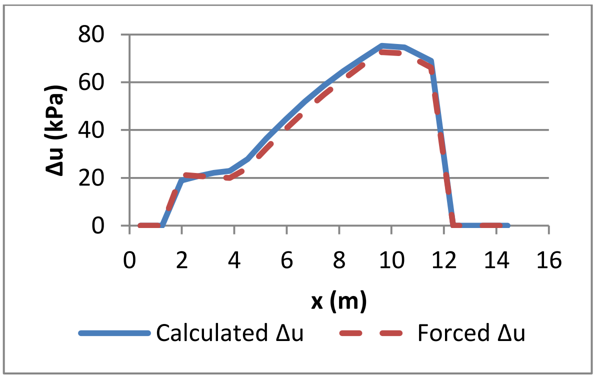

Figure 11. Calculated Δu at failure, HSU undrained effective stress analyses

As the shear strength distribution along the slip surface is exactly the same between the φ = 0 -analysis and the undrained φ'-c' analysis with the forced approach, the calculated factor of safety is exactly F = 1 in both cases. The Δu_f distribution based on the calculated stress changes is only slightly larger, and therefore results in a slightly lower shear strength and a factor of safety of F = 0.93.

4 DISCUSSION & CONCLUSIONS

This paper briefly presents the newly developed calculation method "Hybrid s_u" (HSU). The HSU method (Lehtonen 2015) is intended for calculating undrained shear strength or excess pore pressure at failure of clay in LEM stability calculations.

By a process of back-calculation, the HSU method parameters were fitted to the case of the Saint-Alban A test embankment failure (La Rochelle et al 1974). The model fitting is quite straightforward, as the parameters can be chosen directly based on the available basic soil data. The value of the control parameter λ/κ is obtained via back-calculation.

The results of the back-calculation are generally quite plausible. Failure is obtained with λ/κ = 2.53. This represents a typical value in back-calculations and model fits (see Lehtonen 2015), and as such the result is quite plausible. While such a ratio is typical for failure test conditions, natural slope failures might proceed at a much slower rate, which then requires a higher λ/κ ratio to be able to simulate the higher failure induced pore pressure/lower undrained shear strength.

A FEM analysis using the MIT-E3 effective stress soil model (Zdravkovic et al 2002) is used as a benchmark to compare the results.

The mobilized shear strength profile at failure, as predicted by HSU, compares quite favourably to the one predicted by the more advanced soil model. However, when predicted strength profiles against depth are compared, the HSU model underestimates the active (compression) shear strength. This happens because "strength" in the HSU framework is defined as the stress point where the effective stress path reaches the critical state line.

When shearing occurs on the dry side of critical (as it does for active shearing at OCR = 2.2), the model exhibits volumetric softening of the yield surface. This behaviour occurs with all Modified Cam Clay derivatives, such as the S-CLAY1 model from which HSU is developed.

The HSU method is generally intended for use with normally consolidated or lightly overconsolidated clays (e.g. up to OCR ≈ 2). In this case the underestimation of dry side active strength did not affect the end result very much. This is due to the particular slip surface geometry, where only a small portion of the slip surface corresponds to active shearing in the clay layer.

While it is obvious that for OC clays, the HSU method may underestimate active strength compared to measured peak strengths, in design this may actually be beneficial. HSU is not intended for modelling soil, but as a stability calculation tool. As HSU does not include strain softening, it could be dangerous to use it for predicting peak strengths. This is obviously due to the fact that the soil does not achieve peak strengths simultaneously along the slip surface. Instead, the definition of strength in

HSU is a post-peak, or critical state strength for shearing on the dry side of critical.

The method can still be fitted to produce peak strengths, but this requires conscious fitting to peak strength data. In this example, the model fit (namely, the choice of the λ/κ value) was achieved through back-calculation of a failure, which results in an average mobilized shear stress profile at failure.

It may be worthwhile to include the option of changing the definition of strength in subsequent iterations of the HSU. Instead of defining strength as a point on the critical state line, the overconsolidated (dry side of critical) strength could be defined as the peak strength that is obtained when the effective stress path reaches the initial yield surface (Figure 12). This would simplify model fitting to peak strengths to some extent. The choice between the two definitions would then depend on the engineering judgment of the user.

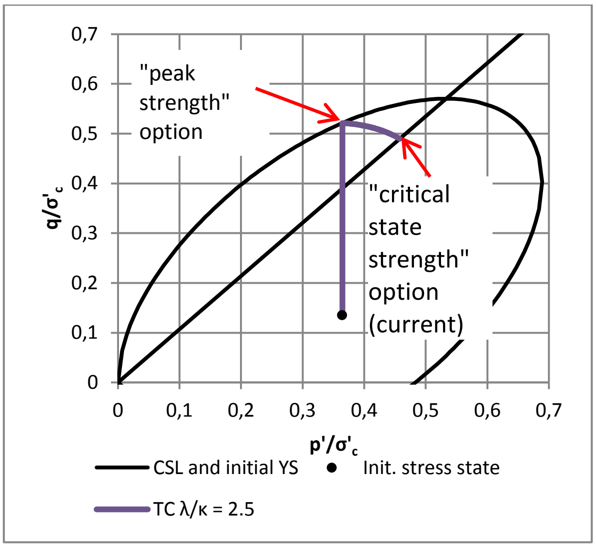

Figure 12. Possible option for the choice of dry side shear strength definition in HSU

The use of the HSU method to calculate Δu_f in undrained effective stress calculations is presented only as a proof of concept. As the goal in the formulation is to obtain results that are similar to corresponding total stress HSU calculations, the use of undrained φ'-c' can be considered redundant. Additionally, the calculations involved in obtaining Δu_f are unnecessarily complicated when compared to the calculation of s_u with the HSU method.

Overall, the HSU method shows promise as a simple yet reasonably accurate design tool in LEM stability calculations. The new method is currently being implemented to a geotechnical design software commonly used in Finland.

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