SUMMARY. - Simple shear tests on saturated kaolin have confirmed the existence of a unique state boundary surface enclosing all possible states of the clay. Stress paths lying on this surface result in volumetric yielding, while paths beneath the surface produce an elastic volumetric response. Angular strains associated with volumetric yielding are small in magnitude, with total angular strains being adequately represented by a strain contour pattern which is independent of stress path.

I. - INTRODUCTION

Over the past 15 years, an idealised model for clay (hereafter referred to as Cam-Clay) has been developed at Cambridge and refined in the light of available experimental data. Cam-Clay is assumed to behave mechanically as an elasto-plastic, isotropic, non-viscous continuous medium, and has been analysed mathematically by Roscoe and Schotield (1963) and Roscoe, Schofield and Thurairajah (1963) for the case of triaxial compression tests. The parameters used in these analyses are simple derivatives of the stress and strain invariants applied to the case of the triaxial test, and are defined in detail in the above papers. In the following discussion, general reference is made to the volumetric strain $v$ (directly related to void ratio $e$), shear strain $e$, mean effective pressure $p$, and deviator stress $q$.

The Cam-Clay theory aims to predict the small strain increments caused in a soil element by the application of a probing stress increment. Plastic-yielding of the material is assumed to satisfy the stability criterion of Drucker (1959), which is applied mathematically to an energy equation relating input energy to stored (elastic) and dissipated (plastic) energy. Central to the development of the theory is the assumption of a unique relationship between void ratio $e$ and effective stresses $(q, p)$ which defines a succession of "critical states" of the clay. It is postulated that all clay samples, regardless of stress history, will reach a point on this critical state line when deformed under continually increasing shear stresses.

A full development of the theory is presented by Schofield and Wroth (1968), and leads to the mathematical definition of a surface in $e$, $p$, $q$ space which encloses all possible states of Cam-Clay, and which is referred to as the state boundary surface.

The primary purpose of this Paper is to investigate experimentally the nature of the state boundary surface under plane strain test conditions in the simple shear apparatus. After describing the experimental procedure, the Paper discusses the significance of the state boundary surface as is understood in the development of the Cam-Clay theory. The region enclosed by this surface is then investigated using tests with a number of stress paths. The experimental data are presented in two sections, the first dealing with volumetric strains as a function of applied stresses (section IV), and the second analysing the observed angular strains (section V). Conclusions derived from both sections are then summarised.

II. - EXPERIMENTAL PROCEDURE

The experimental programme consisted of incremental stress-controlled shear tests on saturated kaolin. Samples were preconsolidated from a slurry which was initially mixed under vacuum at a moisture content of 160%, about twice the liquid limit. The shear tests were performed in a model of the simple shear apparatus (S.S.A) whose basic design and mode of operation has been described by Roscoe (1953). The apparatus was designed to test 6 cm x 6 cm x 2 cm samples of clay under conditions of plane strain, while ensuring that a uniform distribution of stress and strain throughout the sample is maintained. All tests were performed in a constant temperature room at 25°C ($\pm 1^\circ$C).

The vertical stress $(\tau_y)$ was applied through the S.S.A. piston by means of a hanger and dead weights. The horizontal shear stress $(\tau_x)$ was applied by means of a dead loading system of tanks, being transmitted to the top of the sample through the base of the piston. The shear stress was varied by pumping water from one tank to the other. This method of shear stress application minimizes
inertial loading effects which can occur with the sudden dead-loading of hangers.

Preliminary tests in the S.S.A. established the errors caused by friction in the shearing mechanism, and by vertical yield of the apparatus under load. Corrections were applied to the experimental data to account for these factors. The effect of side friction on the samples was considerably reduced by the presence of rubber (lubricated with silicone grease) between the sample and the enclosing walls of the apparatus. Volume change measurements were derived both from burette readings and from vertical dial gauge readings, thus minimising errors caused by water leakage through the membrane enclosing the sample. The angular strain during shear was determined from the ratio of horizontal displacement to sample height.

The stress and strain parameters used in the analysis of results are thus volumetric strain $\varepsilon$, angular strain $\psi$, effective vertical stress $\sigma_v$, and shear stress $\tau$.  

III. - THE STATE BOUNDARY SURFACE

(a) General

The experimental data will be discussed in the following section with reference to current concepts of the Cam-Clay model. As considerable use will be made of the properties of the state boundary surface, this surface will now be discussed in greater detail with reference to shear tests performed in the S.S.A.

The state boundary surface (shown diagrammatically in Figure 1) encloses all possible states of specimens tested in the simple shear apparatus, and is made up of two portions - the volumetric yield surface Y and the Hvorslev surface H (Roscoe et al. 1958). The volumetric yield surface is bounded by the virgin consolidation line in the plane ($\tau = 0$), and by the 'critical state' line $FF'$, which represents the limiting equilibrium states attainable by normally consolidated specimens. The Hvorslev surface contains all possible states of failure (represented by peak shear stress ($\tau_{\max}$)), and includes the 'critical state' line as one of its edges. In fact the critical state condition is not achieved in plane strain tests, due to the intervention of a Mohr-Coulomb rupture criterion (Henkel and Wade 1966). For this reason, the term 'n.c. rupture' line will be used in the remainder of this paper to denote the failure state of all normally consolidated (n.c.) samples. Results to be presented confirm the assumption that a planar Hvorslev surface (including the n.c. rupture line) results from S.S.A. tests using a variety of stress paths.

The full state boundary surface can conveniently be represented by a two-dimensional plot (Burland 1965) involving the parameters $\tau_{\max}$ and $\sigma_v$, $\sigma_v$ being the equivalent consolidation pressure defined by Hvorslev (1960). Such a plot is illustrated in Figure 2, the volumetric yield surface being represented by the portion $AX$, the Hvorslev surface by $BX$ with the n.c. rupture line represented by the point $X$. The region beneath the surface can be divided into two sub-regions (termed "wet" and "dry") by the line $XX'$ in Figure 2. Samples on the wet side of $XX'$ ultimately fall on the n.c. rupture line at $A$, and if drained show a volume decrease under increasing shear stress. Samples on the dry side of $XX'$ fail on attaining some point on the Hvorslev surface and increase in volume as the point $X$ is approached.

![Fig. 1. The State Boundary Surface](image1)

![Fig. 2. Two-Dimensional Representation of the State Boundary Surface](image2)
(b) Volumetric Strains

The majority of S.S.A. tests performed by the author were drained tests, and therefore some consideration should be given to the general pattern of volumetric strains predicted by the Cam-Clay model. Irrecoverable (plastic) volumetric strains occur only when the sample state moves across the volumetric yield surface due to a change in the applied stresses. The magnitude of such strains is related directly to the magnitude and direction in stress space of the applied stress increment.

If an applied stress increment is such that the sample state lies always beneath the state boundary surface, then only recoverable (elastic) volumetric strains occur. In such a case, the strains are confined to a vertical elastic wall whose projection on the $- \log p$ plane is parallel to an elastic swelling line. A typical elastic wall has been represented in Figure 1. Each of the many possible elastic walls is defined by the combination of $\sigma_y'$ and $\tau_y'$ at which yielding last occurred. During elastic deformation, the void ratio is thus solely a function of the mean effective pressure, and is independent of possible variations in the applied shear stress.

Specific illustrations of the above are provided by two types of test used in the experimental programme. The first is a "constant $\sigma_y'$" test, i.e. a drained shear test during which the vertical applied load is held constant. Such tests can be performed with various initial over-consolidation ratios as represented on Figure 2 by idealised paths such as MN and M'N'. The second test, a "constant $\tau_y'$" test involves three stages. Consolidation to a value of $\sigma_y'$ is carried out, followed by an increase of $\tau_y'$ from zero up to the selected value. Failure is then induced by a successive step-wise reduction in $\sigma_y'$, with drained conditions at all stages. An idealised stress path for this type of test is given by AST in Figure 2.

A variety of tests can be selected using either constant $\sigma_y'$ or constant $\tau_y'$ stress paths and these provide evidence against which the basic assumptions of the Cam-Clay model can be assessed.

(c) Shear Strains

The theoretical Cam-Clay model presented thus far associates shear strains only with irrecoverable volumetric strains; this is a logical extension of the application of Drucker's stability criterion. All shear strains (angular strains in the S.S.A. test) therefore occur as a result of movement of the sample state across the volumetric yield surface. Thus a stress increment which involves sample states lying wholly beneath the state boundary surface should cause no shear strains.

This idealised specification of shear strains in the Cam-Clay model is incomplete as (for example) it has been established that significant shear strains occur during triaxial tests on overconsolidated samples (Bishop and Henkel 1962). Attempts have been made recently (Wroth and Loudon 1967, Roscoe and Burland 1968) to include into the framework of the Cam-Clay theory additional shear strains which overcome the obvious objections to the simplified model. This has been done essentially by considering shear strains as being of two types - the first being a function of plastic volumetric strains as discussed above and a second involving shear strains which are independent of yielding on the state boundary surface.

The second category of shear strain is the subject of current interpretation. On a two-dimen-

sional representation of the state - boundary surface, Wroth and Loudon (1967) presented contours of equal shear strain for standard undrained triaxial compression tests on overconsolidated clay specimens, including one normally consolidated sample. They showed that the use of these contours enable an accurate prediction to be made of the total shear strains occurring in a drained over-

consolidated test.

The author has used this strain contour concept to present results from a number of tests involving different stress paths. While this approach is empirical in its nature, some more fundamental basis for its use is contained in the work by Roscoe and Burland (1968). For the present, the empirical approach does enable a significant assessment to be made of one area in which the overall theory is deficient.

(d) Time Effects

Time-dependent deformations of the idealized clay model have thus far not been considered. It has been assumed in the past (Roscoe, Schofield and Thurasirajah 1963) that such deformations are the result of a diffusion process, and that the effective soil structure is non-viscous. This assumption has been studied in some detail (Walker 1969), as the viscous properties of real clays prove to be of significance under some conditions. In essence it has been shown that the mean stress-strain curve obtained from an incremental stress-controlled shear test is independent of the increment duration time, provided that pore pressure equalization throughout the sample is achieved in each increment.

The stress increments in the author's drained S.S.A. tests were permitted to act for the 100% consolidation time defined during preliminary consolidation of the sample. This time varied between 20 and 90 minutes for different samples. For the few undrained tests reported herein, an increment time of 10 minutes was generally used.
In this way, pore pressure equalization was achieved during all increments, and secondary deformations were kept to a minimum. Only the 'equilibrium' data at the end of each increment are presented below.

IV. EXPERIMENTAL DATA - VOLUMETRIC STRAINS

In this section, the experimental data obtained from various S.S.A tests will be presented with the intention of studying the volumetric yielding of one particular clay, and its relation to the idealized behaviour discussed in the preceding section. Presentation of the angular strain data from some of these tests will be made in the following section.

(a) Compression Curve

The compression curve from a typical one-dimensional consolidation test performed in the S.S.A. is shown in Figure 3. The particular sample for this test was consolidated in increments to 5.5 kg/cm², unloaded to a vertical stress of 0.3 kg/cm², and reconsolidated to 5.5 kg/cm². The virgin compression curve is linear on the log. pressure plot, while the unloading curve is markedly nonlinear at low stresses. The swell-back and reconsolidation curves together define a significant hysteresis loop. The assumptions of the Cam Clay theory are thus approximations when large cyclic stress changes are involved. The important assumption of a straight virgin consolidation line is, however, justified by the data.

(b) Normally Consolidated Shear Tests

A series of standard drained and undrained S.S.A. tests on normally consolidated samples was carried out to establish the shape of the volumetric yield surface. Samples were consolidated to various vertical pressures ($\sigma_{yy}$) and tested by increasing the horizontal shear stress ($\tau_{xy}$) in steps until failure occurred.

Fig. 3. Compression Curve ($\tau_{xy} = 0$)

Two-dimensional plots of the yield surface are shown both for drained tests (Figure 4) and for undrained tests (Figure 5). The mean line from the drained tests has been transferred onto the undrained test results as a broken line. Taken together the Figures show that the volumetric yield surface is reasonably independent both of consolidation pressure and type of test. Normally consolidated drained and undrained tests in the S.S.A. thus define a unique yield surface, a conclusion which was reported by Roscoe and Thuralrajah (1966) for strain-controlled tests in the S.S.A.

Failure states in stress-controlled shear tests are difficult to define precisely; the failure stress ratio obtained from Figures 4 and 5 is approximately

$$\frac{\tau_{xy}}{\sigma_{yy}} = 0.55 \approx 0.52$$

Now in their application of the Mohr-Coulomb rupture criterion to plane strain tests, Roscoe and Burland (1968) predicted that, for S.S.A. tests on normally consolidated specimens, the failure stress ratio is given by

$$\frac{\tau_{xy}}{\sigma_{yy}} = \frac{4 \gamma \sigma_{yy} (\sigma_{yy} - \gamma)}{2 \times 1 + 2 \gamma \sigma_{yy} - \gamma^2}$$

Fig. 4. Drained Volumetric Yield Surface

Fig. 5. Undrained Volumetric Yield Surface
where \( \gamma_k = \frac{3M}{2G} \) and \( M \) is the critical state strength defined by triaxial compression tests. Using Roscoe and Burland's value of \( M = 0.9 \) for re-moulded kaolin gives a predicted S.S.A. stress ratio at failure of 0.37 i.e. in good agreement with observation, thus confirming the validity of the Mohr-Coulomb rupture criterion.

The end points of each of the tests shown in Figures 4 and 5 have been plotted in an e vs log \( \sigma_{yy} \) diagram in Figure 6. The results from two long-term (one day increment) undrained tests have been added to broaden the spread of the data. The initial points of all samples have been corrected in void ratio to lie on the virgin consolidation line of Figure 3, and the final void ratio corrected by a similar amount. A line can be drawn bounding the points in Figure 6, being approximately parallel to the normal consolidation line. This n.c. rupture line, which plots as a point on the state boundary surface (X in Figure 2) defines the states at which normally consolidated samples reach a peak shear stress.

![Fig. 6 Compression and Rupture Lines for n.c. tests](image)

(c) Constant \( \sigma_{yy} \) Tests

A series of drained S.S.A. tests on over-consolidated samples was carried out as a first step in the investigation of the region beneath the state boundary surface. Over-consolidation ratios of 1.3, 1.9, 3.5, 8.3 and 19.2 were used with a constant preconsolidation pressure of 5.06 kg/cm². Volumetric strains occurring in these tests have been interpreted in terms of a two-dimensional plot of the state boundary surface.

The experimental data are presented in Figure 7, the volumetric yield surface (AX) in this figure being the mean curve of the drained tests plotted in Figure 4. Beneath the state boundary surface, all over-consolidated state paths are closely vertical, thus confirming the idealised behaviour shown in Figure 2. The lightly overconsolidated specimen (test No. 6.6) undergoes significant volumetric strains once the yield surface is reached, and shows a state path close to the idealised path MNX shown in Figure 2; the yield surface is reached by a gradual curve rather than at the ideal discontinuity N. All initially dry samples undergo very little volume change before failing (cf. M'N' in Figure 2). Despite the difficulty of determining failure stresses accurately in the S.S.A. tests, a reasonable straight line (BX) can be drawn through all failure points. This line defines the Hvorslev surface, and completes the experimental definition of the state boundary surface AXB.

(c) Constant \( \tau_{xy} \) Shear Tests

A series of drained constant \( \tau_{xy} \) tests was carried out on normally consolidated samples. Four of the tests (Nos. 5.3-5.6) were initially consolidated under the same vertical stress (4.1 kg/cm²) but with values of \( \tau_{xy} \) varying from 0.11 kg/cm² to 0.69 kg/cm². Experimental data from the tests are presented in two figures. The state paths are plotted on the state boundary surface representation in Figure 8, and on the e vs log \( \sigma_{yy} \) diagram in Figure 9.

The state boundary surface drawn in Figure 8 has been taken from Figure 7. For each test represented in Figure 8, the first point (labelled by

![Fig. 7. State Boundary Surface - Constant \( \sigma_{yy} \) Tests](image)

![Fig. 8. State Boundary Surface - Constant \( \tau_{xy} \) Tests](image)
letters D to H) represents the equilibrium state under the constant value of $\tau_{xy}$. The final point represents the final equilibrium state before failure occurred. To a fair approximation, the same line BX governs the failure conditions for both constant $\sigma_{yy}$ and constant $\tau_{xy}$ tests. In addition state paths follow the general shape given by the idealised curve ST in Figure 2.

Fig. 9 Void Ratio Changes - Constant $\tau_{xy}$ Tests

The changes in void ratio accompanying the reduction of vertical stress in the constant $\tau_{xy}$ tests are shown in detail in Figure 9. The broken line (SS') in this figure is the rebound line taken from Figure 3. The line TT' has been drawn parallel to SS' and closely represents the average of tests No. 5.3, 5.4 and 5.5 which commence at approximately the same point on the volumetric yield surface. The results indicate that the state paths for the three tests have almost the same projection on the $\sigma_{yy}$ plot despite the significant differences in $\tau_{xy}$ values, i.e., three state paths lie approximately in the same vertical elastic wall, whose projection is parallel to that established for the consolidation test. This behaviour is in accord with the theoretical model.

Data from the remaining tests (5.2 and 5.6) produce lines which deviate in slope from SS' as $\sigma_{yy}$ approaches its failure value. These two tests were performed with higher values of $\tau_{xy}$, and it might therefore be concluded that, as failure on the Hvorslev surface is approached, volumetric strains exceed those to be expected from elastic behaviour, i.e., small plastic volumetric strains occur as the state boundary surface is approached.

V. EXPERIMENTAL DATA - ANGULAR STRAINS

Angular strain data have been interpreted using the strain contour concept introduced by Wroth and Loudon (1967). Their geometric contour pattern consists of a set of radial lines on the dry side of XX' (Figure 2) connecting to a set of horizontal lines for wet clays. It can be shown (Walker 1967) that the full contour pattern for both normally and over-consolidated samples can be usefully represented by

$$\left(\frac{\tau_{xy}}{\tau_{y}}\right) = F(\delta_{xy})$$

For a point represented by co-ordinates $\tau_{xy}$, $\frac{\sigma_{yy}}{\tau_{y}}$, the value of $\tau_{xy}$ is obtained by projecting the value of $\sigma_{yy}$ onto the Hvorslev surface.

Data from drained over-consolidated tests are presented in Figure 10. This figure shows a plot of $\left(\frac{\delta_{xy}}{\delta_{y}}\right)$ against angular strain, with over-consolidation ratios varying from 1 to 19. Of the six tests shown, only two provide stress paths which reach the volumetric yield surface before failure is achieved. Test No. 6.1 is a normally consolidated test, while Test No. 6.6, by reference to Figure 7, undergoes a limited amount of volumetric yielding before failure.

Fig. 10 Angular Strains - Constant $\sigma_{yy}$ Tests

Within experimental scatter, the six tests define a curve which supports a unique pattern of strain contours independent of overconsolidation ratio. In general, the lightly overconsolidated test results lie above the mean line, while the heavily overconsolidated tests give points below the mean line. The exception to this pattern is the normally consolidated test result which lies below the mean curve. This variation is, however, of secondary significance despite the fact that volumetric yielding occurs throughout the n.c. test. It must therefore be concluded that angular strains in all constant $\sigma_{yy}$ tests in the S.S.A. are predominantly of the second type discussed in section III above, i.e., strains not associated with volumetric yielding.

An analysis of angular strain data from undrain and constant $\tau_{xy}$ tests has also been made, using a similar approach to that outlined above. These additional results are closely represented by the mean curve of Figure 10 and thus confirm that a unique strain contour pattern exists for S.S.A. tests regardless of the stress path adopted. In no S.S.A. test have angular strains associated with volumetric yielding been shown to be a significant proportion of the total.
This latter conclusion is in marked contrast to the deformation characteristics of the same clay in the triaxial compression test. Published data show conclusively that shear strains in drained triaxial shear tests on normally consolidated clays are largely associated with volumetric yielding of the clay. As a consequence, shear strain prediction using the Cam-Clay theory is quite accurate (Burland 1965). Roscoe and Burland (1968) have discussed the stress probes possible in plane strain shear tests, and the S.S.A. test in particular, and have justified the small shear strains observed in such tests.

It is apparent from the test results presented above that the mathematical Cam-Clay theory as presently constituted is deficient to the extent that shear strains which are not associated with volumetric yielding can dominate under certain stress conditions. Such strains cannot as yet be quantitatively assessed within the framework of the theory.

VI. - CONCLUSIONS

Experimental data from simple shear tests with a variety of stress paths have been analysed in terms of current concepts associated with Cam-Clay model. The stress paths involved both an increase and decrease in vertical stress, although at no stage was a reduction in horizontal shear stress applied, hence conclusions must be restricted only to those paths which have been investigated.

The data confirm the existence in $\sigma_{xy}, \sigma_{yy}$, a space of a unique state boundary surface which, together with the co-ordinate axes, enclose all possible states of the clay studied. The surface is composed of two parts - the volumetric yield surface and the Hvorslev surface - which are joined by a line which contains all possible failure states of normally consolidated samples. The yield surface defines the surface on which irrecoverable volumetric strains may occur, while the Hvorslev surface contains all possible failure states of the clay. Within the limitations of the experimental programme, the state boundary surface as a whole was found to be independent of the stress path chosen.

Void ratio changes beneath the surface are essentially recoverable and are confined to a series of vertical elastic walls, whose projections on the $\epsilon_{3} = \log \sigma_{yy}$ plane are parallel to the swelling curve obtained from one dimensional consolidation tests. Angular strains associated with stress paths beneath the state boundary surface define a family of strain contours which appear to be independent of the chosen stress path. The test data indicate that angular strains associated with volumetric yielding are relatively small, and consequently the determination of a strain contour pattern is of considerable importance in the interpretation of plane strain test results.

VII. - ACKNOWLEDGEMENTS

The experimental work reported above was performed in the Engineering Laboratories at Cambridge University under the direction of the late Prof. K. H. Roscoe.

REFERENCES


