Proceedings of the XVII ECSMGE-2019
Geotechnical Engineering foundation of the future
ISBN 978-9935-9436-1-3

doi: 10.32075/17ECSMGE-2019-0345

XVII European Conference
ECSMGE 2019 Reykjavik
on Soil Mechanics and Geotechnical Engineering

Upper and lower bounds for the bearing capacity of foundations on soft soils with reinforcement columns

Limites supérieurs et inférieurs de la capacité portante de fondations sûr sols mous avec colonnes de renfort

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ABSTRACT: The finite element program *mechpy*, developed within the authors' team, which implements the upper and lower bound theorems of limit analysis, is applied to the calculation of the bearing capacity of foundations on soft soils improved with reinforcement columns. The problem is analysed in two and three-dimensions, considering the soil and the columns responding under undrained conditions, using different replacement area ratios and different column-to-soil shear strength ratios. The upper and lower bound results are found to be in a narrow range and in good agreement with experimental results found in the literature. A simple equation is proposed to evaluate the bearing capacity of soil reinforced with columns.

RÉSUMÉ: Le programme aux éléments finis *mechpy*, développé dans l'équipe de recherche des auteurs, le quel implémente les théorèmes statique et cinématique de l'analyse limite, est appliqué dans le calcul de la capacité portante de fondations sur sols mous renforcés avec colonnes. Le problème est analysé dans des conditions à deux et trois dimensions, considérant le sol et les colonnes répondant dans des conditions non drainées, en utilisant des différents rapports de surface de remplacement et des différents rapports de résistance non drainée. Les résultats des limites inférieures et supérieures se trouvent dans un intervalle étroit et en accord avec des résultats expérimentaux trouvés dans la littérature. Une équation simple pour déterminer la capacité portante du sol renforcé avec colonnes est proposée.

Keywords: limit analysis; bearing capacity; soil reinforcement with columns; tridimensional effects

1 INTRODUCTION

Soil improvement of deposits of soft soils is a common practice, particularly in coastal areas, where the construction in such foundations is justified by the needs arising from economical development. Amongst the techniques of soil improvement is in-situ deep mixing. With this technique, the in situ soil is mixed with a stabilizing material, resulting in significant increase of strength and reduced deformability. The usual procedure is to perform stabilizing columns using equipment specifically developed and built to this purpose.

Several contributions have been made to the determination of bearing capacity of soil reinforced with columns, such as the work of Bouassida et al. (1995), Bouassida and Porbaha (2004) and Rashid et al. (2015a,b).

In the present paper, the problem is addressed using a numerical implementation of the upper- and lower-bound theorems of limit analysis, the finite element program *mechpy*. The results obtained from the numerical calculations are compared with available test results and, from the results of these numerical calculations, a simple equation for the determination of bearing capacity of soft soils reinforced with columns is proposed.

2 THE FINITE ELEMENT PROGRAM

The finite element limit analysis code is implemented in *mechpy*, which is a platform written in Python language for the development of non-conventional finite element formulations (Deusdado, 2017). The limit analysis module of this software has its origins in the *sublim3D* program, also developed within the authors' research team (Vicente da Silva and Antão, 2008; Vicente da Silva, 2009; Antão et al., 2012).

From a numerical point of view, the determination of plastic collapse loads using limit analysis theorems leads to a non-linear optimization problem. The optimization technique used in *mechpy* is the Alternating Direction Method of Multipliers (ADMM). Its iterative solution scheme is based on an operator splitting algorithm, which is suitable to efficiently solve large-scale variational problems with parallel processing. It has been applied successfully to several geotechnical problems, including bearing capacity problems (Antão et al, 2010; Antão et al., 2012; Franco et al., 2017).

This software is capable of producing both upper and lower bounds. It assumes a rigid plastic behaviour, with associated flow rule, for the materials. In this work, 6-noded triangles (with the usual 3 corner nodes and 3 midside nodes) are used to obtain strict upper bounds. In these elements, the velocity field approximation is assumed to be quadratic and the approximation of both the strain rate and stress fields to be linear. To obtain strict lower bounds, a 7-noded triangular element is used (with 2 nodes associated to each edge of the element, plus an additional node located at the barycenter). In this element the strain rate and stress fields approximations are assumed to be linear. Conversely, the velocity field is incoherent and discontinuous. In the current formulation the velocity, strain rate and stresses fields are independently approximated.

3 TWO-DIMENSIONAL NUMERICAL ANALYSES

The finite element program *mechpy* was applied to the determination of the bearing capacity of a rigid block on soft soil reinforced with columns, initially under two-dimensional conditions. The geometry of the problem analyzed is represented in Figure 1, where the cases analyzed (reinforcement with 1, 2, 3, 4 and 5 columns) are also schematically represented. The columns were assumed to be prolonged to a rigid stratum, 1.6B deep. This depth and the horizontal size of the problem analyzed were chosen in order to not interfering with the developed mechanism. Calculations took advantage of the symmetry of the problem, as suggested in the figure. Both the soil and the columns were assumed responding under undrained conditions, with undrained shear strength c_{us} and c_{uc} (for soil and columns, respectively). The unit weight of both materials were assumed to be the same (γ_c=γ_s). For each case (1 to 5 columns), several c_{uc}/c_{us} ratios (ranging from 2 to 50) were considered, as well as several replacement area ratios A_c/A (ranging from 0.04 to 0.80).

Upper and lower bound calculations were performed to determine the bearing capacity of the rigid footing under vertical centered loading, q_r.

Figure 2 shows an example of a coarse 2D finite element mesh similar to those used in the calculations, which are much more refined, with the size of the elements equal to 1/5 of the ones presented in the Figure.

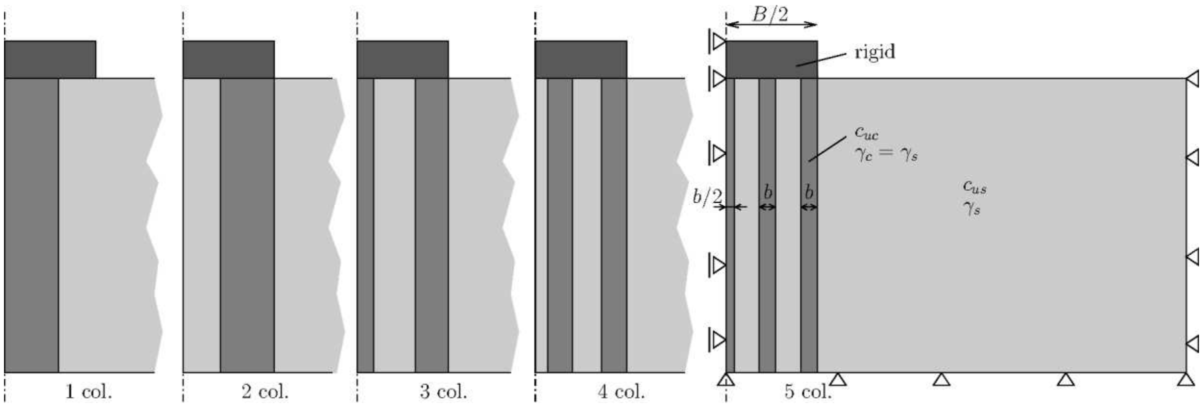

Figure 1. Schematic representations of the geometries of the two-dimensional calculations; different values of the columns widths were used, to achieve different replacement area ratios A_c/A

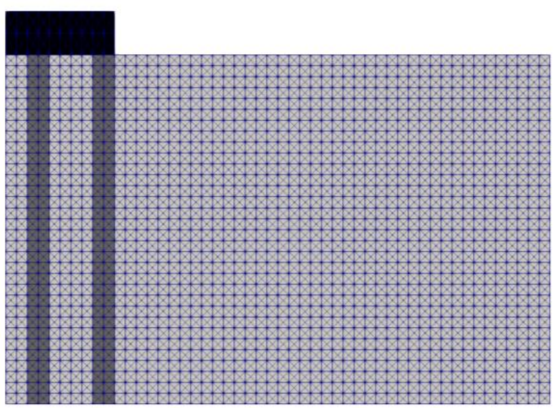

Figure 2. Coarse example of a 2D finite element mesh

Part of the results are represented in Figure 3. In this figure, the bearing capacity is represented by the dimensionless parameter θ, equal to:

$$\theta = \frac{q_r}{(2+\pi)c_{us}} \qquad (1)$$

where q_r is the bearing capacity stress obtained from the upper- and lower-bound limit analysis calculations. This parameter θ is shown as a function of an improvement ratio, ρ, given by:

$$\rho = \frac{A_c}{A}\frac{c_{uc}}{c_{us}} \qquad (2)$$

which reflects both the effects of the replacement area ratio and of the ratio between the undrained shear strength of the columns and of the soil. Figure 2 shows the results obtained for four different c_{uc}/c_{us} ratios - 2, 8, 15 and 50.

From

Figure *3* it can be seen that: a) upper- and lower-bound results are very close and in most cases indistinguishable; b) the results obtained in the present study improve the previous limit analysis results presented by Bouassida and Porbaha (2004); c) there is a linear relationship between θ and ρ (for each c_{uc}/c_{us} ratio the fitting equation is presented). The following equation is proposed for this relationship:

$$\theta = m\rho + 1 \qquad (3)$$

and m increases with the c_{uc}/c_{us} ratio and seems to be stabilizing for the greater values of this ratio. This can be seen in Figure 4, where the obtained values of *m* are represented as a function of c_{uc}/c_{us} ratio. The following equation was found to fit the results presented in Figure 4 quite accurately:

$$m = 0.386\left(1 - \frac{1}{\left(\frac{c_{uc}}{c_{us}}\right)^{1.113}}\right) \qquad (4)$$

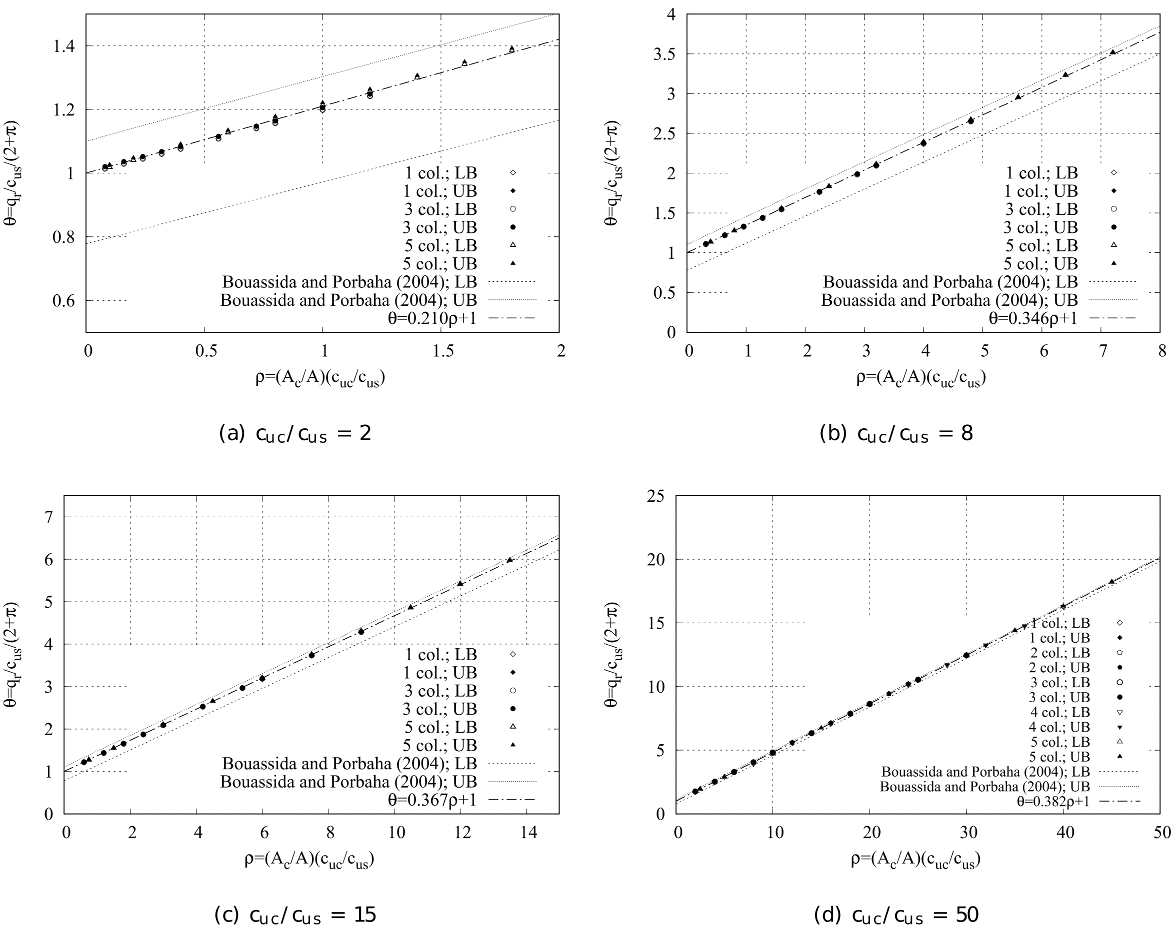

Figure 3. Dimensionless bearing capacity parameter θ as a function of the improvement ratio ρ for four different c_{uc}/c_{us} ratios – 2, 8, 15 and 50. Comparison with other limit analysis solutions

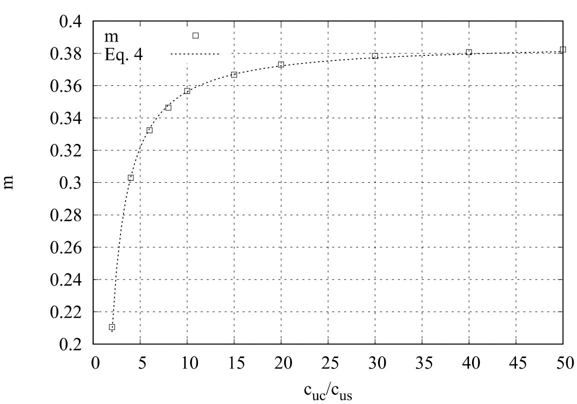

Figure 4. Values of parameter m as a function of the c_{uc}/c_{us} ratio

Figure 5 shows the plastic deformation zones obtained from the upper-bound calculations for A_c/A=0.40, three values of c_{uc}/c_{us} ratios (2, 15 and 50) and for 1, 3 and 5 columns. These plastic deformation zones allow the mechanisms in failure to be inferred, and it can be observed that there are significant differences between the mechanisms obtained for c_{uc}/c_{us} ratios of 2 and 15, but not between the mechanisms obtained from this ratio equal to 15 and 50, which is in agreement with the bearing capacity results (similar mechanisms leading to similar bearing capacities).

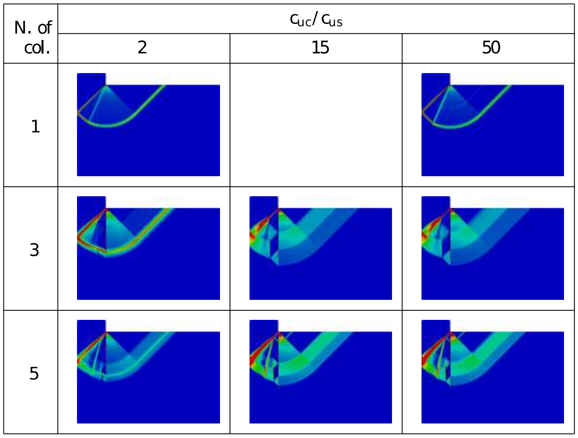

Figure 5. Plastic deformation zones obtained from upper-bound calculations for c_{uc}/c_{us} ratio =15, different A_c/A ratios and for 1, 3 and 5 columns

Figure 6 shows the same type of results for a given c_{uc}/c_{us} ratio (of 15) and for different A_c/A ratios. It can be observed an increase of the complexity of the mechanisms when the A_c/A ratio increases, particularly for 3 and 5 columns.

This greater complexity involves larger dissipation areas and greater volume of soil involved in the mechanisms and results in greater bearing capacity. The complexity of the mechanisms can also be seen in the plastic hinges formed in some of the columns; some of those columns have several of them.

4 COMPARISON WITH LOAD TESTS

In Figure 7, equations 3 and 4 are compared with the results presented by Bouassida and Porbaha (2004) and Rashid et al. (2014) A reasonable agreement can be observed for the first set of tests, whereas some significant differences are found between the equations and the second set of tests.

To check if the three-dimensional nature of the tests could somehow be responsible for the differences shown in Figure 7, a set of three-dimensional calculations was then performed using the same finite element program *mechpy* and with a geometry and mechanical properties replicating the one from Rashid et al. (2014).

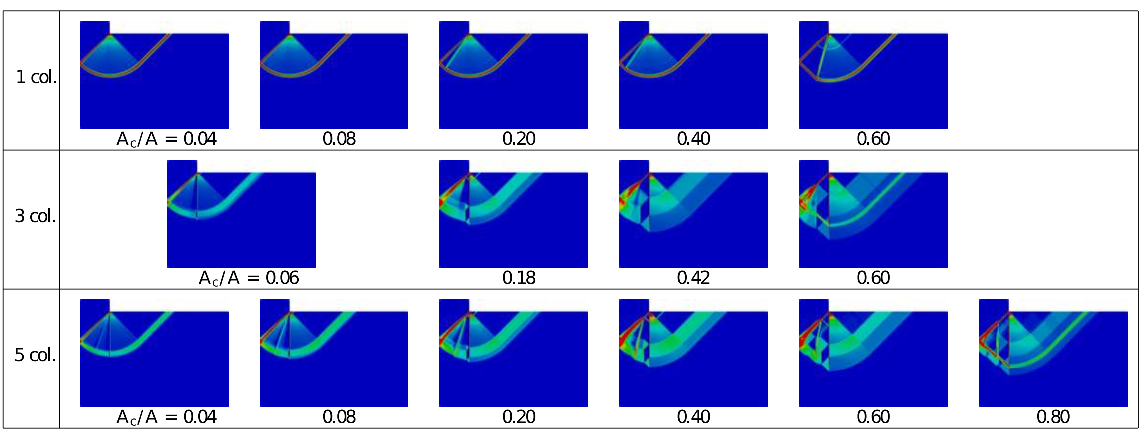

Figure 6. Plastic deformation zones obtained from upper-bound calculations for A_c/A=0.4, c_{uc}/c_{us} equal to 2, 15 and 50 and for 1, 3 and 5 columns

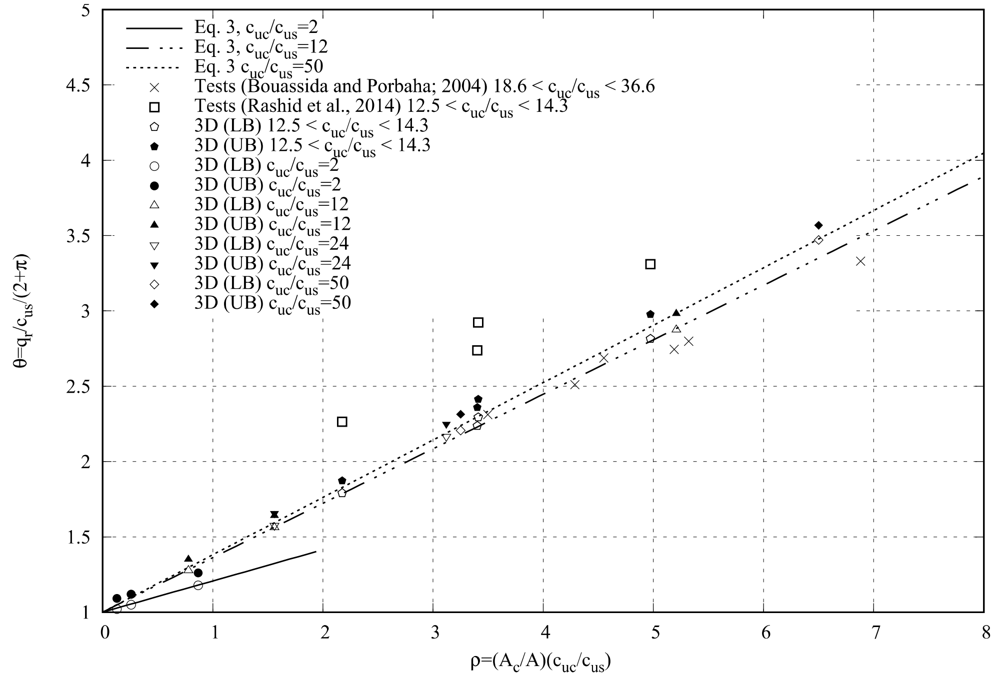

Figure 7. Comparison between the results from equations 3 and 4 with load tests and three-dimensional calculations

5 THREE-DIMENSIONAL NUMERICAL ANALYSES

The tests consisted in a rigid block of 2 to 4 rows of 3 columns each. The columns reach the lower border of the model. Figure 8 shows a coarse example of a three-dimensional finite element mesh similar to the ones used to model one of the tests presented by Rashid et al. (2014), corresponding to the case of the 2 rows of columns. As in the 2D case, the finite element mesh used in the calculations is much more refined. The calculations took advantage of both the symmetry and the periodicity of the rows of columns (Figure 9). The c_{uc}/c_{us} ratios were between 12.5 and 14.3.

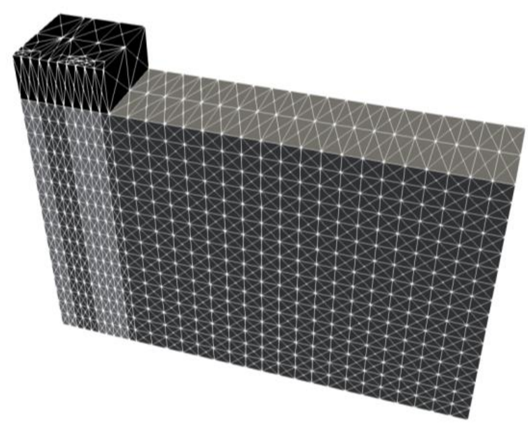

Figure 8. Coarse example of a three-dimensional finite element mesh

The results obtained are also included in Figure 7. It can be seen that they match quite well the two-dimensional results (represented by equations 3 and 4) and therefore no significant 3D effect was found.

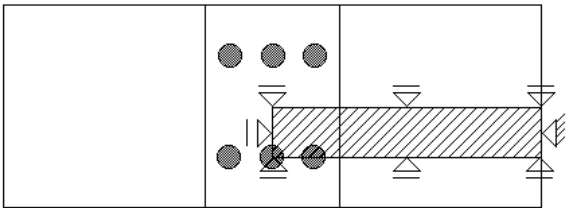

Figure 9. Scheme of the plan view of the tests with two rows of columns with indication of the volume analysed in 3D calculations

There are therefore some differences between 3D calculations and the corresponding test results from Rashid et al. (2014). This can be explained by the fact that the boundary conditions of the calculations do not replicate the friction between the soil mass and the walls of the model, as demonstrated by Rashid et al. (2014).

Finite element meshes of the same type of those used for the 3D calculations to model the tests were also used for values of c_{uc}/c_{us} ratio and of A_c/A in wider range. The results are also shown in Figure 7 and, again, a good agreement between the 3D calculations and the previously mentioned equations was found.

Figure 10 presents two views of the plastic deformation zones for one of the 3D calculations and it can be seen that the behavior is mainly two-dimensional, due to the effect of the rigid footing. This explains the low influence of the 3D geometry of the results.

6 CONCLUSIONS

Results of the bearing capacity of soil reinforced with columns obtained from two and three-dimensional finite element analysis calculations are found to accurately be written using a simple relationship between a dimensionless bearing capacity parameter, θ, (equation 1) with a dimensionless improvement ratio, ρ, (equation 2). This relationship is expressed by equations 3 and 4 and can be written as

$$\theta = 0.386\left(1 - \frac{1}{\left(\frac{c_{uc}}{c_{us}}\right)^{1.113}}\right)\rho + 1 \qquad (5)$$

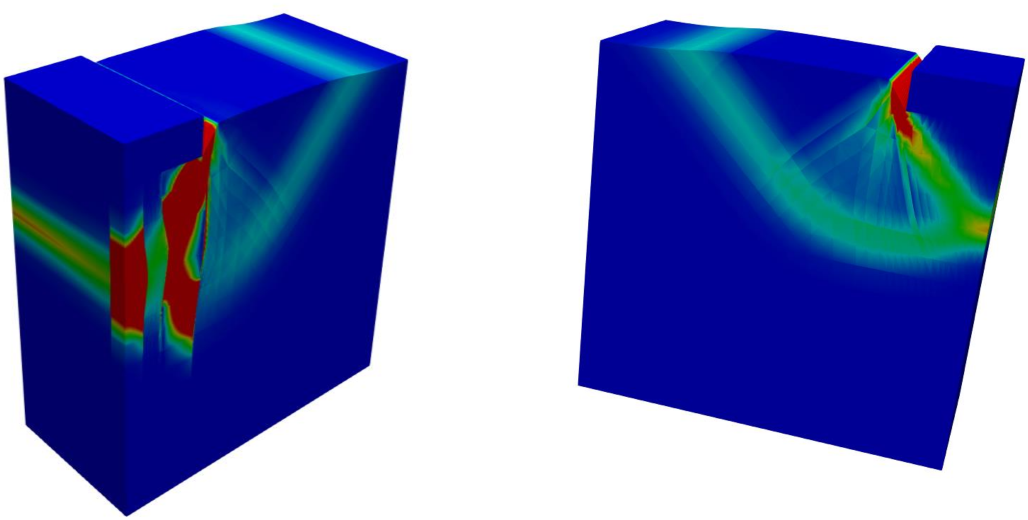

Figure 10. Two views of the plastic deformation zones for an example of a 3D calculation (c_{uc}/c_{us}=12; A_c/A=0.065)

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