The Collapse of Sands Upon Saturation

By

(Senior Lecturer in Civil Engineering, University of Melbourne)

AND

D. V. MILLAR, B.E.
(Graduate Student, Department of Civil Engineering, University of Melbourne)

SUMMARY. - The following possible mechanisms of collapse of sandy soils following saturation have been examined experimentally and theoretically:
(a) dissipation of initial capillary stresses,
(b) reduction in normal force at particle contact by increase in pore pressure,
(c) contact instability due to decrease in shearing resistance on saturation.
The magnitudes of the initial capillary stresses were estimated to be quite low and the influence of their dissipation upon collapse behaviour did not appear to be as marked as the decrease in shearing resistance following saturation. This decrease in resistance was quite significant particularly at void ratios which were large in comparison with the critical void ratio. It was at these high void ratios that the largest collapses were observed. The increase in pore air pressure during saturation was found to be negligible.

I. - INTRODUCTION

The principle of effective stress which states that:
(a) changes in volume and shearing resistance are due exclusively to changes in effective stress,
(b) effective stress is the difference between the total stress and pore water pressure

\[ \sigma' = \sigma - u_w \]  

has been of fundamental importance in explaining the behaviour of saturated soils. For partially saturated soils Bishop (Ref. 1) proposed a modified form for the effective stress equation

\[ \sigma' = \sigma - u_a + \chi (u_a - u_w) \]  

where the parameter \( \chi \) varies between 0 and 1, and it was assumed that statement (a) remained valid.

In studies on the phenomena of volume decrease or collapse of partially saturated soils upon saturation some doubts have been raised about the validity of applying the principle of effective stress. Workers at M.I.T. (Ref. 2) showed that the collapse behaviour of partially saturated soils implied negative values of the \( \chi \) parameter and they concluded that Bishop's form of the effective stress equation was inadequate in explaining collapse. Jennings and Burland (Ref. 3) concluded from their studies on collapsing soils that statement (a) could not be validly applied to partially saturated soils because of the essential difference between the internal and external stresses involved. Both of these conclusions, however, are based upon the assumption that collapse is a result of the dissipation of initial capillary stresses. In this paper some other possible collapse mechanisms are considered through an examination of the causes of instability at particle contacts.

II. - NOTATION

\( \sigma \)  total stress
\( \sigma' \)  effective stress
\( u_w \)  pore water pressure
\( u_a \)  pore air pressure
\( \chi \)  parameter in effective stress equation
\( N \)  normal force at particle contact
\( T \)  shear force at particle contact
\( \phi_a \)  angle of friction at particle contact
\( \phi_m \)  mobilized angle of friction
\( \beta \)  inclination of particle contact to vertical
\( H \)  horizontal force at particle contact
\( V \)  vertical force at particle contact
\( T_s \)  surface tension for water
\( w \)  water content

III. - CAUSES OF CONTACT INSTABILITY

From considerations of contact equilibrium (Fig. 1) three possible causes of instability could be considered:
(a) An increase in the shear force \( T \) without a corresponding increase in the normal force \( N \). As it is difficult to visualise the process of saturation leading to an increase in \( T \) this possibility was not considered further.
(b) A decrease in \( N \) without a corresponding decrease in \( T \). The force \( N \) could be decreased either by the dissipation of any initial capillary stress or by an increase in the pore pressure.
(c) A decrease in the angle of friction \( \phi_a \) following soaking.
Fig. 1 Forces between Two Particles

V.- FORCES ACTING AT CONTACTS BETWEEN PARTICLES

The forces acting between two particles is presented in Fig. 1, in which $\phi_m$ is the mobilized angle of friction. By means of resolving forces it may be shown that

$$\frac{V}{H} = \tan (\beta + \phi_m)$$

which reaches a maximum value for any contact when the friction has been fully mobilized, that is, when $\alpha$ becomes $\phi_u$. The relationship between $\phi_u$ and $\beta$ for various values of $\phi_m$ when the friction has been fully mobilized is shown in Fig. 2. For contacts which are not on the point of instability the points would plot to the right or below these lines. In other words these lines represent envelopes, points to the right of which indicate stable contacts and points to the left indicate unstable contacts.

If contact instability is going to result from soaking the assemblage of particles in water then sliding will commence first at those contacts for which the friction is fully mobilized before soaking. That is the contacts of immediate interest will be those which plot on the lines in Fig. 2.

In order to examine the effect of soaking upon the stability of the assemblage the behaviour of one contact will be studied in detail. It will be assumed for purposes of discussion that $\phi_u$ is 20° (before soaking). Assume that point A in Fig. 2 represents a contact at the point of sliding before soaking. If the angle of friction $\phi_u$ is increased by, say 10° as a result of soaking then the limiting envelope will move from the 20° line to the 30° line in Fig. 2. This means that as contact A the particles will no longer be on the point of sliding. The assemblage would become more stable and no movement should result from soaking. On the other hand should soaking reduce the angle of friction $\phi_u$ by 10° the envelope will be moved from the 20° line to the 10° line. This means that contact A will be outside the envelope, the contact will be unstable and sliding will result. Some of the possible ways in which this sliding would occur are represented by the dotted lines in Fig. 2. This movement will cause a redistribution of forces and rearrangement of surrounding particles. Similarly all of the contacts which originally plotted between the 10° and 20° lines would become unstable and particle rearrangement would continue until all contacts plotted below the new 10° envelope.

On the basis of the foregoing model it is seen that the prerequisite for settlement or "collapse" of an assemblage of particles through soaking is that the angle of friction $\phi_u$ is decreased by the process of soaking. More generally, any effect of soaking that would induce contact instability could lead to sliding, particle rearrangement and a consequent compression of the sample.

Fig. 2 Maximum Force Ratio at Particle Contacts

V.-COLLAPSE SIMULATION IN OEDOMETER

In order to identify the causes of particle contact instability several series of laboratory tests were carried out. The first series consisted of the preparation of a variety of samples of sand-sized materials in an oedometer followed by
saturation of each sample while subjected to a stress.

The oedometer which was designed and built for these tests is shown in Fig. 3. The sand sample is placed in the bottom half of the device in a cylindrical chamber to which a water inlet is connected. The two halves of the device are firmly bolted together, the sand sample being separated from the pressure chamber in the upper half of the device by a rubber membrane which was sealed circumferentially with two O-rings. The vertical displacement of the sample was measured by means of a Schaeveitz linear variable differential transformer (LVDT) which had a working range of ±0.030 in. The excitation for this displacement transducer was a voltage amplitude of 20 volts at a frequency of 20 kilo-hertz, which was provided by a model 200 CD Hewlett Packard oscillator, the output being displayed on the screen of a Tektronix 502A oscilloscope. The pressure was controlled by means of a pressure regulator the pressure being read on a Bourdon pressure gauge. This oedometer is similar in many respects to the one described by Moore (Ref. 4) but it possesses a major advantage over the earlier model, namely the LVDT can be reset to the null position while the sample is under

Fig. 3 Sketch of Oedometer

![Sketch of Oedometer](image_link)

Fig. 4 Grain Size Distributions

![Grain Size Distributions](image_link)
Tests in the oedometer were carried out on four samples - two quartz sands and coarse and fine samples of spherical glass beads. The grain size distributions of these four samples are shown in Fig. 4. The samples were prepared at 1% water content and in the oven dry state at various void ratios. The samples were spooled into the oedometer and Levelled with a spatula. The pressure was increased to 10 lb/in² at a rate of approx. 1 lb./in²/min. The sample was then inundated with water and any vertical compression was observed. The results of this series of tests are indicated in Fig. 5.

As expected the amount of collapse increases as the void ratio prior to saturation increases. For many of the dry samples little collapse was observed following saturation. Collapse was observed, however, for some of the dry Earlston sand samples. As the results for the dry samples lie within the general scatter of results for the moist samples, this suggests that large collapse is not observed with the dry samples simply because of the difficulty of preparing these samples at higher void ratios. Larger collapses are observed with the moist samples because of the relative ease of preparing these samples in a looser condition. Since no initial capillary stresses were present with the dry samples, these foregoing observations suggest that particle contact instability due to dissipation of capillary stresses could not be the sole cause of collapse. Capillary effects may be a contributing factor in the collapse of initially moist samples but an examination of the capillary stresses in regular packings of spherical particles suggests that these stresses are quite small in magnitude.

Jennings and Knight (Ref. 6) have observed increasing amounts of collapse with increasing vertical overburden stress. Confirmation of this observation was not obtained with the present series of tests since the majority of the tests was carried out for one value only of the vertical stress.

VI.- CAPILLARY STRESSES IN REGULAR PACKINGS

Consider two equal spheres in contact at point A (Fig. 6) and connected by a film of water CADEABC. Since the mean curvature of the air-water interface is constant (Ref. 5) the pressure difference \( u_a - u_w \) across this interface may be found from the known mean curvature at point F.

\[
u_a - u_w = T_s \left( \frac{1}{c} - \frac{1}{b} \right)
\]

(4)

<table>
<thead>
<tr>
<th>EARLSTON SAND:</th>
<th>NO. 650 GLASS BEADS:</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRY</td>
<td>DRY</td>
</tr>
<tr>
<td>1% WATER</td>
<td></td>
</tr>
<tr>
<td>( \sigma = 10 \text{ lb./in.}^2 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WHITE SAND:</th>
<th>NO. 070 GLASS BEADS:</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRY</td>
<td>DRY</td>
</tr>
<tr>
<td>1% WATER</td>
<td></td>
</tr>
<tr>
<td>( \sigma = 10 \text{ lb./in.}^2 )</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5 Collapse of Sands Following Saturation
where \( T_s = 0.005 \text{ lb/ft. at 670°F} \) is the surface tension of water.

The radii \( b \) and \( c \) may be related to the sphere radius \( r \) and subtended angle \( \theta \) to yield

\[
\frac{u_a - u_w}{r} = \frac{T \cos \theta (\sin \theta + 2 \cos \theta - 2)}{r(1 - \cos \theta)(\sin \theta + \cos \theta - 1)}
\]

(5)

By the use of equation (5) the equivalent effective vertical stress \( \sigma' \) produced by these capillary effects in various packings of uniform spheres may be calculated. For the simple cubic packing the effective stress assuming that the \( u \) is identical with the ambient air pressure may be shown to be

\[
\sigma' = \pi T \tan \theta (\sin \theta + \cos \theta - 1)
\]

(6)

The water content \( w \) may also be determined as a function of \( \theta \) for simple cubic packing assuming the specific gravity of solids as 2.65.

\[
w(\%) = 113.2 \left[ \frac{\tan \theta - 1 - \cos \theta}{\cos \theta} \right]^{\frac{1}{4}}
\]

(7)

The relationship between the effective stress and water content may be expressed as in Fig. 7. This relationship may be expected to change as the water content exceeds 6.25\% (\( \theta=45^o \)) since the water films begin to merge at this point. At a water content of 19.35\% air will be present only in the form of separated bubbles. If it is assumed that the pore air pressure is still equal to the atmospheric pressure the effective stress is as indicated in Fig. 7. For the water content to increase further to the saturation value of 34.3\% the pore air pressure must be increased to permit the solution or escape of the bubbles.

Similarly the relationships between effective stress and water content may be determined for other regular packings of uniform spheres and these are also shown in Fig. 7. For these packings which cover a range of void ratios from 0.91 to 0.35 the effective stress is seen to decrease as the void ratio increases. This conclusion should also be applicable to irregular packings. For the Erlston and white sands that were used in the tests previously described the equivalent effective stress for the moist samples (12\% water content) would be of the order of 0.2 to 0.3 \( \text{lb/in}^2 \). It appears that capillary effects are of importance only at relatively shallow depths in the field. For the oedometer tests previously described the dissipation of the capillary stress following soaking would reduce the effective stress and hence the normal force at particle contacts by roughly 32\% or less. With this relatively small effect only those contacts near the point of sliding would be rendered unstable by soaking.

VII.- MEASUREMENT OF PORE AIR PRESSURE DURING COLLAPSE

As previously discussed contact instability upon soaking could be induced by an increase in pore air pressure which would reduce the normal force between contacts. In order to check upon the likelihood of this possibility measurements of pore air pressure during the soaking of samples were carried out. The measurements were made by means of a miniature pressure transducer which was placed within the sand sample inside the oedometer (Fig. 3). The transducer was a Micro-Systems model 1017.

---

Fig. 7 Relation between Effective Stress and Water Content for Uniform Packing
Fig. 8 Typical Collapse Patterns

Implantable blood pressure transducer. The transducer, which consists of four semiconductor train gauges bonded to the interior surface of a 0.3 mm diameter pressure sensing diaphragm, has a pressure range 0 to 8 lb/in². The diaphragm was inclosed in a small permeable case so that pore air pressures only would be measured.

Two typical results for tests on the white sand are shown in Fig. 8. In some tests the collapse occurred suddenly, in others it occurred gradually. The cause of the difference in the rates of collapse was not satisfactorily identified although it appeared to be related to the occurrence or non-occurrence of a sudden compression of the sand during loading. In all tests the change in pore air pressure during saturation was either negligible or barely detectable.

III.- SHEARING RESISTANCE BEFORE AND AFTER SATURATION

In order to obtain a measure of the influence of saturation upon the angle of contact friction $\phi$, a series of drained direct shear tests was carried out on the white sand and Earlston sands. Horn and Deere (Ref. 7) have shown that the frictional resistance that can be developed between smooth surfaces of quartz increases as the surface moisture increases. However, this antilubricating action of water was found to diminish rapidly as the surface roughness increased. The particle surfaces of the two sands which were examined under low magnification could not be described as smooth. The conclusions of Horn and Deere were therefore considered to be inapplicable.

A typical set of results is shown in Fig. 9. The shear strength of the moist sand exceeded that of the saturated sand over the entire range of void ratios tested, the strength difference increasing as the void ratio increased. This finding confirms that, in terms of total stresses, the shearing resistance of the moist sand is decreased by saturation. The volume change behaviour of the moist sand during shearing which is shown in Fig. 10 possesses marked similarities with the volume change behaviour following saturation as shown in Fig. 5. The processes of saturation and shearing of the moist sand produce similar decreases in volume. The collapse or volume decrease following saturation becomes very small as the void ratio approaches the critical void ratio, which is the void ratio at which the sample exhibits no volume change following shearing.

---

**Fig. 9 Shear Strength of Earlston Sand**

- White Sand
- Saturation
- Oven Dry

- Normal Stress 10 lb/in²

- Pre-Shear Void Ratio

- $\phi$ 26°

- $\phi'$ 26°

- Direct shear

- Earlston Sand

---
IX. CONCLUSIONS

From the results of tests on two sands the major reason for collapse or volume decrease upon saturation of moist sands was found to be contact instability related to a shear strength decrease following saturation. Rise in pore air pressure changes do not appear to be a factor influencing collapse since the increase in pore air pressure following saturation was found to be negligible. Reductions in the normal force due to dissipation of capillary stresses may be a contributing factor in collapse at shallow depths but it appears to be of less importance than the changes in shear strength.

REFERENCES


