Cyclic consolidation response of clays using Bounding Surface Plasticity model

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ABSTRACT: This paper presents an elasto-plastic constitutive model called Bounding Surface Model (BSM) which can predict the behaviour of porous material under various conditions. Application of the BSM for unsaturated soils under dynamic loading is investigated. Main components of the model such as yield surfaces, hardening rules, and plastic flow rule are described in detail. Effect of suction is considered by defining suction hardening and examples are solved in different suction values to compare the response of soil under various suctions. Besides the mentioned constitutive model, Finite Element method is employed as the global solution to propose a mathematical framework which solves boundary value problems. Soil Consolidation problem is simulated in saturated and unsaturated state and all required initial and boundary conditions are defined. Results are verified with some existing experimental data and good agreement is achieved. A comprehensive discussion is presented and it is concluded that the BSM is capable to simulate the dynamic behaviour of unsaturated soils to a good extent.

1 INTRODUCTION

In many scientific fields such as geotechnical engineering, reservoir engineering, and material science, it is important to correctly predict the response of geo-materials. Fluids such as air or water exist in the pore spaces and interaction between fluid phases and solid particles have to be considered to develop a realistic constitutive model. This is essential for all numerical methods that predict the behaviour of geo-materials. BSM is a successful constitutive model among many constitutive models that are developed (1-8).

BSM was first developed by Dafalias and Popov (9) for steels. The model was later extended to simulate the behaviour of cohesive soils by Dafalias and Herrmann (10). There were no separated purely elastic and plastic region and the whole stress states, elastoplastic behaviour was assumed and this was the advantage of BSM comparing to classic Cam Clay model. In fact, the stress point always lies on the loading (yield) surface and gradually moves towards the bounding surface as the stresses are increased. The main hardening parameter is related to the distance between the loading surface and the bounding surface. Due to this, the stiffness of the material changes as the loading surface is enlarged. More attempts were made later to extend the BSM to non-cohesive soils (3, 4).

Russell and Khalili (11) proposed a bounding surface model (UNSW model) which had a simpler bounding surface equation comparing to the previous models, and included particle rearrangement and particle crushing effects for monotonic loading paths. The model was later extended to include cyclic loading by Khalili, Habte (5). The fully coupled version of this model was later proposed by Khalili, Habte (12) including both hydraulic and mechanical hysteresis. As the original mapping rule was complicated and time consuming in cyclic loadings, Kan, Taiebat (13) suggested a simplified mapping rule for the model.

Mathematical framework proposed by Khalili, Habte (5) and Kan, Taiebat (13) is extended in this study to add the effect of load angle ($\theta$) in the formulation. The presented model is capable of capturing the behaviour of soils under various loading conditions. Application of bounding surface model on predicting the consolidation response of clays is studied in particular.
2 METHODOLOGY

The model presented in this study is an extension of the model previously presented by the Geotechnical group at UNSW (5, 11-13). Because of the limitation of space and to avoid unnecessary repetition, readers are referred to those references for the complete description of the UNSW bounding surface plasticity model. Compresive stresses are assumed negative, however the mean effective stress, pore air pressure and pore water pressure are assumed positive in compression. The formulation of the bounding surface model is derived in $p' \cdot q \cdot \theta$ space:

\[
p' = \frac{l'}{3} \tag{1}
\]
\[
q = \sqrt{3} I_2 \tag{2}
\]
\[
\theta = \frac{1}{3} \sin^{-1} \left[ -3\sqrt{3}/2 (I_2/I_3^2) \right] \tag{3}
\]

where $l'$ is the first invariant of the effective stress tensor, and $I_2$ and $I_3$ are the second and third invariants of the deviator stress tensor respectively. The main components of BSM are a bounding surface, a loading surface, a plastic potential, and a hardening rule which are described in detail in the following section:

It is required to define a surface which encompasses all the possible stress states for a specific material. This surface is called Bounding surface and its shape is determined from laboratory tests. In fact lowest state of the soil for different stress paths in undrained loading condition determines the shape of the bounding surface (Fig. 1). Bounding surface formula is derived in $p' \cdot q \cdot \theta$ and expressed in Eq. 2.

\[
F(p', q, \theta, p'_c) = \left[ \frac{q}{M_{cs}(\theta)} \right]^{p'_c} - \ln(p'_c/p') / \ln(R) = 0 \tag{4}
\]

where $p'_c$ controls the size of the bounding surface, $N$ and $R$ are material parameters, and $M_{cs}(\theta)$ is the slope of the critical state line.

A loading surface is also defined in a way that it is homogenous with the bounding surface about a point. This point is named as Centre of Homology (CoH) and is the last point of stress reversal at each loading/unloading (Fig. 2). The current stress point always locates on the loading surface. Stress directions are determined from the normal vector of the loading surface at the desired point.

It is required to define the direction of plastic strains and this is achieved by using non-associative flow rule. The normal vector of the proposed plastic potential surface at the current stress point determines the direction of plastic strain at each step. Based on the type of loading (compression or extension), there are two functions for the plastic potential (Eq. 3 and 4).

For $A \neq 1$;

\[
g(p', q, \theta, p_0) = \bar{\tau} + A \cdot M_{cs}(\theta) \cdot p'/A - 1[(p'/p_0)^{\bar{A}-1} - 1] \tag{5}
\]

and for $A = 1$;

\[
g(p', q, \theta, p_0) = \bar{\tau} + M_{cs}(\theta) \cdot p' \cdot \ln(p'/p_0) \tag{6}
\]

$A$ is a material parameter, $\bar{\tau}$ is +1 when the soil is under compression and -1 when there is extension loading. $p_0$ controls the size of the plastic potential surface on the hydrostatic axis. The plastic potential surface for compression loadings is shown in Fig. 3.
3 APPLICATION

A dynamic framework is presented to capture the cyclic consolidation response of clays. Finite element method is used as the global solution and the constitutive relations are derived from bounding surface model. A 100m column of soil is modelled in various conditions to investigate one dimensional consolidation response of soils. Step loading and harmonic loading are applied to the top of the soil column. Effect of degree of saturation on the behaviour of the soil is examined. Deformation of the column and also stress changes are recorded. Moreover, Changes in pore pressures and effect of suction is studied.

Fig. 4 shows the geometry and boundary condition of the soil column. The mesh pattern of $1 \times 100$ is considered as the default mesh configuration for the analyses. The finite element model presented in this study was verified by comparing the numerical results with some analytical solutions using a linear elastic constitutive model (14). First a step load and then a cyclic load is applied to the top of the soil column and the results are presented below:

Fig. 3: Schematic shape of 3D plastic potential in compression state.

3.1 Step loading:

In this section, results of a uniform step load applied to a 1D column of Kaolin clay are described. The load is increased from zero to reach maximum value of 500 kPa at time of 100 sec; it then remains constant. There is a confining pressure of 100 kPa applied to the soil column. Material and bounding surface parameters are gathered in Table 1.

Fig. 4: Boundary conditions of the soil column.
3.2 Harmonic loading

A harmonic load with amplitude of $q = 400 \text{ kPa}$ is also applied to the soil column. The loading period ($d$) is 20 sec and Five cycles of harmonic loading are applied to the soil which takes 100 sec in total.

$$f(t) = q \cdot \left[1 - \cos\left(\frac{2\pi t}{d}\right)\right]$$  \hspace{1cm} (7)

Fig. 6 shows the changes of top surface displacement during loading time. At each time step, the maximum displacement is captured at the top surface and it reached a peak value at the end of loading/reloading period. Some part of this deformation is due to elastic behaviour and reverses during the unloading. Less amount of displacement is observed during next re-loadings which show that soil becomes stiffer as each loading cycle passes and more elastic behaviour is displayed. It can be seen that soils is close to reach its quasi stationary state after five cycles. It should be noted that although the amplitude of the displacement or pore pressure change becomes constant after reaching the stationary state, its quantity is different for points in different depths of the soil column.

![Fig. 6: Displacement of the top surface versus loading time.](image)

Fig. 7 compare changes of pore water pressure at different depth respectively. Pore water at peak of loading is maximum at the bottom of the column which is due to defined boundary conditions i.e. drainage is allowed from top surface; however, all other boundaries are impermeable.

![Fig. 7: Comparison of pore water pressure changes for a point at the bottom and middle of the soil column.](image)

Table 1: Material and BSM parameters for Kaolin clay

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tbody>
<tr>
<td>Poisson’s ratio</td>
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<tr>
<td>Specific volume</td>
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<tr>
<td>Slope of compression line ($\lambda$)</td>
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<tr>
<td>Slope of recompression line ($\kappa$)</td>
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<td>Bulk modulus ($K$)</td>
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<td>Shear modulus ($G$)</td>
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<td>Lame modulus ($\Lambda$)</td>
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<tr>
<td>$M_{cs}$</td>
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<tr>
<td>$N$</td>
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</tr>
<tr>
<td>$R$</td>
<td>2.90</td>
</tr>
<tr>
<td>$A$</td>
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</table>

The soil column is simulated at dry, saturated, and unsaturated conditions and results are compared in Fig. 5.

![Fig. 5: Consolidation response of dry, saturated, and unsaturated Kaolin clay.](image)

It can be seen that the maximum deformation is the same for all cases; however the dry soil more quickly reaches the maximum settlement. In fact, there is a smoother trend in samples with less values of suction. In the samples with some water content, pore water pressure increases suddenly and the load is increased and dissipates as le load is kept const-
Fig. 8 and 9 show the distribution of pore water pressure and suction respectively through the depth of the column at different time steps during the first cycle.

At first, the values are the same for the whole column; however, it changes as the applied load is increased. Pore water pressure at top surface of the column are always zero as the top side is completely drained. Maximum absolute values of pore water pressure is observed at the bottom of the column. Comparing the curves corresponding to $t = 40$ sec (middle of the loading stage), and $t = 120$ sec (middle of the unloading stage) verifies that rate of pore pressure dissipation is higher than pore pressure generation. The greatest negative values are recorded at the end of the cycle.

Fig. 8: Pore water pressure distribution through the depth of the column in various time steps of the first cycle.

Fig. 9: Suction distribution through the depth of the column in various time steps of the first cycle.

4 CONCLUSION

Simulation results for consolidation response of cohesive clays under monotonic and cyclic loadings are presented in this study. Bounding surface plasticity model is developed to define the constitutive relations and Finite Element method is used for the global solution. Results for dry, saturated and unsaturated soil are compared to each other and change in various parameters such as pore water pressure, displacement and stress are investigated. It is discussed that the presented mathematical framework can describe the behavior of soils under various conditions.

REFERENCES


