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Critical length of stone columns

Longueur critique des colonnes ballastées

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ABSTRACT: The concept of critical length for stone columns is equivalent to that in piles. Thus, for columns longer than the critical length, the improvement achieved with the stone columns does not notably change. Sometimes in the literature, critical column length is expressed as the ratio between column length and column diameter. The authors have previously shown that this slenderness parameter of each individual column has a minor influence, and critical column length is better expressed using the footing width. In this paper, the critical column length is analyzed in detail through simplified two-dimensional axisymmetric finite element analyses. A uniform soft soil layer is considered for the sake of simplicity. For the studied cases, the values of the critical column length are around 1.2-2.2 times the footing width. The value of the critical length is related to the extent of plastic deformation and that may be used to decide the column length in the design phase without the need for a parametric analysis of the column length. As a first approximation, a general value of the critical column length of 2 times the footing width may be considered for ordinary stone columns.

RÉSUMÉ: Le concept de longueur critique pour les colonnes ballastées est équivalent à celui des pieux. Ainsi, pour les colonnes plus longues que la longueur critique, l'amélioration obtenue avec les colonnes ne change pas sensiblement. Parfois, dans la littérature, la longueur critique de la colonne est exprimée par le rapport entre la longueur de la colonne et son diamètre. Les auteurs ont précédemment montré que ce paramètre d'élancement de chaque colonne a une influence légère et que la longueur critique de la colonne est mieux exprimée par la largeur de la semelle. Dans cet article, la longueur critique de la colonne est analysée en détail par d'analyses simplifiées à deux dimensions en éléments finis. Pour des raisons de simplicité, une couche uniforme de sol est envisagée. Pour les cas étudiés, les valeurs de la longueur critique de la colonne sont d’environ 1,2-2,2 fois la largeur de la fondation. La valeur de la longueur critique est liée au étendue de déformation plastique et il peut être utilisée pour déterminer la longueur de la colonne lors de la phase de conception sans nécessiter d'analyses paramétriques de la longueur des colonnes. Comme première approximation, une valeur générale de la longueur de colonne critique de 2 fois la largeur de la semelle peut être considérée pour les colonnes ballastées.

Keywords: stone columns; column length; critical length; finite element analyses; settlement reduction

1 INTRODUCTION

Ground improvement using stone columns, also known as granular piles or aggregate piers, is a popular technique to improve soft soils for foundation of embankments or structures (e.g. Barkdale and Bachus 1983; Kirsch and Kirsch 2010; Han 2015). Stone columns may reach a rigid substratum (end-bearing columns) or may be embeded just on a soft soil layer (floating columns). For the latter case, the length of the columns is an important design parameter to be chosen. In some cases, it may be more cost-effective to add additional columns than increase the length of the columns. In this way, the concept of the critical length of stone columns appear. For columns longer than the critical length, the improvement achieved with stone columns does not notably change or increase. Although the load transfer mechanisms are different from piles, the concept of critical length is equivalent.

Table 1. Values of the critical column length

Reference	Type	L_{cr}
Black et al. (2011)	Lab.	8-10 d_c
Hughes and Withers (1974)	Lab.	$4d_c = 4D$
McKelvey (2002)	Lab.	$6 \cdot d_c$
Muir Wood (2000)	Lab.	1.5 B
Najjar et al. (2010)	Lab.	6 d_c
Wehr (2004)	Num.	1.7 B
Zhou et al. (2017)	Num.	2 B

Lab.: Laboratory tests; Num.: Numerical analyses

A review of different proposals for the critical column length (L_{cr}) may be found in Table 1 and, for example, in Babu et al. (2013). The values show some range of variation and in many cases are given as a function of the column diameter (d_c), but in others as a function of the footing width or diameter (B, D). Castro (2014, 2017a) showed that the column length to diameter ratio, L/d_c (slenderness of the column), has a minor influence (second order effect) on the ground improvement achieved with stone columns and the ratio L/B plays an important role. Thus, L_{cr} should be better expressed in terms of the footing dimensions (B, D). In McKelvey (2002), d_c=25 mm, D=100 mm, L_{cr}=150 mm, and then, L_{cr}=1.5 D. In Najjar et al. (2010), d_c=20 mm, D=71 mm, L_{cr}=120 mm, and then, L_{cr}=1.7 D. The accuracy of the critical length obtained from laboratory testing is limited because few column lengths are normally considered. The source of confusion could be in the original proposal by Hughes and Withers (1974), where the footing diameter coincides with the column diameter.

Muir Wood (2000) pointed out the importance of the deformation mechanism beneath the footing. Castro (2017b) conceptually showed that the critical column length for settlement reduction is related to the extension of the pressure bulb beneath the footing (Figure 1) and the critical column length for bearing capacity depends on the failure mechanism (Figure 2). Since the critical length is larger for settlement reduction, this is the one that it is normally used.

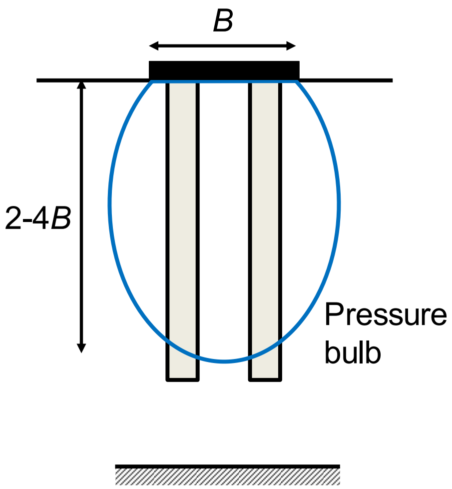

Figure 1. Justification of critical column length in a homogeneous soil layer for settlement reduction

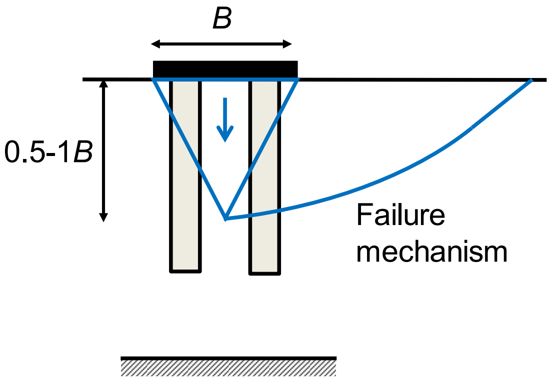

Figure 2. Justification of critical column length in a homogeneous soil layer for bearing capacity

The critical column length depends on the loading width and then, for embankments and large loaded areas, there is no critical length in practise because the soft soil layer thickness is smaller than the critical length.

This paper presents detailed numerical calculations to evaluate the value of the critical column length for settlement reduction. The numerical model (Section 2) and the results of the numerical simulations (Section 3) are presented. The analysis of the numerical results (Section 4) shows that the critical column length may be related to the depth of plastic deformation. Finally, some conclusions are provided.

2 NUMERICAL MODEL

The finite element code Plaxis 2D 2017 (Brinkgreve et al. 2017) was used to represent a simplified 2D axisymmetric model of only one centered column beneath a rigid circular footing. This 2D simplified model reproduces similar values of the footing settlement than those of a full 3D model of a group of columns with the same area replacement ratio (Castro 2014). The area replacement ratio (a_r) is the ratio between the area of the columns and the loaded area. The study started with a simple reference case and parametric studies were performed to analyze the influence of several parameters on the critical column length. All the numerical simulations were performed using a small strain formulation.

Perfect bonding between soil and column at their interface was modelled, as it is common practice (e.g. Castro 2014) because stone columns are tightly interlocked with the surrounding soil. The rigid footing was assumed as perfectly rough and modelled as a very stiff plate that produces uniform settlements.

For the reference case, the footing diameter, D, is 2.5 m and the column diameter (d_c=1.37 m) was chosen to give an area replacement ratio of a_r=30%. The soil profile was simplified to only one homogeneous soil layer, with a thickness of H=10 m (Figure 3). Both the soil and column were modelled as linear elastic-perfectly plastic materials using the Mohr-Coulomb yield criterion and a non-associated flow rule (Mohr-Coulomb model). Common values were chosen for the soil and column parameters (Table 2).

The footing and the column were "wished-in-place", ignoring the changes in the natural soil due to column construction (e.g. Castro and Karstunen 2010). Drained conditions were assumed for all the process, i.e. no excess pore pressures were generated. Geostatic initial stresses were generated using the effective soil unit weight (γ') and the coefficient of lateral earth pressure at rest, K_0=0.6. For simplicity, the ground water level was assumed to be at the ground surface and an effective unit weight of γ'=10 kN/m^3 for soil and column was directly considered without modelling pore water pressures. The applied pressure p_{app} is 100 kPa.

The column length was varied from L=10 m (end-bearing column, L/H=1) to L=0 m (no column) in steps of 0.5 m. The critical column length was identified from the variation of the settlement reduction factor with the column length. The settlement reduction factor (β) is the ratio between the settlement with columns and the settlement without columns.

Parametric analyses were performed using the reference case and varying the value of the parameters as shown in Table 3. The Young's modulus of the column (E_c) was also varied between 20 and 80 MPa, but very minor differences were found.

From the calculations, some mesh sensitivity (of around 3%) was observed between different cases, but the same mesh was used to identify each critical column length, i.e. for each case with different column lengths (Figure 3).

Table 2. Soil and column properties for reference case

Material	E (MPa)	ν (-)	c (kPa)	ϕ (º)	ψ (º)
Soil	2	0.33	5	25	0
Column	30	0.33	0.1	45	10

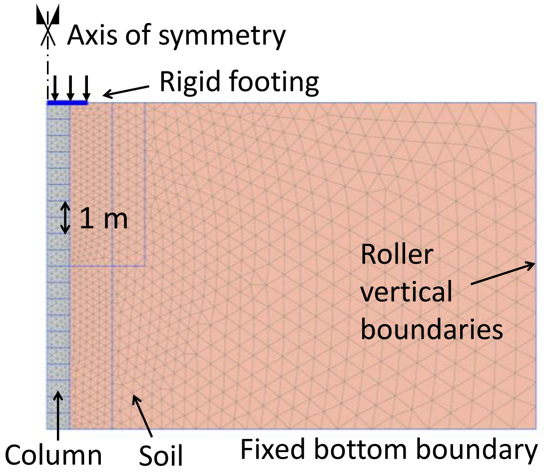

Figure 3. Finite element model. Reference case.

Table 3. Summary of parametric analyses

Parameter	Values
$\boldsymbol{a_r}$	10, **30**, 50, 70, 100%
$\boldsymbol{c_s, \phi_s}$	(3, 23) **(5, 25)** (7, 27) (9, 29) (11, 31)
$\boldsymbol{p_{app}}$	20, **100**, 200 kPa
$\boldsymbol{D}$	2, **2.5**, 5 m

3 NUMERICAL RESULTS

3.1 Area replacement ratio

Figure 4 shows the variation of the settlement reduction factor with the column length for different a_r, i.e. different d_c (D=2.5 m). The settlement for the unimproved soil is 12.3 cm. Obviously, the settlement is greatly reduced for longer columns and larger area replacement ratios. However, for columns longer than 1.6 D (i.e. L=4 m), the settlement reduction does not notably change. Therefore, that is the critical column length for these cases. In this simulations, no differences in the value of the critical column length with the area replacement ratio are noticeable. For higher area replacement ratios, the pressure bulb should be deeper because of the higher average stiffness (e.g. Muir Wood 2000), but in this case, that is compensated by less deep plastic strains (see further details in Section 4).

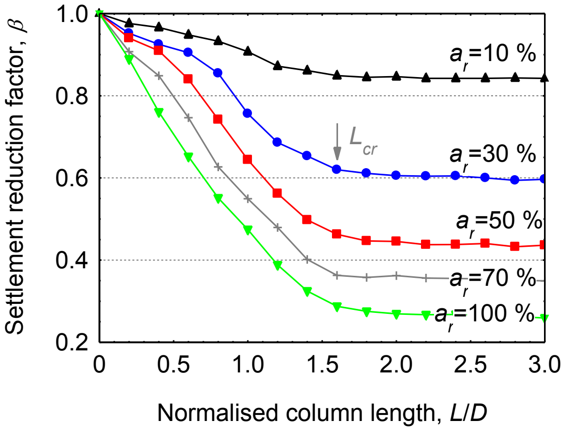

Figure 4. Critical column length for different area replacement ratios

3.2 Soft soil strength

The settlement of the footing is obviously larger in a soil with a lower strength (Figure 5). For a better comparison, the settlement reduction factor is depicted in Figure 6, which shows its variation with the column length for different soft soil strengths. The improvement achieved with the stone column is usually larger in softer soils. Besides, the critical column length notably varies with the soft soil strength. Its value approximately varies from L_{cr}=1.2 D for (c_s=11 kPa, ϕ_s=31º) up to L_{cr}=2.2 D for (c_s=3 kPa, ϕ_s=23º).

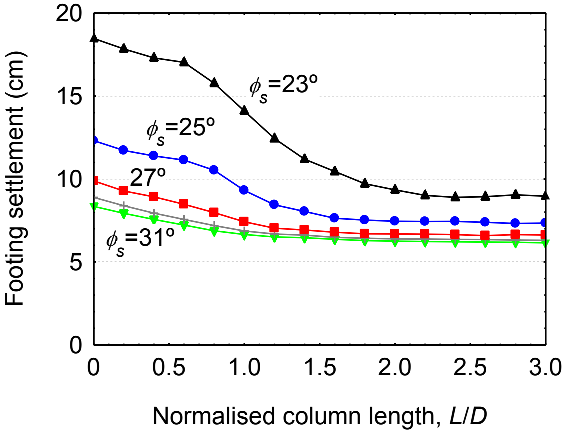

Figure 5. Settlement for different soft soil strengths

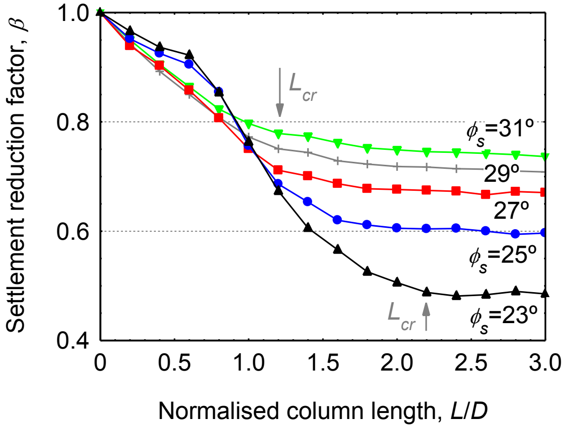

Figure 6. Critical column length for different soft soil strengths

The variation of the soft soil strength notably affects the relevance and extension of plastic deformation. As it will be shown in Section 4, the critical column length is related to the extension of plastic deformation, and therefore, L_{cr} is larger in softer soils.

3.3 Applied pressure

Figure 7 shows the variation of the settlement reduction factor with the column length for different applied pressures on the footing. Interpretation of the results is complex because there are not any clear trends.

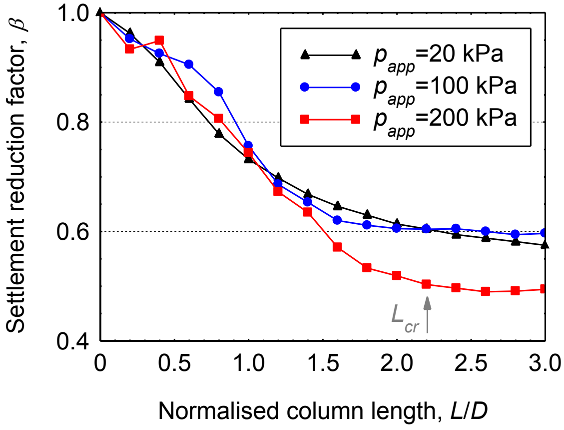

Figure 7. Critical column length for different applied pressures

For larger applied pressures (i.e. p_{app}=200 kPa), the critical column length is larger (around 2.2 D) because the pressure bulb and plastic strains reach deeper zones, analogous to what happens for lower soft soil strengths (Figure 5, ϕ_s=23º).

For low applied pressures (p_{app}=20 kPa), plastic strains concentrate in the shallow part and elastic strains are proportionally more relevant and it is difficult to define an exact value of the critical column length.

3.4 Diameter of the footing

Theoretically, the critical length is proportional to the diameter of the footing and that is why it is used to normalize it. In this case, that is observed for D=2 m and 2.5 m (Figure 8). However, when the diameter of the footing is large (e.g. D=5 m), the position of the rigid bedrock (H=10 m) influences the critical column length (Figure 8). Thus, the critical column length for D=5 m in terms of the footing diameter is smaller (1.4 D).

Figure 9 shows the results for D=5 m and several area replacement ratios. The results show that for a_r=50-100% there is a critical column length, but reaching the rigid bedrock (L=2D) provides a further reduction of the settlement.

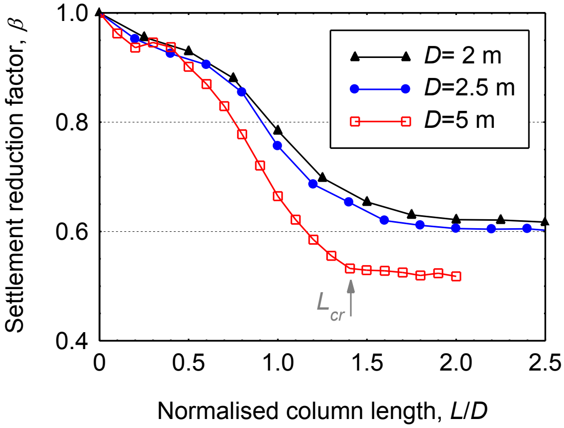

Figure 8. Critical column length for different footing diameters

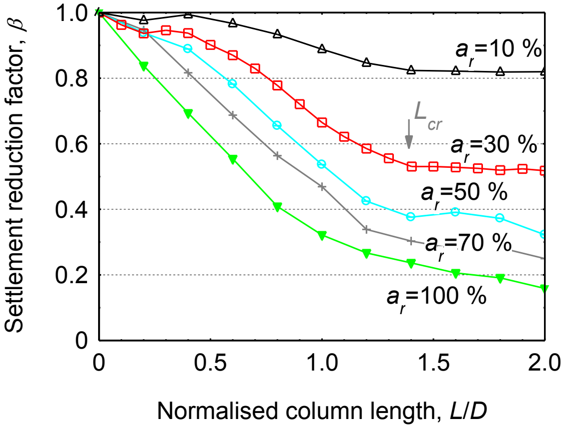

Figure 9. Critical column length for different area replacement ratios and D=5 m

4 IMPROVEMENT MECHANISMS

Although the conceptual model of the pressure bulb under the footing (Figure 1) is useful to justify the dependence of the critical column length on the footing size, it is strictly valid only for an elastic and uniform soil. Stone columns cause stress concentrations and there are usually plastic strains beneath the footing, mainly in the column and in the soil surrounding the upper part of the columns.

An analysis of the plastic points for the reference case (Figure 10) shows 4 different zones: (1) a small area beneath the footing (z<0.3 *D*) where plastic strains are restrained by the footing roughness; (2) a region (z<1.3 *D*) where plastic strains appear in the surrounding soil due to the lack of lateral confinement in the column; (3) a region where plastic strains appear only in the column (z<1.8 *D*); and (4) a region where there are only elastic strains (z>1.8 *D*).

These four different zones are reflected in the variation of the settlement reduction with the column length (Figure 11): (1) the settlement is slightly reduced because the column affect mainly the rigid soil wedge beneath the footing; (2) an important reduction of the settlement is visible because plastic strains are reduced both in the soil and column; (3) the settlement is reduced by a smaller amount because the column reduces plastic strains mainly near the column tip; and (4) the settlement is marginally reduced because the column reduces only elastic strains.

A simple linear elastic perfectly plastic model (Mohr-Coulomb) was used for the soil and the column, and Figure 10 shows plastic point history, i.e. all the integration (stress) points that have ever been in a plastic state. Similar results are expected when using more advanced models, but then, failure or critical state points should be analyzed.

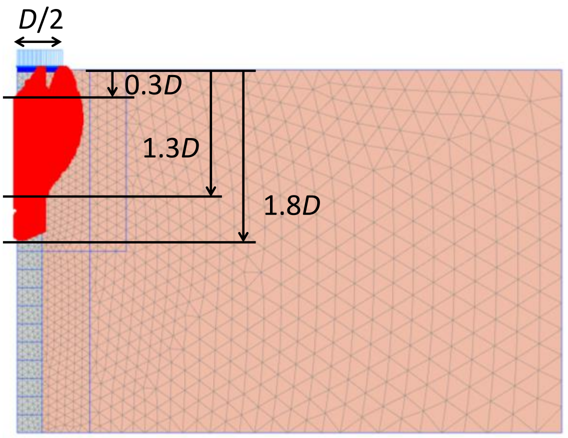

Figure 10. Plastic points for reference case (a_r=30 %; H=10 m; D=2.5 m; L=10 m)

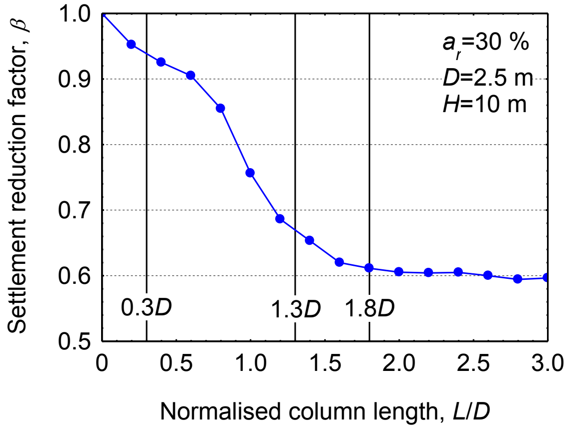

Figure 11. Variation of the settlement reduction with column length for the different plastic zones case (a_r=30 %; H=10 m; D=2.5 m)

Similar results are found for other cases, for example, for twice the footing diameter (D=5 m) (Figures 12 and 13). In this case, the extension of plastic strains (Zones 2 and 3) is reduced due to the proximity of the rigid bedrock (Figure 12), but the correlation between plastic strains zones and variation of the settlement reduction with the column length is kept (Figure 13).

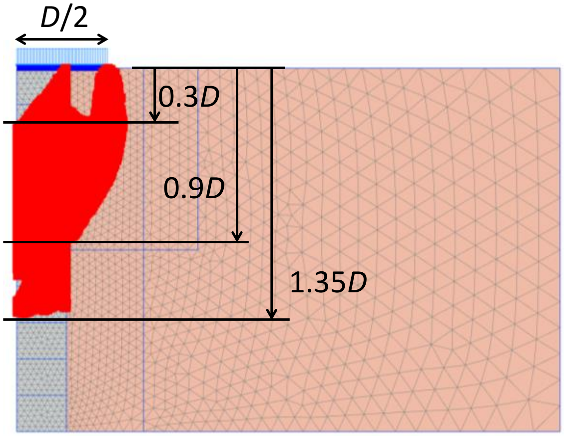

Figure 12. Plastic points for (a_r=30 %; H=10 m; D=5m; L=10 m).

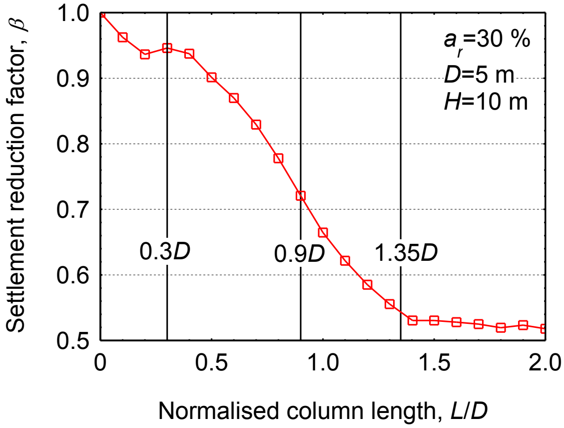

Figure 13. Variation of the settlement reduction with column length for the different plastic zones (a_r=30%; H=10 m; D=5 m)

There are some cases where plastic strains are less important and elastic strains play an important role (e.g. p_{app}=20 kPa in Figure 7; ϕ_s=31° in Figure 6; a_r=100% in Figure 9). In these cases, the settlement for zone (4) is proportionally more important and the slope of the final part of the settlement reduction vs. column length does not look horizontal. However, the absolute value of those elastic strains is not usually important, see for example Figure 5 for ϕ_s=31° where the absolute value of the settlement is plotted.

The advantage of considering the plastic zones is that their extension is approximately constant for columns longer than the critical column length. Thus, it is not necessary to perform a parametric analysis varying the length of the columns and it is enough to study the case with end-bearing columns or columns longer than L_{cr} to identify the extension of the plastic zones.

As a first approximation, a conservative general value of L_{cr}=2D is here proposed for ordinary stone columns. Please, note that for encased stone columns, the critical length is usually larger (e.g. L_{cr}=2-4D, Castro 2017a). When a more specific analysis may be performed for the design of a group of columns, it is enough to identify the extension of the plastic zone with columns longer than L_{cr}, using for example, finite element analyses (Figure 14).

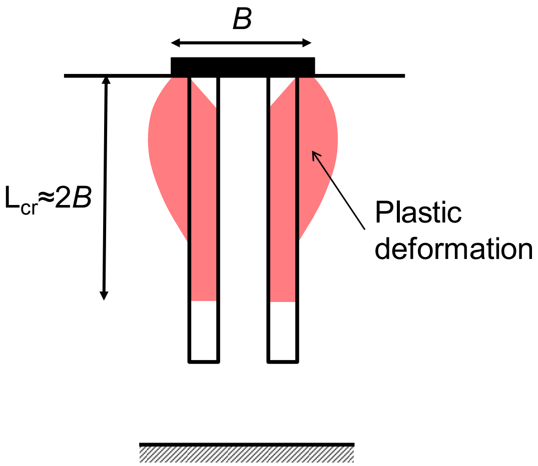

Figure 14. Conceptual justification of the critical column length as the depth of plastic deformation

5 CONCLUSIONS

For stone columns, there exists the concept of critical column length. Thus, for columns longer than the critical length, the improvement achieved with the stone columns does not notably change or increase. Sometimes in the literature, critical column length is expressed as the ratio between column length and column diameter. The authors have shown in previous publications that this slenderness parameter of each individual column has a minor influence, and critical column length is better expressed using the footing width. The reanalyses of a couple of previously published critical column lengths (L_{cr}=6 d_c) in terms of the footing diameter provides satisfactory results (L_{cr}=1.5-1.7 D).

Here, simplified finite element analyses show values in the range L_{cr}=1.2-2.2 D, being its value for common cases around 1.8 D. In softer soils, the critical column length is larger. The analysis of the finite element simulations shows that the value of the critical length is related to the extent of plastic deformation and that may be used to decide the column length in the design phase without the need for a parametric analysis of the column length. As a first approximation, a general value of L_{cr}=2 D may be assumed for ordinary (non-encased) stone columns.

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