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Evaluating the reliability of back analysed shear strengths in slopes

Evaluation de la confiance à la résistance au cisaillement résultant d'une rétro-analyse

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ABSTRACT: The back analysis of slope close to failure to gain shear strength parameters of the soil layers along a shear plane is a common tool in practise and is a suggested method by various Norms such as (SIA, 2013) where laboratory test data of the soil is not available. This paper investigates the effectiveness and limitations of this procedure for two unstable slopes. One of the major assumptions in conducting a back analysis of shear strength parameters is that the slope remains stable close to the limit equilibrium state. It is mostly neglected that the limit equilibrium state is often a peak state at the dry side of critical due to water infiltration and loss of suction, i.e. a decrease in the mean effective stress. Soils with stress conditions on the dry side of critical tend to soften due to visco-plastic behaviour close the yield surface. This behaviour leads eventually to a potential loss of shear strength until the critical state is reached. As soon as this happens, stable slopes might suddenly fail. Therefore, it is suggested that the results of this method (back analysis of shear strength parameters) can lead to over-estimation of the critical state shear strength parameters of the soil.

RÉSUMÉ: La rétro-analyse de la stabilité des pentes est un outil souvent utilisé en practice quand on veut identifier la résistance au cisaillement d'un certain sol. La contribution ci-présente s'appuye sur l'étude de deux cas de pentes différentes qui ont atteintes la rupture. Il est le but de cette contribution de souligner les limites de la méthode de rétro-analyse: notamment la validité des valeurs de la résistance au cisaillement qui sont obtenues par rétro-analyse. Quand on effectue une rétro-analyse, on suppose que la pente reste stable jusqu'à l'équilibre ultime. Toutefois il faut tenir compte du fait que dans beaucoup de cas on néglige le fait que l'équilibre ultime est souvent un état «on the dry side of critical». Les sols qui sont soumises à des états de contraintes «on the dry side of critical» ont la tendance de perdre en rigidité en raison du comportement viscoplastique dans la proximité de la surface de rupture. Ce comportement conduit finalement à la perte de résistance au cisaillement. Dans ces cas les pentes premièrement stables peuvent devenir instables tout d'un coup. En conséquence il vaut mieux se poser la question de l'état critique au lieu d'effectuer une rétro-analyse.

Keywords: back calculation, shear strength, slope stability, critical State, softening

1 INTRODUCTION

The shear strength parameters of the soil in stable slopes are sometimes estimated using the back analysis method (SIA, 2013). It is assumed that there must be a shear strength which is able to keep the slope in equilibrium and it is usually assumed that this shear strength will not change with time. It is also assumed that the analysed slope is close to the limit equilibrium state. Back analysis is a common method to evaluate slopes on their stability especially when designing a new construction on/in the slope (e.g. buildings, bridge foundations, etc.). A back analysis is often used to gain feasible parameters for the design of slope stabilising systems. In these situations the back-calculated internal friction angle φ'_k (°) is assumed to be the characteristic value (e.g. SIA 267, 2013). For the design of the stabilising systems, a design value of the internal friction angle φ'_d (°) is set as follows:

$$\varphi'_d = arctan\left(\frac{tan\varphi'_k}{\gamma_m}\right) \quad (1)$$

Where γ_m (-) is the partial factor to ensure that the newly built geotechnical construction has a certain level of safety compared to the situation beforehand. The partial factor γ_m is usually set to 1.2.

On the other hand, stable slopes fail suddenly due to severe rainfall periods (e.g. Rickli, 2001). These failures are often quite shallow (less than 1 m deep). Due to those incidents the question comes up whether such back calculations provide reliable values of shear strength parameters and therefore allow to set the design values for newly built constructions.

2 BACK CALCULATION

The basic assumption of a back calculation for a natural slope or a cut is, that the slope or cut is stable and the factor of safety F_S (-), defined as follows (Lang et al., 2007):

$$F_S = \frac{\tau_{failure}}{\tau_{mobilised}} \geq 1.0 \quad (2)$$

is at least 1.0 and close to 1.0 as the slope is meant to be near the limit equilibrium state. Based on those assumptions the mobilised shear strength $\tau_{mobilised}$ (kPa) can be calculated based on several considered failure mechanisms. Knowing the mobilised shear strength and assuming that F_S would at least be equal to 1.0 leads to the ultimate limit shear strength in the slope ($\tau_{failure}$).
Generally the Mohr-Coulomb failure criterion (Coulomb, 1776) is assumed as the simplified soil shear strength model:

$$\tau_{failure} = \sigma'_n \cdot \tan(\varphi') + c' \quad (3)$$

where σ_n' (kPa) is the normal effective stress, φ' (°) is the friction angle and c' (kPa) is the cohesion. For undrained conditions in saturated soils, Equation (3) will result in

$$\tau_{failure} = s_u \quad (4)$$

as φ' is assumed to be zero for total stress analysis (Wood, 1990). Only the undrained shear strength s_u (kPa) is left as a total shear strength. The parameter s_u is dependent on φ'_{crit} (friction angle at the critical state) and the current water content w (-) (Atkinson, 2007, Arnold et al. 2018).
The shear strength at failure is dependent on 2 parameters, φ' and c', as Equation (3) shows. The back calculation itself allows only calculating one of the two parameters directly. To solve this problem, Yang (2014) suggested a dimensionless paramter to relate c' to φ' dependent on the depth of the failure mechanism. According to the theory of critical state soil mechanics (Schofield and Wroth, 1968), at the ultimate limit state no cohesion is left between the grains of the soil and φ'_{crit} is the only shear strength parameter to estimate by assuming F_S= 1.0. Back calculations may also be performed on the dry side of critical with peak values φ'_p, higher than φ'_{crit} as shown in Figure 1 after Atkinson (2007).

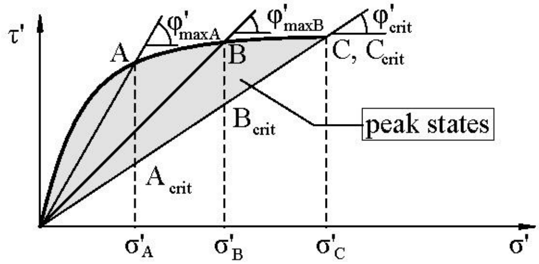

Figure 1: Peak states of soils with different states (after Atkinson, 2007)

The swiss code SIA 267 (2013) for geotechnical engineering also mentions the possibility of performing a back calculation. It is defined in the code SIA 267 as follows: „method of arithmetical determination of characteristic subsoil parameters on the precondition of analysing a limit equilibrium system.“ It is also mentioned that the back calcultion to get soil parameters has to be done on the precondition of using the same calculation model as it will be used for the strengthening of the geotechnical structure.

3 MOBILISATION OF SHEAR STRENGTH: NORMALLY CONSOLIDATED AND OVERCONSOLIDATED BEHAVIOUR

The mobilisation of shear strength is dependent on stress-strain-relations of the soil. The initial conditions of a subsoil are given by the loading history as shown in Figure 2.

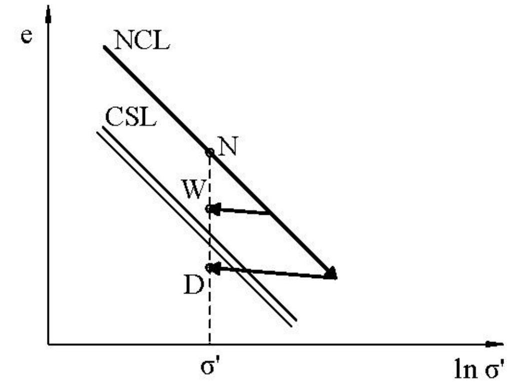

Figure 2: Defining the initial conditions depending on the loading history. N: stress point on the normal compression line (NCL). W:stress point at the wet side of critical (normally consolidated or slightly overconsolidated). D: stress point on the dry side of critical (heavily overconsolidated) (after Atkinson, 2007)

Overconsolidated or dense soils can mobilise more shear strength on the dry side of critical (D) due to dilation than normally consolidated or loose soils on the wet side of critical (W). However, with increasing shear deformations a certain soil will reach the critical state, which is for both, overconsolidated- and normally consolidated soils the same as mentioned in Figures. 1 and 3, where p‘ is the mean effective stress and q is the deviatoric stress. This is an important aspect of the critical state soil mechanics: soils ultimately shear at constant volume at their critical state no matter what they experienced before. Therefore, it can be concluded that the soil reconstitutes at the shear zone to the same void ratio as normally consolidated soil has at the critical state (Atkinson, 2007).

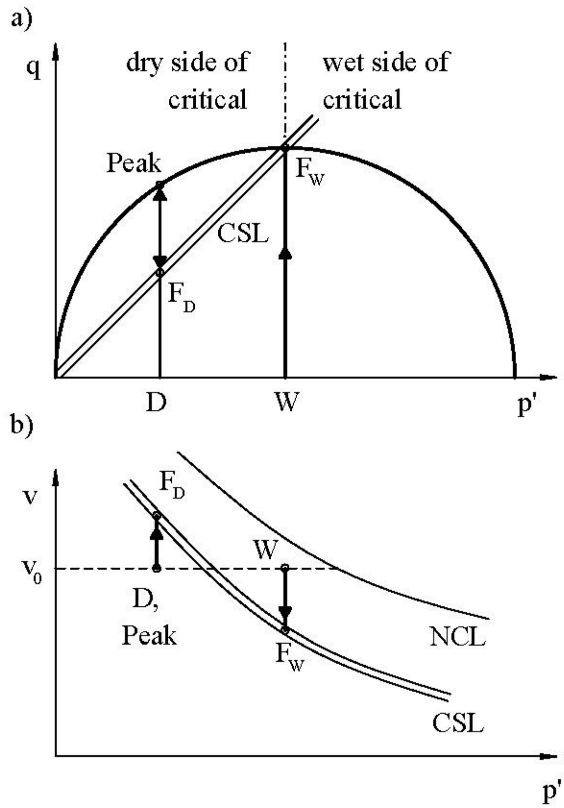

Figure 3: Soil shearing on the wet and on the dry side of critical. Reaching the critical state with shearing at constant volume. a) Peak values of q for stress conditions on the dry side. b) Development of volume with p‘; softening means increase of volume, hardening means decrease of volume after Atkinson (2007).

Bearing this basic concept of critical state and mobilisation of shear strength in mind, the question comes up, if one would be able to know the current stress-state-condition in the soil of a slope which is going to be back analysed? And furthermore: is it important to know the current stress-strain-condition to gain strength-parameters for safe design of new structures in or above current slopes?

Leroueil (2001) mentions that visco-plastic strains develop near, but still inside the yield surface (Zone 3). The basic elasto-plastic Cam Clay Model (Roscoe and Burland, 1968) can be extended as given in Figure 4.

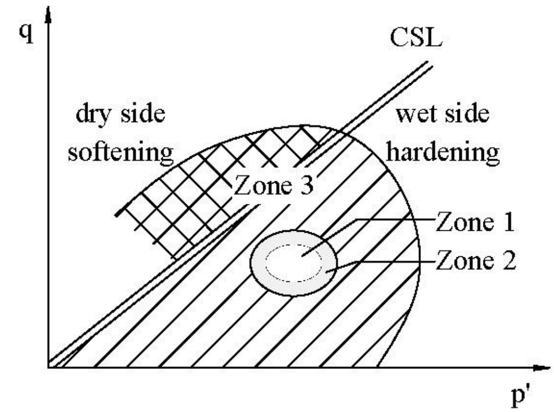

Figure 4: Basic elements of soil behaviour. Zone 1: linear-elastic behaviour; Zone 2: non-linear elastic behaviour; Zone 3: visco-plastic behaviour, after Leroueil (2001).

As one can see in Figures. 3 and 4, the critical state line divides the yield surface in volumetric hardening (wet side of critical)- and volumetric softening (dry side of critical) behaviour. As long as certain stress conditions of slopes lie on the wet side of critical, soil will harden and gain more strength, but if the stress conditions lie on the dry side, soil will soften inside the yield surface and therefore loose strength until the critical state line is reached.

4 INVESTIGATION OF A CLAY SLOPE: TAKE & BOLTON (2011)

Take and Bolton (2011) investigated the behaviour of a heavily overconsolidated clay 36º slope subjected to various dry and wet seasons in a geotechnical centrifuge. They found that the slope is going to fail progressively due to softening behaviour at the dry side of critical down to the critical state. They counted the extra-strength at the dry side of critical as a cohesion which will be lost step by step due to the softening behaviour in wet seasons. They also showed the typical stress path due to infiltration of water in wet seasons and how the soil reaches finally the dry side of critical (Figure 5).

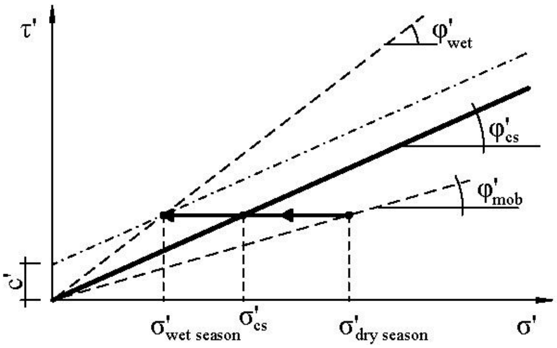

Figure 5: Qualitative stress path for water infiltration to the soil after Take & Bolton (2011).

In Figure 5 one can see, that due to negative pore pressure, a very small angle of friction is needed to keep the slope stable in dry seasons. Due to infiltration of water, the negative pore pressure and hence p‘ is going to decrease while the value of q is almost constant. With rising pore pressure the stress path reaches the dry side of critical where „cohesion“ due to overconsolidation is needed to keep the slope stable. With every change in dry to wet season, the available cohesion due to overconsolidation decreases due to softening behaviour inside the yield surface at the dry side of critical (Leroueil, 2001). With completed softening the soil reaches the critical state shear strength and must therefore fail since the slope is steeper than the critical state would allow.

5 INVESTIGATION OF THE RUEDLINGEN SLOPE: ASKARINEJAD (2013)

Askarinejad et al. (2018) conducted a full scale landslide triggering experiment on a natural silty-sandy slope subjected to an artificial rainfall event which resulted in mobilisation of 130 m^3 of soil mass. This experiment was done on a forrested slope in Ruedlingen (CH). Roots in the soil were cut before the test was started to rule out effects of root reinforcements (Świtała et al., Yildiz et al., 2015).

By analysing the behaviour of the slope, one can see, that basically decreasing negative pore pressure leads the stress path to the dry side of critical, where some sort of „cohesion“ maintained the stable slope for a certain while. The fact that the critical state friction angle of the silty sand is 32.5º and the slope angle is 38º leads to this conclusion, since the slope remained stable for a certain while without negative pore pressures (Tang et al., 2018). This leads to the stress path given in Figure 6.

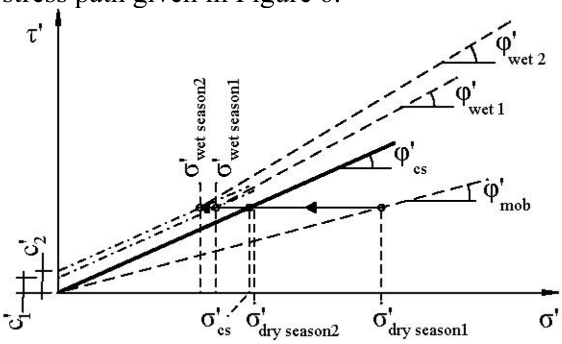

Figure 6: Qualitative stress path for water infiltration to the soil.

In contrast to the investigation of Take and Bolton (2011) no climatic cycles were driven to provoke failure of the slope. The two main triggering mechanisms of this slope were reported to be rain water infltration as well as water exfiltration from the bedrock (Askarinejad et al., 2014).

However, one can see that similar behaviour of two rather different slopes lead to unstable situations. It is concluded that infiltration of water on the very upper parts of the slopes (top-down infiltration) would lead to failure modes near to the surface whereas bottom-up infiltration (e.g. bedrock exfiltration) would rather lead to deeper failure mechanisms as shown on the Ruedlingen site.

6 BACK ANALYSIS OF THE BY TAKE AND BOLTON (2011) AND ASKARINEJAD (2013) INVESTIGATED SLOPES

Take and Bolton (2011) do a back analysis of the investigated clay slope at the end of a wet season which leads to the values of 24 degrees for the friction angle and 7.5 kPa for the cohesion. They calculated a cohesion by knowing that the friction angle at the critical state would certainly be 24 degrees. Usually a back analysis is done to estimate the friction angle by analysing the geometric boundary conditions. The results presented here are basically gained by analysing the geometric boundary conditions and assuming that there are no negative pore pressures and no water table in the slope. Assuming that there would be no water table in the slopes is feasible in terms of back-calculating slopes which seem not to have water tables available in normal climatic conditions. If one would assume a water table in the slope, greater values of φ'_k would result in the back calculation because water pressure would enforce the slope to slip whereas greater friction would enable stability. The back analysis for the clay slope (Take and Bolton, 2011) was done by using Bishop‘s method (1955). The back calculation for the Ruedlingen slope (Askarinejad, 2013) was done by using Janbu‘s method (1954). The investigated slope geometries and failure mechanisms in the back calculations are given in Figure 7. The requirement of having a slope at the limit equilibrium state should be given as one can see that both slopes are rather steep compared to expected friction angles of the involved soils. The fact that both analysed slopes failed indicate also that they were near the limit equilibrium state.

If we now compare the measured critical friction angle φ'_{crit} to the design value φ'_d assumed based on the back calculation, it can be seen that the back calculated friction angles are larger on their design level than the friction angle at the critical state. This makes clear that the back analysis presented here do not fullfill their purpose by estimating feasible values of shear strength for the design of geotechnical structures.

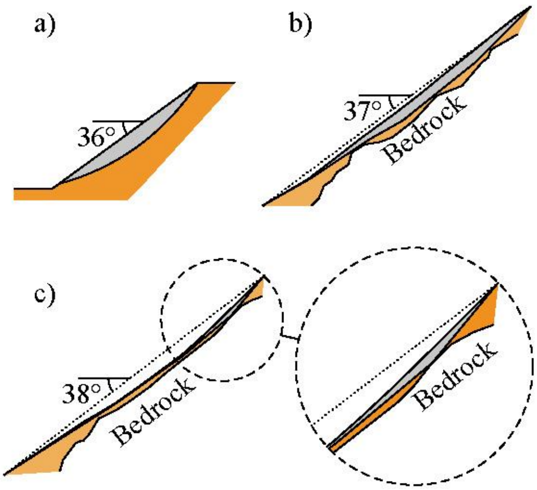

Figure 7: Investigated slopes: a) clay slope given by Take & Bolton (2011); b) Ruedlingen slope on the left side (Askarinejad, 2013); c) Ruedlingen slope on the rigth side (Askarinejad, 2013)

Table 1. Back calculation of the friction angle for the investigated slopes

Slope	**φ'_k**	**φ'_d (SIA 267 2013)**	**Measured $\varphi'_{crit.}$**
Clay slope in Centrifuge	34°	29.3°	24°
Ruedlingen slope	38° (b) 40° (c)	33.1° 35.0°	32.5° 32.5°

7 SOFTENING BEHAVIOUR AT THE DRY SIDE OF CRITICAL

In both investigated slopes, the soil was slightly or heavily overconsolidated. Therefore it is obvious that the dry side of critical is reachable due to water-infiltration process and therefore decreasing p‘. Figure 8 shows, that also normally consolidated soil can reach the dry side of critical in terms of water-infiltration process and decreasing p‘.

This makes clear, that also slopes with normally consolidated soils will have the potential of subsequent softening behaviour leading to first-time failure of the slope as long as the slope is steeper than the critical state friction angle.

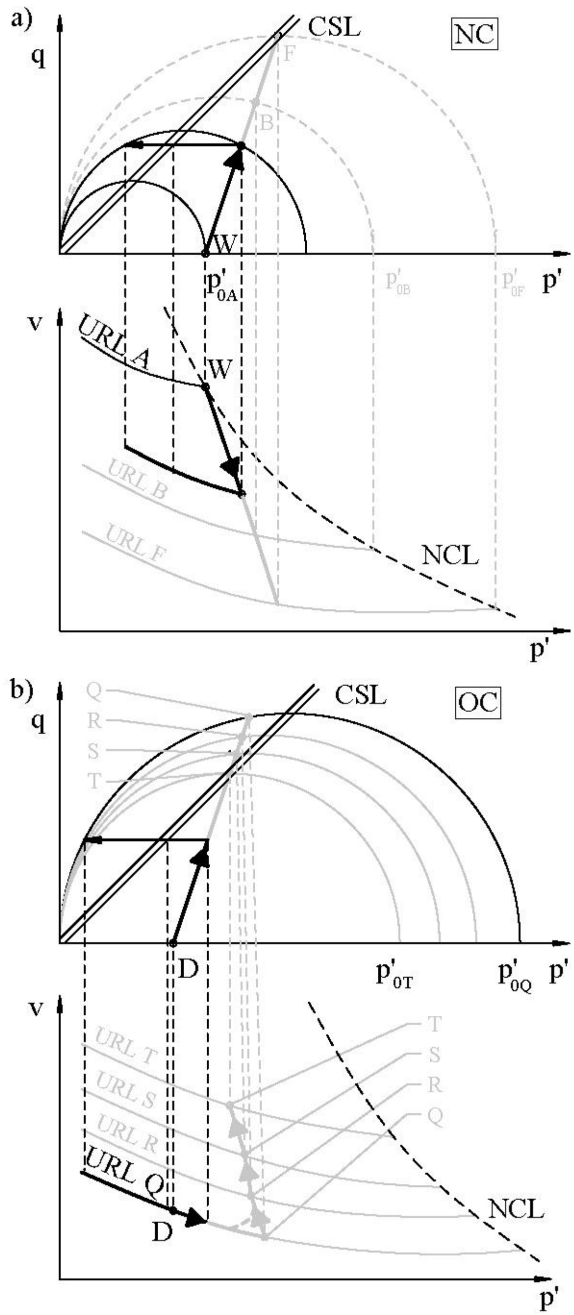

Figure 8: a) NC-soil: expansion of yield surface (hardening). b) OC-soil: shrinkage of yiel surface (softening) after Wood (1990).

The stress paths mentioned here need to have rather large negative pore pressures in dry seasons which tend to vanish due to rainfall.

8 CONCLUSIONS

The back analysis of stable slopes might lead to an overestimation of the friction angle, since the stress-strain-relations of the soil in the slope remains unknown. Therefore it is uncertain whether the soil sustains softening behaviour in wet seasons on the dry side of critical which eventually lead to failure. Instead of performing a back calculation to estimate the friction angle one should determine the critical friction angle of the involved subsoil using laboratory tests. If the slope is steeper than the critical friction angle it will certainly be a matter of time until the slope will produce a failure assuming that there are no effects of reinforcements such as roots or vegetation. It is also of great importance that some stabilizing effects which will lead to a rather large angle of friction in the back analysis might not be available anymore due to construction processes. One should question the different stabilizing effects in slopes and also the boundary conditions such as soil layers and water table while performing a back analysis.

9 OUTLOOK

If we have a look at construction stages of nailed or anchored walls or excavations, the softening of the soil might not be finished during a certain construction stage. There, it might be useful to analyse the current stress state of the soil for predicting a peak stage of shear strength. As long as such construction stages only last for a short time with a reasonable monitoring, one might not design those stages using the critical state shear strength. Some of the stages probably need a short term peak shear strength to maintain the progress of construction. More investigation would be needed on that field of soil behaviour to better understand peak stages of shear strength.

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11 REFERENCES

Arnold, A., Zhang, W., Askarinejad, A. 2018. *Undrained Shear Strength Profile of Normally and Over-Consolidated Kaolin Clay.* International Conference on Physical Modelling in Geotechnics, London. 119-124

Askarinejad, A., Akca, D. & Springman, S. M. 2018. Precursors of instability in a natural slope due to rainfall: a full-scale experiment. *Landslides,* **15**, 1745-1759

Askarinejad, A., Laue, J. & Springman, S. M. *Effect of bedrock shape and drainage properties on the stability of slopes.* International Conf. of Physical Modelling in Geotechnics, 2014 Perth. 1211-1217.

Askarinejad, A. 2013. *Failure mechanicsm in unsaturated silty sand slopes triggered by rainfall,* ETH Disseration Nr. 21423, vdf Hochschulverlag AG an der ETH Zürich, Zürich.

Atkinson, J. 2007. *The mechanics of soils and foundations***,** Taylor & Francis, Oxon.

Bishop, A.W. 1955. The use of the slip circle in the stability analysis of slopes. *Géotechnique* **3**, 1-5.

Coulomb, C.A. 1776. *Essai sur une Application des Règles des Maximis et Minimis à quelques Problèmes des Statique Relatifs à l'Architecture,* Mém. acad. roy. prés. divers savants, Bd. 7. Paris.

Janbu, N. 1954. *Application of the composite slip surfaces for stability analysis*, Proceedings of the European Conference, Stockholm.

Leroueil, S. 2001. Natural slopes and cuts: movement and failure mechanisms, *Géotechnique* **51**, No. 3, 197-243.

Rickli, C. 2001. *Vegetationseinwirkungen und Rutschungen – Untersuchung zum Einfluss der Vegetation auf oberflächennahe Rutschprozesse in Sachseln OW am 15. August 1997*, Birmensdorf, Bern: Eidg. Forschungsanstalt WSL, Bundesamt für Umwelt, Wald und Landschaft, 97p.

Roscoe, K.H., Burland, J.B. 1968. *On the generalised behaviour of wet clay,* In: Heyman, J., Leckie, F.A. (eds) Engineering Plasticity, Cambridge University Press, 535-610.

Schofield, A.N. and Wroth, C.P. 1968. *Critical State Soil Mechanics*, McGraw Hill, London.

SIA 267, 2013. *Geotechnik.* Schweizerischer Ingenieur- und Architektenverein, Zürich.

Świtała, B. M., Askarinejad, A., Wu, W. & Springman, S. M. 2017. Experimental validation of a coupled hydro-mechanical model for vegetated soil. *Géotechnique*, **68,** No. 5, 375-385.

Take, A., Bolton, M.D. 2011. Seasonal ratcheting and softening in clay slopes, leading to first-time failure, *Géotechnique* **61**, No. 9, 757-769

Tang, A.-M., Askarinejad, A., Cui, Y. J., Gentile, F., Gowing, J., Jommi, C., Kehagia, F., Keszeyné Say, E., Ter Maat, H. W., Lenart, S., Lourenco, S., Oliveira, M., Osinski, P., Springman, S. M., Stirling, R., Toll, D. & Viterbo, P. 2018. Atmosphere – vegetation – soil interactions impacts on engineered slopes: A review on recent advances. 2018. *Quarterly Journal of Engineering Geology and Hydrogeology*, **51**, 156-168.

Wood, D.M. 1990. *Soil Behaviour and Critical State Soil Mechanics*, Cambridge University Press, Cambridge.

Yang, H.H. 2014. *Slope Stability Analysis by the Limit Equilibrium Method*, ASCE Press, Virginia

Yildiz, A., Askarinejad, A., Graf, F., Rickli, C. & Springman, S. M. 2015. *Effect of roots and mycorrhizal fungi on the stability of slopes.* XVI Europ. Conf. on Soil Mechanics and Geotechnical Engineering. Edinburgh, UK.