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On the calculation of the settlement of pile foundations including ones containing piles of various lengths by the interaction factors method.

Sur le calcul du tassement des fondations sur pieux, y compris celles contenant des pieux de différentes longueurs par la méthode des facteurs d'interaction.

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ABSTRACT: The article considers the features of settlement calculations of pile foundations using the interaction factors method. Calculations for foundations containing of various lengths also considered. In the course of studying the calculations features, a new approximating function of the pile deformation field was obtained. The approximating function considers, a two-layer base and is also suitable for prediction of settlements of foundations with piles of different lengths. Comparison of the obtained function with published closed form and numerical solutions, as well as performed finite element calculation for a number of cases, is given.

RÉSUMÉ: L'article examine les caractéristiques des calculs de tassement des fondations sur pieux en utilisant la méthode des facteurs d'interaction. Les calculs pour les fondations contenant de différentes longueurs ont également été pris en compte. Au cours de l'étude des caractéristiques de calcul, une nouvelle fonction d'approximation du champ de déformation du pieu a été obtenue. La fonction d'approximation considère une base à deux couches et convient également à la prédiction des tassements de fondations avec des piles de différentes longueurs. Une comparaison de la fonction obtenue avec d'autres solutions fermées et numériques, ainsi que le calcul par éléments finis pour un certain nombre de cas, sont présentés.

Keywords: Pile; Foundation; Pile group; Settlement calculation; Interacton factors method; Finite elements method.

1 INTRODUCTION

Interaction factors method (IFM) is well known pile group settlement estimation method. Initially formulated by (Poulos 1968) method was widely used as at time single method capable for calculations of pile groups consisiting of more than few tens of piles. Due to breakthrough increase of computing power of PC and progress of numerical methods such as FEM, IFM being gradually replaced by FEM.

Despite being most accurate and sophisticated method, FEM by itself has several setbacks. These are long calculation time, inability of modelling of significant number of piles without use of special simplifying elments, difficulties in introducing modifications to finite element mesh.

Design of pile foundations due to the complicated pile soil interaction sometime requires several iterations to achieve optimal pile arrangement. Thus using FEM for routine calculation requires excessive time and effort of skilled geotechnical engineer.

Key advantage of IFM is simplicity and speed of calculation. Linear calculation of pile group consisting of thousand piles may be performed under one minute on office desktop computer with Intel i7 CPU. Calculation time may be further reduced due to utilization of parralelization and optimisation techniques.

IFM also fills the gap between very simple methods such as equivalent raft or equivalent pier and FEM calculations. Which makes method suitable for validation of results of calculatons performed by numerical methods.

Currently there are several issues with IFM that require additional investigation. Namely these are: 1) Questionable accuracy for large pile groups as shown by (Randolph 1994) 2) Inability to account for reinforcing effect of pile group due to the fact that IFM uses results of solution for 2 pile problem. 3) Validation of method by more rigorous numerical methods such as FEM. 4) Applicability of solution for axisymmetrical single pile settlement problem to spatial two pile interaction problem.

Aim of this paper is to adress aforementioned issues

2 INTERACTION FACTORS METHOD

2.1 Applicability of axisymmetrical single pile settlement problem solution for use as a influence function in IFM calculations

In essence IFM based on superposition principle of elasticity theory. Single pile under vertical load due to interaction with soil creates field of vertical displacements around itself. Any unloaded pile within mentioned field will undergo settlement without any load on its top.

IFM based on hypothesys that pile group settlement may be calculated by summation of pile own settlement and additional settlements induced by neighboring loaded piles deformation fields. Thus key element for settlement calculation is mathematical description of deformation field produced by loaded pile or influence function.

As of now, influence function for IFM calculations may be derieved by two distinctive approaches:

1) Calculation of influence function by using approximation formula such as (Федоровский 1974; Randolph and Wroth 1979; Mylonakis and Gazetas 1998) or tabulated values (Poulos and Davis 1980).

This approach usually requires radical simplification of ground conditions.

2) Approximation of results of calculation of two piles interaction spatial problem solved for set distances between piles by numerical methods (BEM, FEM).

Obviously second approach is more accurate but without use of specialised software may require a lot of time and effort.

Several well known implementations of IFM by approach 1 use closed form solution of single pile settlement problem to predict settlement of soil at specified distance. After that it is assumed that unloaded pile settlement is equal to soil settlement at specified depth (for example in (Randolph and Wroth 1979) assumed depth is half of pile length).

Analisys of single pile settlement problem solved by FEM shows interesting feature.

Results are shown on fig.1 and 2. X axis of the graph represents relative distance from pile as distance divided by pile length. Y axis of graph represents settlement of soil normalized by average settlement of soil for given distance. Due to complex form of soil settlement with depth averaging was performed as $w` = 123123123$.

Difference in soil settlement at different depths due to loading of pile in uniform half-space is 30%. In case of 2 layered soil these differences reach almost 100%. These observations show

that in region of 0 to 0.5 length of pile settlement of soil at different depths are non uniform and average soil settlement corresponds to depth between 0,5 and 0,75 lengths of pile.
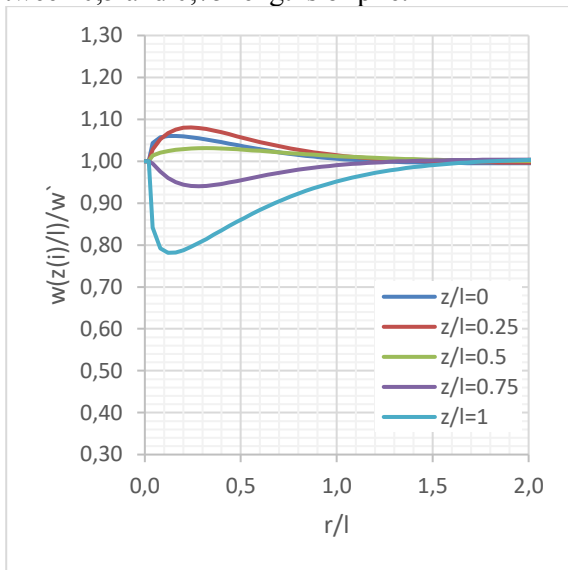

Figure 1. Settlement of soil with distance for given depths normalized by average soil settlement. Uniform half-space.
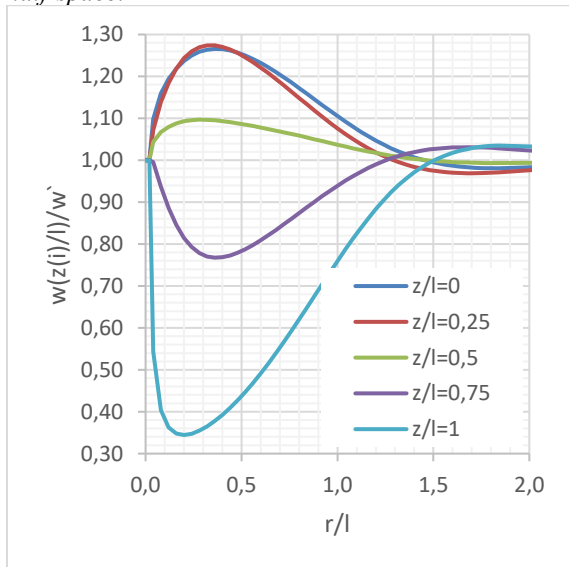

Figure 1. Settlement of soil with distance for given depths normalized by average soil settlement. Two-layer soil, bottom layer 16 times stiffer.

Additional calculation were performed in spatial setting with introduction of second unloaded pile.

Results are shown on fig.3 and 4, for uniform half space and for two layered soil. It was found, that unloaded pile settlement are almost equal to soil settlement at depth z=0.75l.

It is known (Федоровский 1974) that rigid pile settlement largely dependent on poisson ratio, l/d ratio, stiffness of underlaying soil layer in case of two layer soil and location of rigid layer in case one exists.

Unexpectedly observed feature is independent of mentioned factors. Additional investigaton shown that there is no practical reason in attempt to find precise location of depth, as it fluctuates in range of z/l=0.75…0.8.

Interestingly this value is close to one suggested by (Tomlinson and Woodward 2015) z/l=2/3l≈0.66l for location of equivalent raft. Moreover, in a case of observation on response of instrumented pile to soil subsidence reported by (ENDO 1969), location of neutral plane was found on exactly the same depth. Soil subsidence problem is relevant to IFM because settlement of uloaded pile produced by induced soil deformation field.
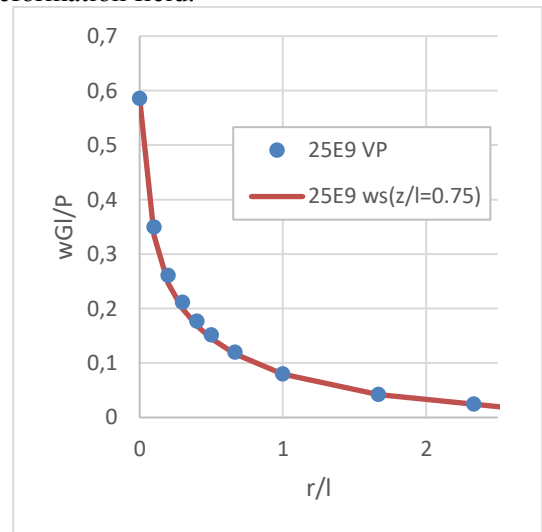

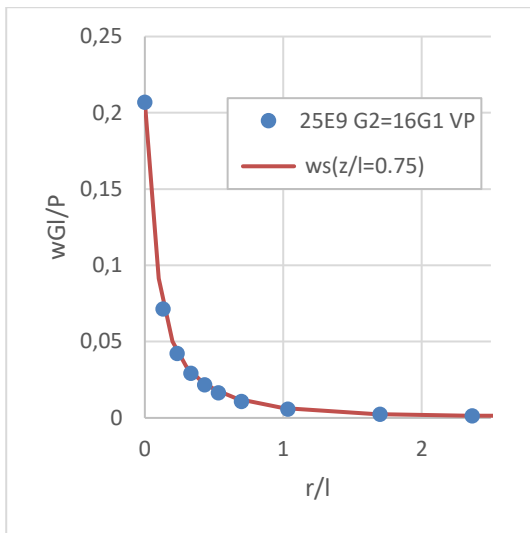*Figure 3. Soil settlement z=0.75l,points - pile settlement. Uniform soil.*

Figure 4. Soil settlement z=0.75l,points - pile settlement. $G_2=16G_1$.

2.2 Soil reinforcing (shielding) effect of pile group

In process of analisys of results for 2 piles problem another feature worth mentioning was found. Settlement of loaded pile in presence of additional unloaded piles is lower than that of isolated pile.

Maximum number of pile considered in problem was 10 and they were located on same line with 2d spacing. It was found that in case of unform halfspace reeduction of settlement of loaded pile was 3.5% and for case of two layer soil ($G_2=16G_1$) 6.7%. As soil reinforcement effect frequently attributed to discrepancies between different calculation methods or observation results additional investigation were performed.

Spatial problem of loading of pile in presence of 1 to 99 additional unloaded piles were solved by FEM. Calculations were performed for rigid piles with l/d=30 located by square grid with spacing of 3d. As before two cases of soil conditions were considered uniform and two layer soil with bottom layer 16 times more stiff.
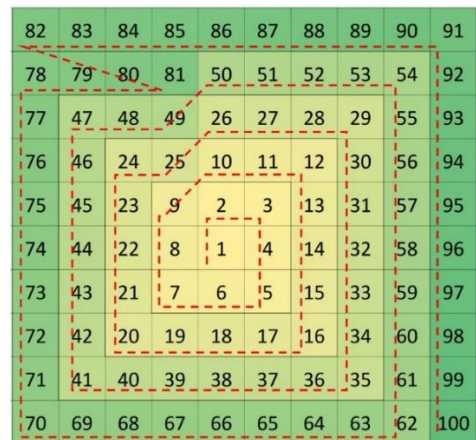

Figure 5. Layout of piles and sequence of activation.

Results are showing that maximum reduction for is 7.5% and 16.5% respectively.
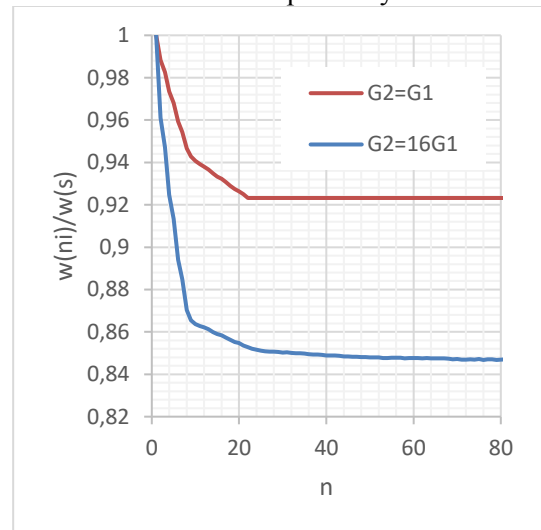

Figure 6. Reduction of single pile settlement in presence of unloaded piles.

Magnitude of reinforcing effect may allow to neglect it in practical calculations. Observed shape of lines suggests that there is a possibility to develop a relatively simple method to account for reinforcing effect.

2.3 Applicability of IFM for calculations of large groups of piles in uniform and two layer soil

Several publications, started with randolph … compared calculation results for pile groups performed by different methods, mostly by IFM with different influence functions.

Comparison of results between different formulations of IFM yeilds two-fold difference for large 400 piles group, and some results tend to fall below shallow foundations stiffnes. These effects even more pronounced for larger pile spacing.

In authors opinion these effects, at least partly, may be attributed to calculation of influence function by simple logarithmic function. While accurate for the inintial part of curve, function will fall below zero at some point, thus deviating from elasticity theory principles. Pile settlement according to Saint-Venant principle at some point will converge with Boussinesq (Boussinesq 1885) point load on surface problem solution.

Simple analisys of pile group geometry shows that for square group of 25^2 piles with l/d=20 at 3d spacing distance between 50% of piles exceeds 2 lengths of pile. For 25^2 pile group with l/d=50 and 6d spacing distance between 30% of piles exceeds 2l thus underestimating interaction between those piles. Simple solutions such as increase of single pile influence radius by adding radius of group may not produce accurate results as it will increase soil settlement at all parts of curve instead of end part.

To overcome mentioned deviaton from elastic solution new formula for soil settlement was proposed in (Боков and Федоровский 2017). To further simplify obtained formula it was updated to following form:

$$s_{ad} = \delta \frac{N}{G_1 l} \tag{1}$$

$$\delta = \frac{a}{\left(b + \frac{r G_2}{l G_1}\right)} \tag{2}$$

$$a = \frac{(1-v)}{2\pi} \tag{3}$$

$$b = (0.3392 - 0.2924v)(\frac{l}{d})^{-0.163} \tag{4}$$

Where s_{ad} (m) is additional settlement of unloaded pile, δ is influence function, a and b influence function coefficients, r (m) is distance from pile, l (m) is pile length, G_1 (MPa) soil shear modulus along length of pile, G_2 (MPa) soil shear modulus below pile bottom, d (m) pile diameter, v is Poisson ratio.
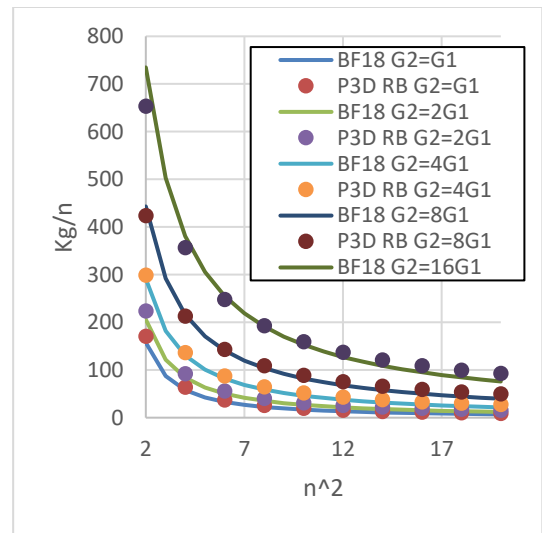

Figure 7. Comparison of IFM result with FEM.

For validation purposes results obtained by IFM with proposed formula were compared against calculation of pile groups by FEM. As it may be seen on fig. 7. Results show good convergence for all considered range further confirming results of reinforcing effect calculations.

To further validation results were compared to published results (Fleming op. 2009; Butterfield and Douglas 1981; Poulos and Davis 1980) (Guo 2012; Basile 1999; Viggiani 1998),for abstract pile group settlement problem. Results are plotted on fig. 8 and 9 for pile spacing s/d=2.5 and 5 respectively.
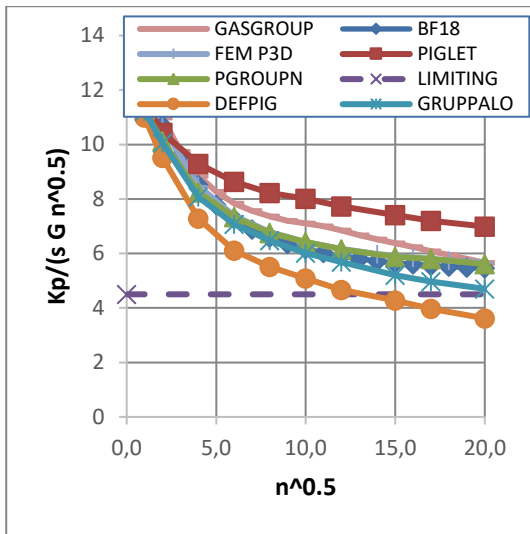
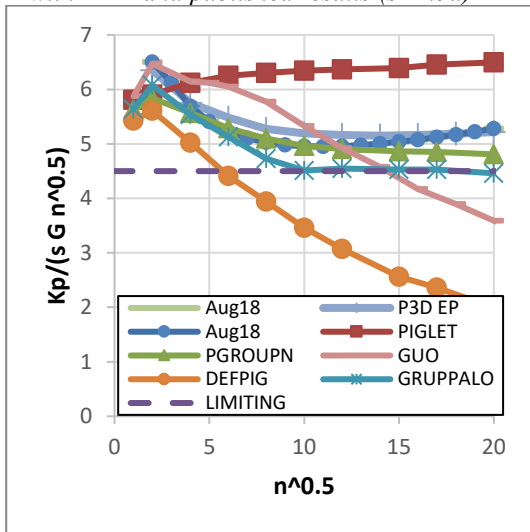*Figure 8. Comparison of pile group settlement by IFM with FEM and published results (s=2.5d)*

Figure 9. Comparison of pile group settlement by IFM with FEM and published results (s=5d)

2.4 Influence function for calculation of settlement of pile group containing piles of different length.

To study applicability of IFM for calculation of pile groups containing piles of unequal length investigation of problem were performed. It was hypothesised that interaction of piles of different length in elastic half-space conditions should follow Maxwell-Betti reciprocal work theorem, ,

stating that for a linear elastic structure subject to two sets of forces $\{Pi\}$ i=1,...,m and $\{Qj\}$, j=1,2,...,n, the work done by the set P through the displacements produced by the set Q is equal to the work done by the set Q through the displacements produced by the set P.

Proposed hypothesis was verified by FEM modelling of 2 pile problem for different pile length. It was found that while settlement of loaded piles are different, additional settlement of unloaded pile are same for both piles.

Based on this observation it was proposed to calculate additional settlement of unloaded pile of different length by taking l in formula 2 as

$$l' = \frac{l_1 + l_2}{2}$$

Where l_1 (m) is length of pile 1, l_2 (m) is length of pile 2.

As it can be seen, proposed approach gives satisfactory convergence of results (fig. 10, 11).
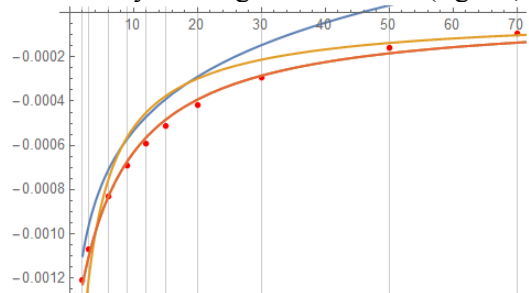

Figure 10. Additional settlement of soil due to interaction of piles of 40 and 50m. Points are FEM results. Red line - proposed approach.
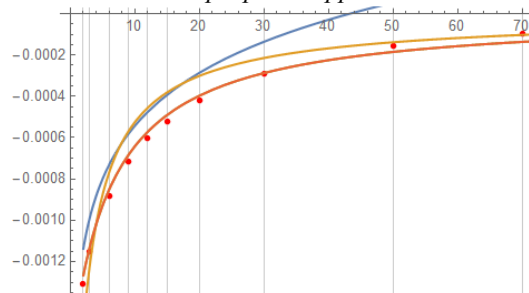

Figure 11. Additional settlement of soil due to interaction of piles of 10 and 60m. Points are FEM results. Red line - proposed approach.

To further validate proposed approach two abstract foundations vere considered.
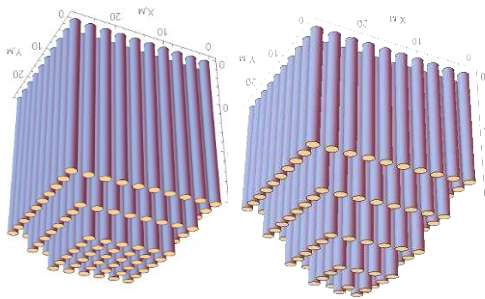
Figure 12. Abstract foundations calculated by FEM and IFM.

It was found that total stiffness of foundation calculated by IFM is 9…9.5 percent lower than calculated by FEM. Range of indinvidual stiffness difference is larger and is about ±40% of FEM result. It is worth mentioning that results of FEM calculation by itself show ±10% difference in individual stiffness between symmetrical piles which may be attributed to mesh features. FEM mesh have boundaries and comparison of results for unlimited half-space conditions with FEM will lead to overestimation of stiffness by FEM. Furthermore, in order to achieve half-space conditions in FEM one would need to extend boundaries to considerable distance, preferrably tens of pile length, but that would lead to increase of average element size and reduction of accuracy if special pile elements are used. Consideration of problem with such dimensions by conventional volume elements is out of the question. Model boundaries for performed calculations were at $8l$ from each side of foundation and special pile elements were used.

3 CONCLUSIONS

Interaction factors method is quick and affordable solution for calculations of pile groups.

It is shown that two pile interaction spatial problem requred to obtain influence function may be approximated by simplier axisymmetry problem by taking settlement of unloaded pile as setllement of soil from single pile problem solution at depth $z=0.75l$.

Analisys of reinforcing effect of pile group was performed. It is shown that settlement of single pile neighboured by 99 unloaded piles reduced by 7,5 to 16,5% for homogenious halfspace and for two layer soil with 16 times stiffer lower layer respectively.

To overcome difficulty in performing calculation of large pile group by IFM new influence function proposed. Comparison with published results and FEM shows good convergence.

Method for accounting for piles of different length is proposed. Verification of IFM results against FEM show good convergence.

4 PUBLICATION BIBLIOGRAPHY

Basile, Francesco (1999): Non-linear analysis of pile groups under general loading conditions. Doctoral. University of Glasgow, Glasgow.

Boussinesq, Joseph (1885): Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques: Gauthier-Villars, Imprimeur-Libraire.

Butterfield, Roy; Douglas, R. A. (1981): Flexibility coefficients for the design of piles and pile groups (108 Monograph).

ENDO, M. (1969): Negative skin friction acting on steel pipe pile in clay. In *Proc. 7th ICSMFE, Mexico, 1969* 2, pp. 85–92.

Fleming, W. G. K. (op. 2009): Piling engineering. 3rd ed. London, New York: Taylor & Francis Group.

Guo, Wei Dong (2012): Theory and Practice of Pile Foundations: CRC Press.

Mylonakis, G.; Gazetas, G. (1998): Settlement and additional internal forces of grouped piles in layered soil. In *Geotechnique 0016-8505* 48 (1), pp. 55–72. DOI: 10.1680/geot.1998.48.1.55.

Poulos, Harry G. (1968): Analysis of the Settlement of Pile Groups. In *Géotechnique* 18 (4), pp. 449–471. DOI: 10.1680/geot.1968.18.4.449.

Poulos, Harry G.; Davis, E. H. (1980): Pile foundation analysis and design. [S.l.], New York, N.Y.: Wiley(IS); Wiley (Wiley series in geotechnical engineering).

Randolph, Mark F. (1994): Design methods for pile groups and piled rafts. In *Proc. 13th ICSMGE* 5, pp. 61–82.

Randolph, Mark F.; Wroth, C. P. (1979): An analysis of the vertical deformation of pile groups. In *Geotechnique 0016-8505* 29, pp. 423–439. DOI: 10.1680/geot.1979.29.4.423.

Tomlinson, M. J.; Woodward, John (2015): Pile design and construction practice. Sixth edition. Boca Raton: CRC Press.

Viggiani, C. (1998): Pile groups and piled rafts behaviour. In *Proc. Deep Foundations on Bored and Auger Piles*, pp. 77–91.

Боков, Игорь Алексеевич; Федоровский, Виктор Григорьевич (2017): О расчете осадки группы свай с использованием коэффициентов взаимного влияния по модели упругого полупространства. In *Основания, фундаменты и механика грунтов* (6), pp. 2–8.

Федоровский, Виктор Григорьевич (1974): Расчет осадок свай в однородных и многослойных основаниях. Кандидатская диссертация. Кандидатская диссертация. НИИОСП им. Н.М. Герсеванова Госстроя СССР, Москва.