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Bearing capacity of shallow foundations that are situated at a varying distance from slopes

Capacité portante des fondations superficielles situées à une distance variable des pentes

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ABSTRACT: Shallow foundations can be placed adjacent to slopes. It may also be necessary to perform open excavation at a varying distance from shallow foundations. Quantifying the effect of the slope at a certain distance from the foundation edge is then a subject of interest. In this paper, bearing capacity formulations that have been developed and taught for several years at NTNU are extended for accommodating effect of slopes situated at varying distances from the edge of the foundation. Both drained and undrained conditions are considered.

RÉSUMÉ: Des fondations superficielles peuvent être placées à proximité de pentes. Il peut également être nécessaire de procéder à une excavation à ciel ouvert à une distance variable des fondations superficielles. Quantifier l'effet de la pente en fonction de la distance au bord de la fondation est alors un sujet d'intérêt. Dans cet article, les formulations de capacité portante qui ont été développées et enseignées plusieurs années à NTNU sont étendues pour tenir compte de l'effet des pentes situées à différentes distances du bord de la fondation. Les conditions drainées et non drainées sont prises en compte.

Keywords: Bearing capacity; Shallow foundations

1 INTRODUNCTION

Evaluating the bearing capacity of foundations adjacent to slopes has practical significance. The problem can well be analyzed using strip line method (eg. Wang, *et al*., 2008) or other advanced methods such as the finite element and the finite difference methods (eg. Georgiadis, 2010, Farzaneh *et al*., 2013, Ahmadi and Asakereh, 2015). However, limit equilibrium methods are still widely in use by the practicing engineer. There have been several attempts to include effect of adjacent slopes into limit equilibrium theories (Meyerhof, 1957, Hansen 1970, Vesic 1975). In this paper, we are going to formulate limit equilibrium-based bearing capacity formulas considering drained and undrained conditions for foundations placed at a varying distance from a slope. For the same purpose, classical bearing capacity formulations that are developed and taught at the Norwegian University of Science and Technology (NTNU) for several years are considered (Grande and Emdal, 2010).

2 DESCRIPTION OF STRESS CONDITION ON ARBITRATY PLANES USING MOHR CIRCLES

The NTNU approach starts by describing the stress condition in a Mohr circle for drained and undrained conditions. The drained shear strength of soils is assumed to be effective stress dependent and constant in the case of the later. We shall introduce the formulations briefly such that developments are followed with ease.

2.1 Effective stress space (drained analysis, aϕ analysis)

Consider the normalized Mohr circle in Figure 1. The shear stress on the ω-plane is given by

$$\tau_\omega = r\tan\varphi N_{\omega\pm}\left(\sigma'_{\substack{3\\1}} + a\right) \tag{1}$$

in which, making use of trigonometric relations, the stress ratio $N_{\omega\pm}$ can be described as

$$N_{\omega\pm} = \frac{\sigma'_\omega + a}{\sigma'_3 + a} = \frac{(1+f_\omega^2)N_\pm}{1+f_\omega^2 N_\pm}, \tag{2}$$

where

$$f_\omega = \frac{\tan\omega}{\tan\alpha_{c+}} = \frac{1}{r}\left(1-\sqrt{(1-r^2)}\right) \tag{3}$$

$$N_\pm = \tan^2\alpha_{c\pm};\ \alpha_{c\pm} = \frac{\pi}{4} \pm \frac{\varphi}{2}. \tag{4}$$

$\sigma'_{\substack{3\\1}}$ is the minor (major) principal effective stress, φ is the mobilized friction angle, a is called attraction and is related to cohesion, c, as $a = c\cot\varphi$, r is the mobilized roughness ratio.

2.2 Total stress space (undrained analysis, c_u analysis)

Consider the Mohr circle in Figure 2. The stress state at any arbitrary plane described by angle ω either with respect to the minor principal stress σ_3 or with respect to the major principal stress σ_1 respectively is given as

$$\sigma_\omega = \sigma_{\substack{3\\1}} \pm \frac{2}{1+f_\omega^2}\tau_c, \tag{6}$$

$$\tau_\omega = r\tau_c, \tag{7}$$

in which τ_c is the critical shear σ_ω is the total stress normal to an arbitrary ω-plane and r is the roughness ratio. Here too, f_ω is related to the roughness ratio r as given in Equation (3).

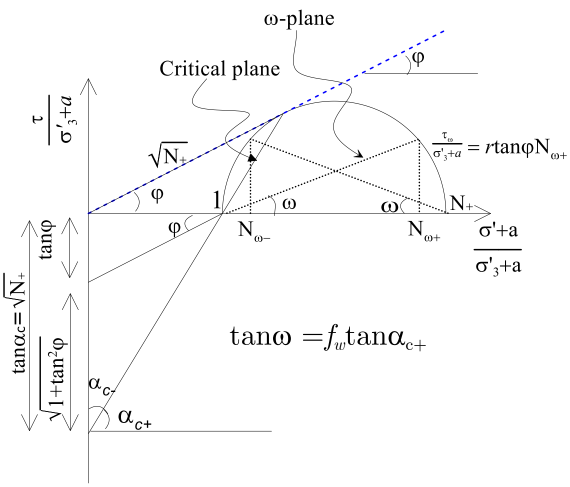


Figure 1. Normalized Mohr-circle for aϕ analysis.

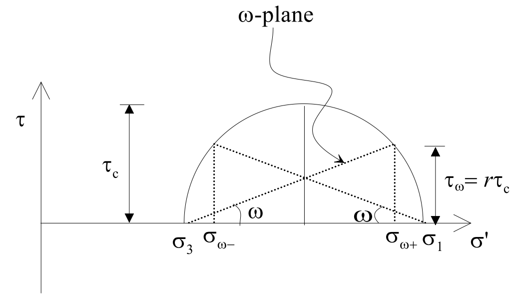


Figure 2. Mohr-circle for c_u analysis.

3 PROPOSED APPROACH

3.1 Drained (aφ analysis)

Let a strip foundation of width B be placed at a distance s from a slope that makes a slope angle of β with the horizontal.

We begin assuming that the load is centric and inclined. Due to the horizontal component of the load, the foundation plane will no longer be a principal stress plane. The principal stress direction will be rotated by an angle ω which is a function of the roughness ratio r and the mobilized shear strength. The passive Rankine elements at the side of the sloping ground are assumed to be rotated by an angle β' to the horizontal. As a result, the Prandtl zone will have an opening angle of $\frac{\pi}{2} - \omega - \beta'$. See Figure 3.

We shall then consider the equilibrium of the three zones.

Zone III

The normal stress σ'_{cb} is assumed to act in a critical stress plane and is found by setting $r = 1$ (and hence $f_\omega = 1$) as

$$\frac{\sigma'_{cb}+a}{\bar{p}'_0+a} = \frac{2N_+}{1+N_+}\,, \qquad (8)$$

in which,

$$\bar{p}'_0 = \left\{\frac{W}{B}\frac{\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\sin\left(\frac{\pi}{2}-\varphi\right)}{\sin\left(\frac{\pi}{2}+\varphi\right)\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}-\omega\right)}e^{-\left(\frac{\pi}{2}-\omega-\beta'\right)}\right\}\cos\beta'\,, \qquad (9)$$

where W is the weight of the soil wedge on top of zone III assuming that it is stable. It is wise to check the wedge stability using, for example, the wedge method of stability analysis. If the wedge is not stable, then it is indicative that a serious slope stability analysis needs to be done.

Note also that $\bar{p}'_0$ is not entirely a minor principal stress although for the sake of simplicity we are regarding it as one. The minor principal stress may be found by considering the shear stress acting the β'- plane and further rotating the passive element.

Zone II

σ'_{cb} is found from moment equilibrium about point O,

$$\sigma'_{ca} + a = e^{2\left(\frac{\pi}{2}-\omega-\beta'\right)\tan\varphi}(\sigma'_{cb} + a), \qquad (10)$$

in which the property of the log spiral is considered.

Zone I

Plane a is a critical plane. Therefore:

$$\sigma'_{cb} + a = \frac{2N_+}{1+N_+}(\sigma'_3 + a)\,. \qquad (11)$$

Combining Equations (8), (9), (10) and (11) the bearing capacity factor, N_q, for a strip foundation on a weightless soil is written as

$$N_q = \frac{\sigma'_v+a}{\bar{p}'_0+a} = \frac{(1+f_\omega^2)N_+}{1+f_\omega^2 N_+}e^{(\pi-2\omega-2\beta')\tan\varphi}. \qquad (12)$$

For $\beta' = 0$, the original bearing capacity factor is recovered. Therefore, $e^{-2\beta'\tan\varphi}$ is the new factor introduced for taking into account effect of a slope at a distance, s from the slope.

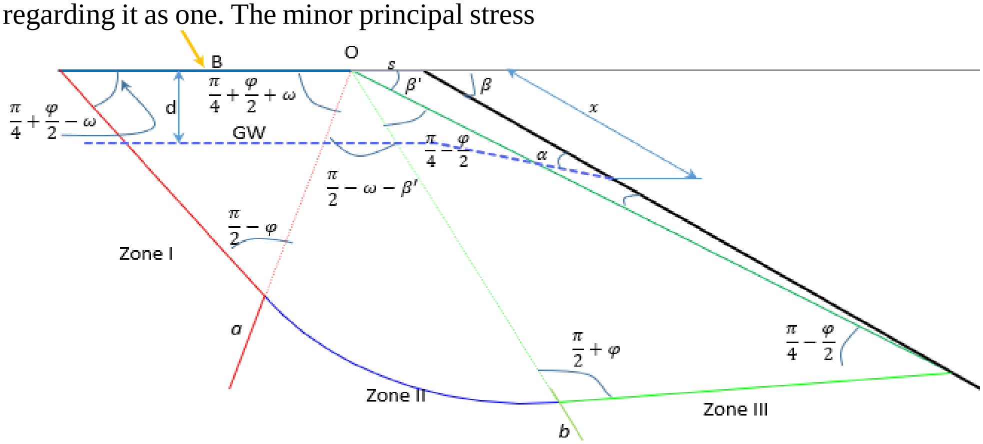


Figure 3. Assumed bearing capacity mechanism for a ϕ bearing capacity analysis.

We will complete our task by specifying β'. From trigonometric relations, the following relation can be derived:

$$\sin(\beta-\beta')\,e^{-\beta'\tan\varphi} = \frac{s}{B}\frac{\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\sin\left(\frac{\pi}{2}-\varphi\right)\sin(\beta)}{\sin\left(\frac{\pi}{2}+\varphi\right)\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}-\omega\right)}\,e^{\left(-\frac{\pi}{2}+\omega\right)\tan\varphi}. \quad (13)$$

Equation (13) is nonlinear and does not have analytical solution. This can be solved numerically. Note the following special conditions:

- If $\beta = 0$, then $\beta' = 0$ is obtained,
- for $s = 0$, $\beta' = \beta$
- The slope has no influence when the foundation setback is greater or equal to

$$s = B\,\frac{\sin\left(\frac{\pi}{2}+\varphi\right)\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}-\omega\right)}{\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\sin\left(\frac{\pi}{2}-\varphi\right)}\,e^{-\left(\frac{\pi}{2}-\omega\right)\tan\varphi}. \quad (14)$$

Note also that if the foundation lies at a distance s from the bottom of the slope, using negative value for β may lead to the corresponding modification.

So far, we obtained the solution assuming a weightless soil. For practical cases, the effect of soil weight needs to be taken into account. In so doing, the bearing capacity of a shallow strip foundation may be written as

$$\sigma'_{vn} = \left(N_q-1\right)(\bar{p}'_0+a)+\frac{1}{2}\bar{\gamma}N_\gamma B_0\,, \quad (15)$$

where $\sigma'_{vn} = \sigma'_v - \bar{p}'_0$, N_γ is the bearing capacity that takes into account effect of soil density, $\bar{\gamma}$. B_0 is the effective foundation width. Equation (15) is thus completed by describing the bearing capacity factor, N_γ. Pure analytical solution does not exist for taking into account effect of soil density and we are not attempting to give one here. In the NTNU bearing capacity formulation, the weight factor is approximated by

$$N_\gamma = 2d_0\left(N_q-1\right), \quad (16)$$

where in

$$d_0 = f_{\beta'}\frac{\sin(\alpha_{c+}-\omega)}{1.25(2-r)}\,e^{(\alpha_{c+}-\omega)\tan\varphi}, \quad (17)$$

in which $f_{\beta'}$ is the modified correction factor. The factor $f_{\beta'} = (1-0.4\tan\beta')^5$ modified from Døssland (1980) may be considered.

Next, we shall demonstrate the bearing capacity formulae so far presented with varying slope angles and setback distances. For the same, a strip foundation of 4 m width is considered. We also assume a foundation soil with an effective friction angle of 33 degrees and a cohesion of 1 kPa. A partial factor of 1.25 is also considered. In Figure 4, the vertical bearing capacity is plotted against setback distance for a varying roughness ratio. It can be seen that the bearing capacity is smoothly (and nonlinearly) interpolated between the minimum (for a setback of 0) and the maximum (for a setback where the slope has no more influence). In Figure 5, the interaction diagram is presented for various mobilized friction angles for a slope angle of 30 degrees and a foundation placed at 2 m from the slope. Furthermore, interaction curves presented in Figure 6a&b show the variation of bearing capacity with setback distance and slope angle respectively. It should be remarked that Equation 9 may require some appropriation such that the resulting bearing capacity is less or equal to bearing capacity on flat terrain. In Figure 7, the rapture mechanism is presented for varying setback distances and a fixed slope angle of 30 degrees. $f_{\beta'} = 1$ is considered in this illustrative example.

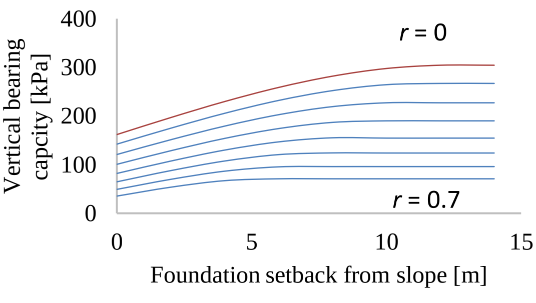


Figure 4. Vertical bearing capacity with foundation setback for different values of the roughness ratio.

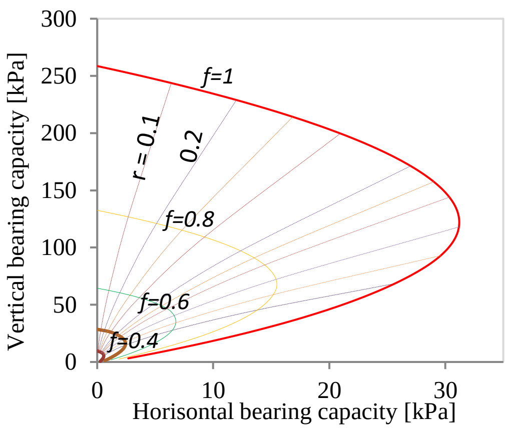
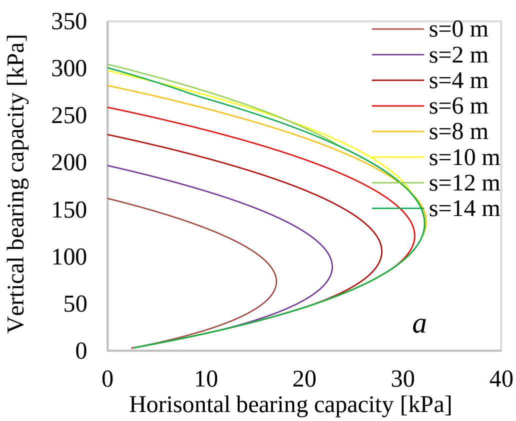


Figure 5. Interaction diagrams for different degrees of shear strength mobilizations, f= tan(ϕ)/$\gamma_{M,}$ 2 m foundation setback and a slope angle of 30 degrees.



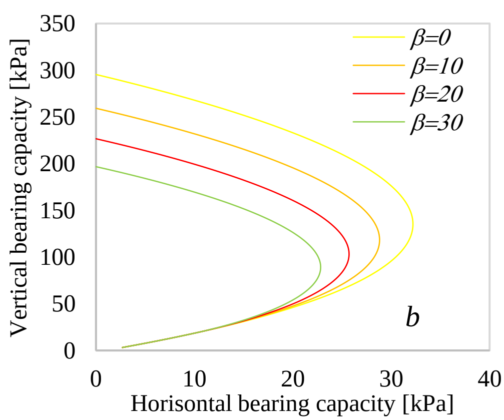


Figure 6. Interaction curves for various foundation a) setbacks and b) slope angles.

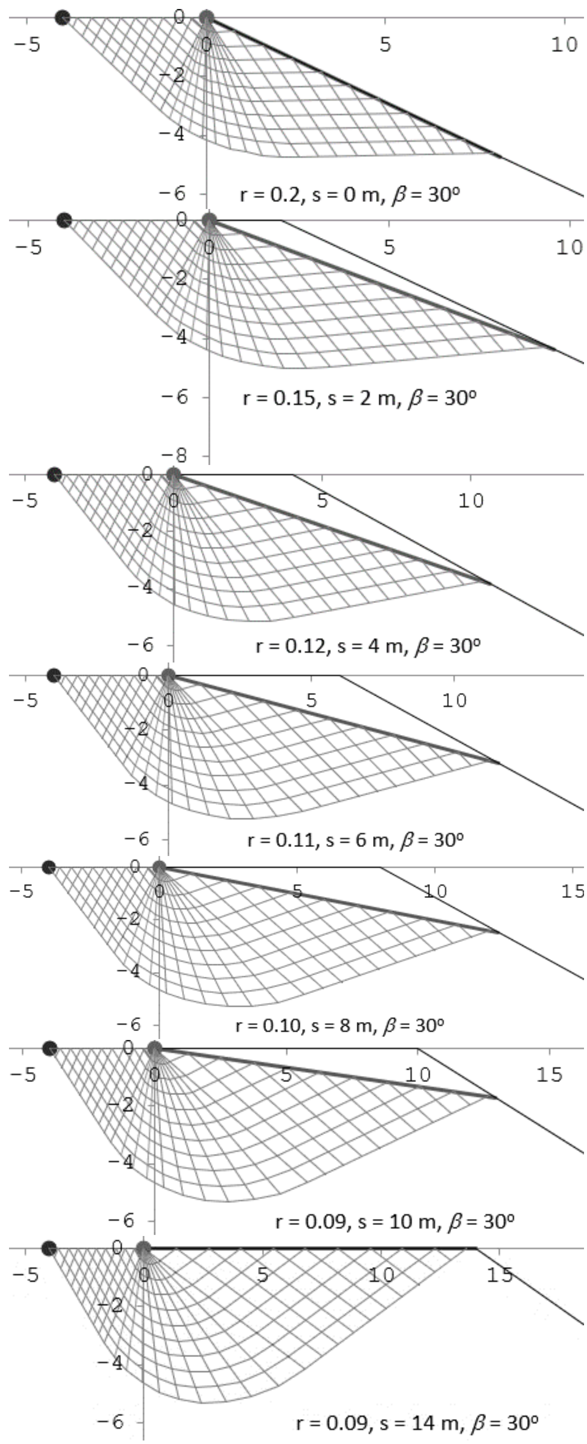


Figure 7. Rotation of the rapture surface for increasing foundation setback.

3.2 *Undrained (c_u analysis)*

Let a strip foundation of width B be placed at a distance s from a slope that makes a slope angle of β with the horizontal. Let the soil have an undrained shear strength of c_u.

Here too, we begin by considering centric and inclined foundation load. For such a case, the foundation plane will no longer be a principal stress plane. The principal stress direction will be rotated by an angle ω which is a function of the roughness ratio r and the mobilized shear strength. The passive Rankine elements at the side of the sloping ground are assumed to be rotated by an angle β' to the horizontal. As a result, the Prandtl zone will have an opening angle of $\frac{\pi}{2} - \omega - \beta'$. See Figure 7.

We shall then consider the equilibrium of the three zones.

Zone III

The normal stress σ_{cb} is assumed to act in a critical stress plane and is found by setting $r = 1$ (and hence $f_\omega = 1$)

$$\sigma_{cb} = \bar{p} + \tau_c, \tag{18}$$

$$\bar{p} = \left\{\frac{W}{B}\frac{\sin\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}-\omega\right)}\right\}\cos^2\beta', \tag{19}$$

W is the weight of the wedge that contains β'. Again, here we are assuming the wedge is stable.

Zone II

σ_{ca} is found considering moment equilibrium about point O as

$$\sigma_{ca} = \sigma_{cb} + 2\left(\frac{\pi}{2} - \omega - \beta'\right)\tau_c, \tag{20}$$

$$\sigma_{ca} = \sigma_3 + \tau_c . \tag{21}$$

Zone I

Plane a is a critical plane. Therefore:

$$\sigma_v = \sigma_3 + \frac{2}{1+f_w^2}\tau_c . \tag{22}$$

Combining Equations (18), (19), (20) and (22) one finds

$$\sigma_v = p + N_c\tau_c \tag{23}$$

where,

$$N_c = 2\left(\frac{\pi}{2} - \omega - \beta' + \frac{1}{1+f_w^2}\right) \tag{24}$$

is the bearing capacity factor for undrained condition.

The angle β' is then derived from geometric relations as

$$\sin(\beta - \beta') = \frac{s}{B}\frac{\sin\left(\frac{\pi}{4}\right)\sin(\beta)}{\sin\left(\frac{\pi}{4}-\omega\right)}. \tag{25}$$

Equation (25) is a degenerate of Equation (13) and can also be derived from Equation (13) by setting $\varphi = 0$.

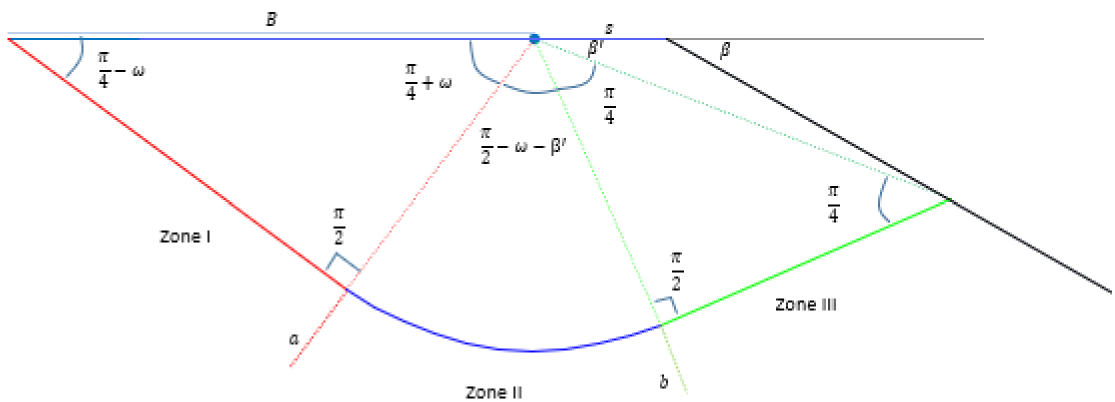


Figure 8. Assumed bearing capacity mechanism for c_u bearing capacity analysis.

Then, the following special conditions hold.

- If $\beta = 0$, then $\beta' = 0$ is obtained,
- for $s = 0,\ \beta' = \beta$
- The slope has no influence when the foundation setback is greater or equal to

$$s = \sqrt{2}\sin\left(\frac{\pi}{4} - \omega\right) B. \qquad (26)$$

According to Equation (26), the rapture mechanism does not span greater than the foundation width, B, from the edge of the foundation.

With the angle β' solved, the solution we wish to find is completed. The result can be enhanced by taking into account effect of depth and area of the foundation and the weight of the soil. These are not considered here. We shall now present some illustrative interaction curves. A strip foundation of 4 m width placed on a soil with characteristic undrained shear strength of 50 kPa is considered. The design shear strength is calculated assuming a partial factor of 1.4. The various plots shown next are plots that illustrate effect of degree of mobilization of undrained shear strength, Figure 9, foundation setback from the slope surface, Figure 10 and interaction curves for different foundation setbacks and slope angles, Figure 11. Furthermore, the rapture surface is illustrated considering a vertically loaded foundation of 4 m width placed at various distances from a slope of angle 30 degrees, Figure 12.

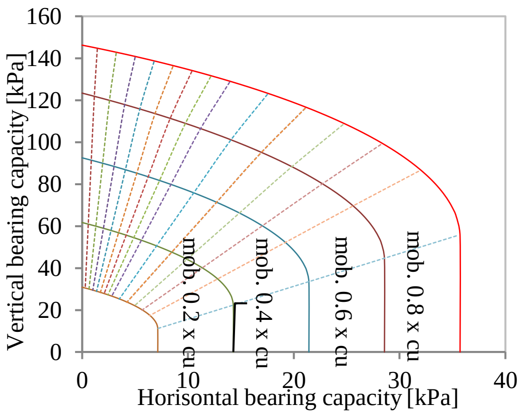


Figure 9. Interaction plots for various shear strength mobilizations for 0 m foundation setback, 30 degrees slope angle.

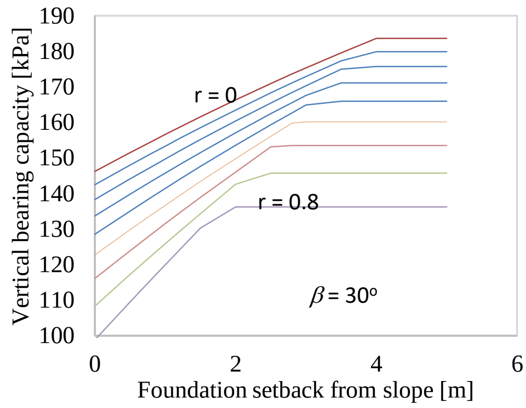


Figure 10. Plot of vertical bearing capacity with foundation setback for different values of the roughness ratio.

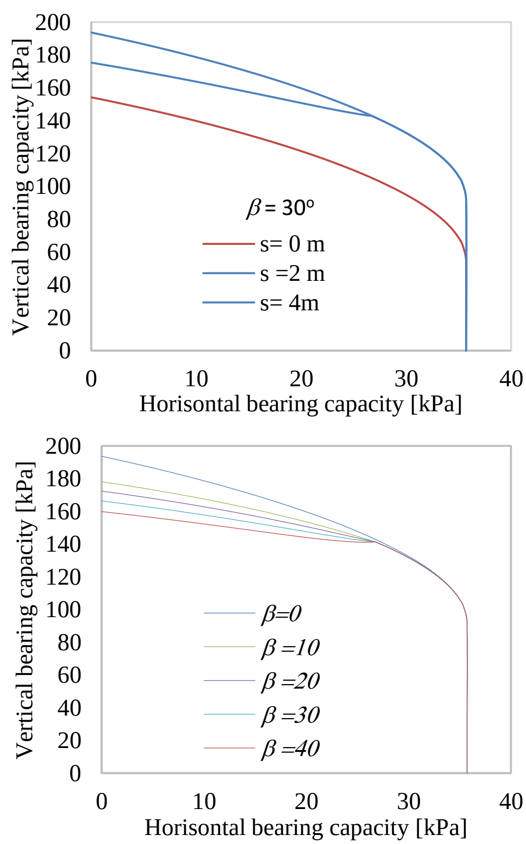


Figure 11. Interaction curves for various foundation setbacks and slope angles.

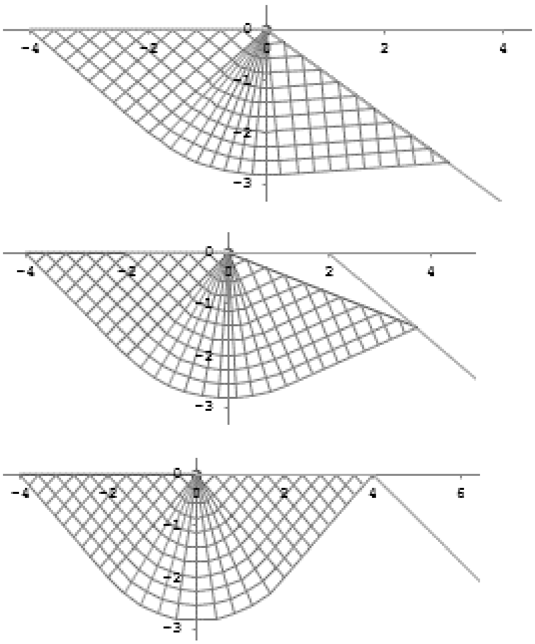


Figure 12. Rotation of the rapture surface with increasing foundation setback.

4 EFFECT OF DEPTH

The depth of the slope is another aspect that should be considered when taking effect of the slope into account. The depth of the slope will have effect when the following inequality holds true ($\varphi = 0$ for the undrained case)

$$\frac{H}{B} < \frac{\sin\left(\frac{\pi}{2}+\varphi\right)\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}-\omega\right)}{\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\sin\left(\frac{\pi}{2}-\varphi\right)} e^{\left(\frac{\pi}{2}-\omega-\beta'\right)}\sin(\beta') \quad (27)$$

5 SUMMARY

Analytical equations have been developed for taking into account effect of nearby slopes on the bearing capacity of shallow foundations. The development is in the NTNU bearing capacity framework. Both drained and undrained conditions were considered. However, the principle can be easily implemented into other similar classical bearing capacity formulas.

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