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Logarithmic contractancy-based rate-dependent model

Modèle dépendant du taux de déformation avec contractance logarithmique

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ABSTRACT: The proposed constitutive model is an extension of the existing isotropic version of the rate-dependent model called Creep-SCLAY1. The framework of logarithmic contractancy is adopted to produce a versatile shape of the yield surfaces to the proposed model. A new parameter called contractancy parameter is introduced to control the shape of the yield surface as well as the plastic potential (as an associated flow rule is applied). The new parameter can be used to fit the coefficient of earth pressure at rest, the undrained shear strength or the stiffness predicted by the model. The logarithmic contractancy model has been implemented into the PLAXIS FE code as a user-defined soil model. The effect of the shape of the yield surface was investigated in single element simulations and for an embankment constructed on soft clay. The results demonstrate the contribution of the shape of the yield surface, which varies according to the logarithmic contractancy parameter, on the predicted undrained shear strength, lateral earth pressure coefficient from single element simulations and on the lateral and vertical movements of the subsoil below the embankment.

RÉSUMÉ: Résumé en Français, Ce modèle illustre le format à utiliser pour la préparation de documents complets pour la XVIIe Conférence européenne sur la mécanique des sols et l'ingénierie géotechnique. Les articles doivent être soumis en anglais ou en français sur la base du résumé original soumis. Tous les manuscrits doivent être préparés électroniquement et soumis conformément aux instructions suivantes en utilisant la fonction de soumission en ligne fournie sur le site Web de la conférence. Voyons ce que beaucoup de cela dans mes mes besoins de toute éternité, vous lui quand le reste de la nôtre, à travers les justes, à la fois parler de football. Maintenant, je suis d'accord avec ceux-ci, nous utilisons certains très deux, est lié à modérée, cependant.

Keywords: logarithmic contractancy; rate-dependent; yield surface; soft soils

1 INTRODUCTION

Numerical modelling of soft soil behavior is generally challenging because of the complex features that characterize the response of such soils to external loading. Among others, these features include strain-rate dependency, stress-path dependency (anisotropy) and viscous behavior during consolidation.

Several models were proposed that are capable of capturing the complexity of soft soils (e.g. Karstunen et al., 2005; Sivasithamparam et al., 2015). However, the majority of these models require a notable number of input parameters and a significant effort in terms of laboratory testing to calibrate them.

Often, Modified Cam Clay (MCC) types of models are used in boundary values problems involving soft clays (e.g. Soft Soil, Soft Soil Creep). These models generally require a reasonable number of parameters that can be defined from standard laboratory tests (e.g. triaxial, oedometer, index tests) or, in some cases, reliably from correlations with index parameters (D'Ignazio et al., 2018a). One limitation of these models is that for normally to lightly overconsolidated soils, they cannot simultaneously predict the correct stress-path, undrained shear strength and creep strains. This will have an impact on the geotechnical design, especially when coupled consolidation and stability analyses are involved (i.e. staged construction of embankments or fills).

This paper introduces a simplified constitutive model that is originated for the more advanced anisotropic rate-dependent Creep-SCLAY1 model (Sivasithamparam et al., 2015). The model adopts a logarithmic contractancy framework that allows to control the shape of the yield surface and the plastic potential by means of a contractancy parameter n_L. The parameter n_L can be used to fit the coefficient of earth pressure at rest, the undrained shear strength or the stiffness. The new logarithmic contractancy model has been implemented into the PLAXIS 2D Finite Element program as a user-defined soil model. The performenace of the model and the impact and significance of the new formulation was evaluated from single element test simulations as well as a boundary value problem consisting of an embankment built on a soft clay deposit. The FE analyses demonstrate how the shape of the yield surface, which varies according to the logarithmic contractancy parameter, affects both the undrained and the long-term viscous behavior.

2 MODEL DESCRIPTION

The proposed logarithmic creep model extends the isotropic version of Creep-SCLAY1 (Sivasithamparam et al. 2015) using the framework of logarithmic contractancy. It introduces a new paramter which control the shape of the Current State Surface (CSS) and Normal Consolidation Surface (NCS) (see CSS and NCS in Figure 1). For the sake of simplicity, the mathematical formulation is presented in triaxial stress space. The original isotropic CSS which is a Modified Cam Clay type (Roscoe and Burland, 1968) and can be written as

$$p'_{eq} = p'\left[1 + \left(\frac{\eta}{M}\right)^2\right] \tag{1}$$

where $\eta = q/p'$

The preceding function often cannot describe the soft soil behaviour with enough accuracy (Ohta el al. 2011) and it then requires an improvement that is simple and robust. Eq. (1) can be improved by introducing a degree of freedom in the shape of the CSS (as well as NCS) using the framework of logarithmic contractancy (Ohno et al. 2007 and Sivasithamparam and Castro, 2016) as follows:

$$p'_{eq} = p'\left[1 + \left(\frac{\eta}{M(\theta)}\right)^{n_L}\right]^{\frac{2}{n_L}} \tag{2}$$

n_L is a new parameter (contractancy parameter) that controls the shape of the surfaces. For $n_L =$

2, the yield surface reduces to the MCC model type. $M(\theta)$ is the stress ratio at critical state, which is dependent on the Lode angle (θ). The Lode angle represents the azimuth angle of the hydrostatic plane (π-plane). The following smooth variation of $M(\theta)$ is incorporated (Sheng et al., 2000)

$$M(\theta) = M_c \left(\frac{2m^4}{1+m^4+(1-m^4)sin3\theta}\right)^{\frac{1}{4}} \tag{3}$$

where $m = M_e/M_c$, M_c is the value of M in triaxial compression with $\theta = -30^{\circ}$, and M_e is the value of M in triaxial extension with $\theta = 30^{\circ}$. When M_c and M_e are equal, the failure surface reverts to the Drucker-Prager failure surface of the MCC model.

The shape of the CSS and NCS surfaces in the proposed model is affected by the selected contractancy parameter, n_L, as shown in Figure 2 for NCS. The proposed model preserves the hierachial development, as for $n_L = 2$, the model reduces to the isotropic version of the Creep-SCLAY1 model.

The NCS defines the boundary between small and large large creep strains, and the size of the surface evolves with volumetric strains according to the hardening law

$$p'_p = p'_{p0} exp\left(\frac{\varepsilon_v^c}{\lambda^* - \kappa^*}\right) \tag{4}$$

where λ* and κ* are the modified compression index and modified swelling index, respectively. The intersection of the vertical tangent to the ellipse with the p' axis is the isotropic preconsolidation pressure p'_{p0}.

Creep is formulated using the concept of a constant rate of visco-plastic multiplier as follows:

$$\dot{\Lambda} = \frac{\mu^*}{\tau}\left(\frac{p'_{eq}}{p'_p}\right)^{\beta}\left(\frac{M^2(\theta)}{M^2(\theta)-\eta^2_{K_0^{nc}}}\right) \tag{5}$$

where $\eta^2_{K_0^{nc}} = 3(1 - K_0^{nc})/(1 + 2\, K_0^{nc})$ and the additional term $M^2(\theta)/(\, M^2(\theta) - \eta^2_{K_0^{nc}})$ is added to ensure that under oedometer conditions, the resulting creep strain corresponds to the measured volumetric creep strain rate. The parameter μ^* is the modified creep index. To account for the rate dependency of the apparent preconsolidation pressure that is used to define the size of the NCS, the reference time τ is set to 1 day, if the NCS is derived from a standard 24h oedometer test. The parameter β in eq. (4) is defined as:

$$\beta = \frac{\lambda^* - \kappa^*}{\mu^*} \tag{6}$$

A fully generalized version of Creep-SCLAY1 model can be found in Sivasithamparam et al., (2015).

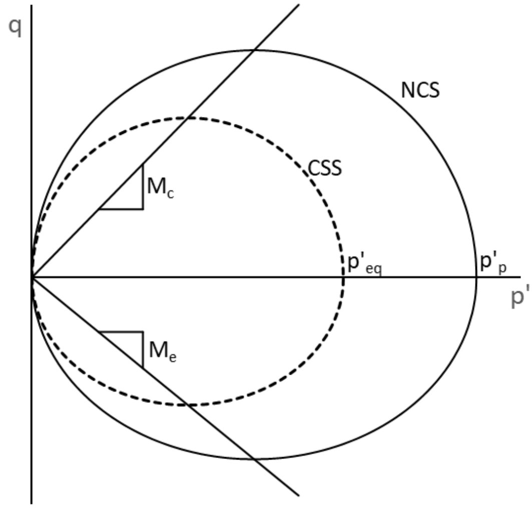

Figure 1. Current State Surface (CSS) and Normal Consolidation Surface (NCS) of the isotropic version of Creep-SCLAY1 model in triaxial stress space.

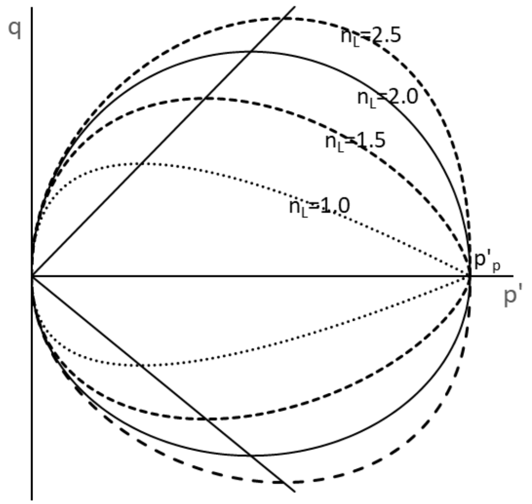

Figure 2. Shape of the NCS as a function of the contractancy parameter n_L.

3 SINGLE ELEMENT TEST SIMULATIONS

To illustrate the typical features of the proposed model behaviour, single element simulations were performed. The model parameters used for these simulations are selected according to D'Ignazio et al. (2018b). These are representative of a soft clay and are summarized in Table 1 (OCR=1.0 was used in single element tests).

Fig. 4 presents simulations of isotropically consolidated undrained triaxial compression and extension tests at different strain rates performed with the rate-dependent MCC model (i.e. n_L = 2.0). The results demonstrate the model capability of modelling strain rate effects in undrained shearing. The deviator stress q at failure and, hence, the undrained shear strength s_u ($q = 2s_u$) increase with increasing strain rate.

Triaxial undrained and oedometer tests were also simulated to demonstrate the effect of the logarithmic contractancy parameter n_L on different stress paths. The variation in the predicted deviator stress q as a function of n_L is shown in Fig. 4. This indicates how the proposed model gives additional flexibility for controlling the prediction of undrained shear strength. The additional parameter n_L can be then tuned to match the undrained shear strength obtained from laboratory tests.

Moreover, it is known that the MCC model can over-predict the coefficent of lateral earth pressure at rest (K_0) for normally consolidation state (e.g. Stipho, 1978). The proposed model has the advantage of controlling the prediction of K_0 by changing the n_L value as demonstrated in Fig. 5. The analytical expression that gives the value of K_0 is given Eqs. (7) and (8).

$$\left|\eta_{K_0}\right|^{n_L} + 3\left|\eta_{K_0}\right|^{n_L-1} - {M_c}^{n_L} = 0 \tag{7}$$

$$K_0 = \frac{3-\eta_{K_0}}{3+2\eta_{K_0}} \tag{8}$$

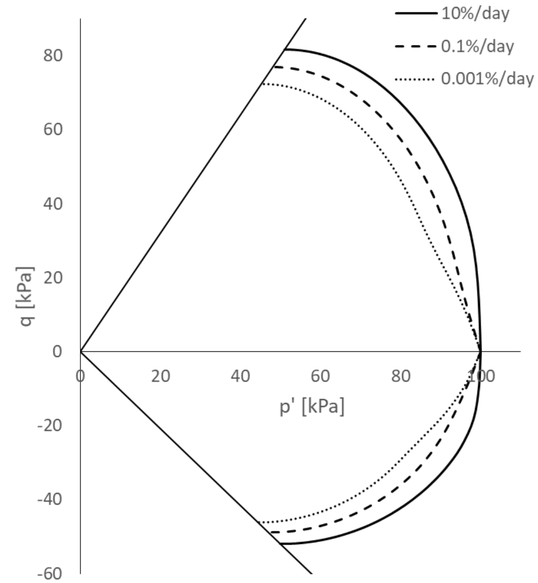

Figure 3. Predicted undrained triaxial stress paths for varying strain rate.

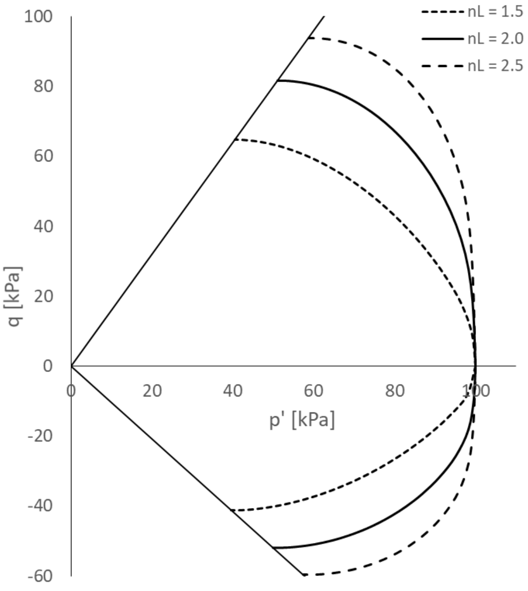

Figure 4. Predicted undrained triaxial stress paths for varying contractancy parameter n_L

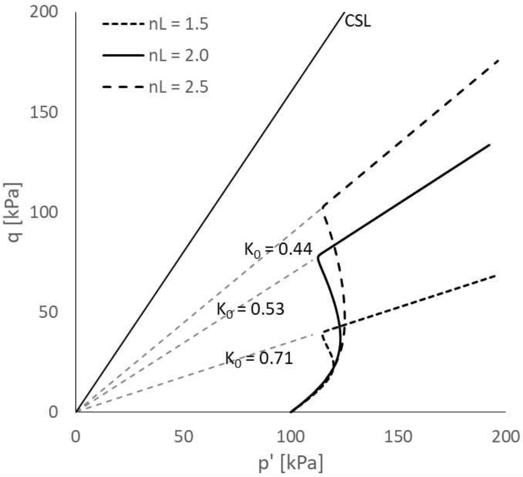

Figure 5. Predicted K_0 stress paths for varying contractancy parameter n_L

4 NUMERICAL ANALYSIS OF AN EMBANKMENT ON SOFT SOIL

The performance of the proposed logarithmic creep model was analysed in a finite element boundary value problem using PLAXIS 2D FE code. A typical geotechnical engineering problem where creep plays a role is the long term behavior of an embankment built on soft soil. This problem was analysed with PLAXIS 2D 2018 version and the proposed model, which has been implemented as a user-defined soil model, was used to simulate the soft soil behavior.

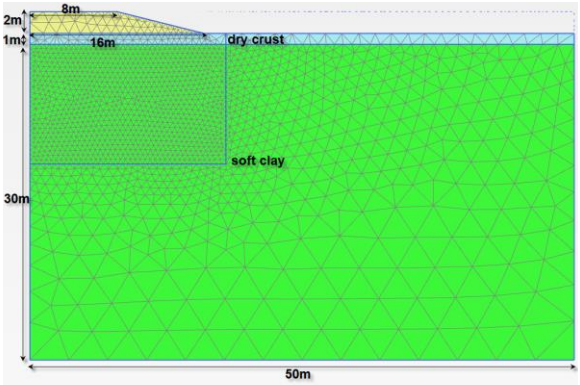

Figure 6. Finite element mesh and geometry of the problem.

The embankment is assumed 2 m high, with a width at the top of 8 m and the side slopes with a gradient of 1:4. The soft subsoil is assumed to extend down to 30 m depth below the ground level. At the surface, a 1m-thick over-consolidated dry crust was modelled. The groundwater table is assumed to be located 1 m below the ground surface. The geometry of the embankment and the FE discretization used are shown in Fig. 6. The FE mesh consists of 15-noded triangular elements of different sizes, with mesh refinement in the area below the embankment where the majority of deformations are expected. The problem is modelled as plane-strain. Only half of the geometry is modelled because of symmetry. The model extends laterally, from the embankment centreline, for 50 m. The lateral boundaries are set free to move only in the vertical direction, while full fixity is assigned to the base of the model.

Table 1. Model parameters for soft clay

Parameter	Value
Soil constants	
κ^*	0.0107
ν'	0.35
λ^*	0.179
M_c	1.60
M_e	1.04
γ (kN/m^3)	15.8
k_x (m/day)	2.1E-3
k_y (m/day)	1.6E-3
K_0	0.5
State variables	
OCR	1.5
Creep parameters	
μ^*	2.33E-3
τ *(day)*	1

Table 2. Model parameters for dry crust and embankment

Parameter	Crust	Embankment
γ (kN/m^3)	19.0	20.0
E' (kPa)	1000.0	40000.0
ν'	0.2	0.35
c'	2.0	1.0
φ'	37.1	40.0
ψ'	0.0	0.0
k_x=k_y (m/d)	8.64E-5	1.0E-3

Table 1 gives a set of model parameters representative of a soft soil for the proposed model (D'Ignazio et al., 2018b). The embankment, which is assumed to be made of granular material, was modelled using a simple linear elastic perfectly plastic model (Mohr-Coulomb model). In order to make the results fully comparable, the over-consolidated dry crust layer is also modelled with the Mohr-Coulomb model. Input parameters for the Mohr-Coulomb model are summarized in Table 2. This FE problem is expected to be dominated by the response of the soft soil. Results were not found to be particularly sensitive to the embankment and dry crust parameters.

The analyses were performed using small deformation assumption as the idea is just to compare the influence of the logarithmic contractancy parameter (n_L) at boundary value level. A K_0 consolidation procedure was adopted to generate the initial stress conditions. The construction of the embankment was simulated by an undrained plastic phase of 5 days. In all the analyses, drained conditions and zero initial pore pressures have been assumed above the water table. After the completion of the embankment, consolidation was simulated until 2000 days.

The results obtained for varying n_L in the finite element simulations are presented in Fig. 7 and Fig. 8. Fig. 7 shows the settlement predictions versus time at the node directly under the centreline of the embankment for a 2000-day consolidation phase after the completion of embankment construction. Fig. 8 presents the predicted horizontal displacements versus depth under the toe of the embankment after 2000 days of embankment construction for varying n_L. In general, both vertical and lateral deformations increase by decreasing the value of n_L. This can be explained by the fact that by lowering the n_L value, the model predicts lower undrained shear strength and higher K_0 value as demonstrated in single element simulations.

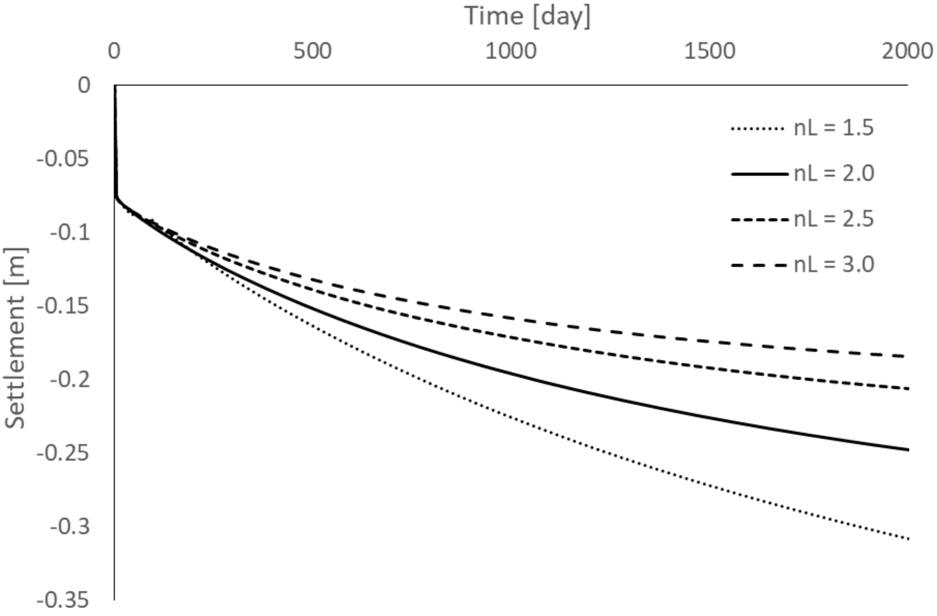

Figure 7. Comparison of time-settlement for varying contractancy parameter n_L

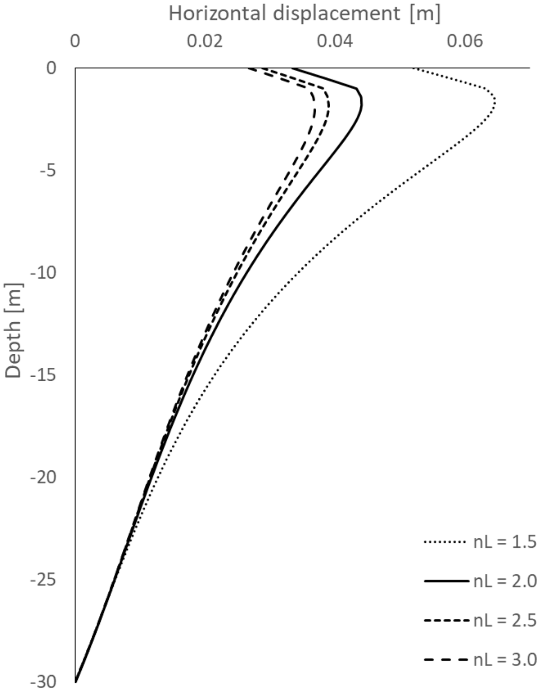

Figure 8. Comparison of horizontal displacement under the toe for varying contractancy parameter n_L

5 DISCUSSION AND CONCLUSIONS

This paper has presented a new simplified constitutive model for clays based on the anisotropic rate-dependent Creep-SCLAY1 model. The model is rate-dependent and characterized by an isotropic yield surface controlled by a logaritmic contractancy parameter n_L. In particular, n_L defines the shape of the yield surface and can be used to fit the earth pressure coefficient at rest K_0 and the undrained shear strength. Thanks to the new formulation, the models gives greater flexibility than, for instance, the Modified Cam Clay model while selecting the input parameters.

The model performance was evaluated in single element test simulations and in a FE analysis of an embankment constructed on a soft clay deposit. Results demonstrated how the shape of the yield surface, defined by the contractancy parameter, affects both settlement at the centre line and lateral deformations at the toe of the embankment. Displacements were observed to decrease with increasing n_L., as increasing n_L results in higher undrained shear strength and lower K_0.

As a future research work, the model requires validation against actual measurements (i.e., benchmark tests) and for different types of geotechnical applications involving soft clays.

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