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# Influence of diameter in piles under lateral loads embedded in stiff clays above the water table

## L'influence du diamètre des pieux sous charge latérale encastrés dans des argiles raides au-dessus de nappe phréatique

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**ABSTRACT:** The design of laterally loaded pile is applicable in many situations, such as offshore structures, bridges, heavy buildings and highway structures. For these cases and many others, the non-dimensional solution proposed by Matlock and Reese (1956) help engineers to predict bending moments, shearing forces and deformed shape of a laterally loaded pile considering that the pile behaves as an elastic beam on a soil behaving as a series of elastic springs.

This paper aims to study the influence of the pile diameter in the soil response and lateral deflections for piles under transverse loading embedded in stiff clays above the water table. In this case, the pile diameter increases 10%, 40% and 60% from the initial value, the moment diagrams and a series of graphs are shown for each case to demonstrate the pile and soil behavior.

When comparing the maximum lateral deformations at the top of each pile analyzed in this paper, it is possible to observe a reduction of more than 50% in the initial value. Obviously, this shows that increasing the flexural stiffness of the pile is a very effective way of reducing lateral deformations.

On the other hand, the bending moment graphs show that piles with greater diameters require more extensive longitudinal reinforcements, since the bending moment approaches zero deeper along the pile length.

**RÉSUMÉ:** Le dimensionnement de pieu chargé latéralement est applicable à des différentes situations, comme pour les structures maritimes, ponts, tours et structures autoroutières. Pour ces cas et certains d'autres, la solution non dimensionnel proposée par Matlock et Reese (1956) permet aux ingénieurs de prévoir moments de flexion, forces de cisaillement et la déformée d'un pieu latéralement chargé compte tenu que le pieu se comporte comme une poutre élastique dans un sol qui se comporte comme une série de ressorts élastiques.

Cet article vise étudier l'influence du diamètre de pieu sur la réponse du sol et déformation latérale pour des pieux sous chargement transversal encastrés dans des argiles raides au-dessus de nappe phréatique. Dans ce cas, le diamètre du pieu augmente 10%, 40 et 60% à partir de la valeur initiale, le diagramme de moment de flexion et des graphiques sont exposés pour chaque cas afin de démontrer le comportement du pieu et du sol.

Lorsqu'on compare la déformation latérale maximale au sommet de chaque pieu analysé dans cet article, il est possible de observer une réduction supérieure à 50% de la valeur initiale. Visiblement ceci représente qu'augmenter la rigidité à la flexion du pieu est une manière effective pour réduire les déformations latérales.

Par contre, le graphique du moment de flexion indique que les pieux possédant diamètre supérieur demande des renforcements longitudinaux étendus, puisque le moment de flexion s'approche de zero plus profondément sur la longueur du pieu.

**Keywords:** Pile; Lateral Loads; Stiff Clay; Deflection

## 1 INTRODUCTION

Several methods have been proposed to analyze laterally loaded piles. Methods based on the theory of elasticity are not generally applicable due to the difficulty to establish constant values to the soil parameters.

Alternatively, methods based on the theory of the subgrade reaction, aligned with some simplifying assumptions, for example the variation of the reaction modulus with depth, are more realistic and because of that, more usual. Nevertheless, these simplifying assumptions imposed on methods based on the theory of subgrade reaction made in order to simplify the solution for the problem, bring a lot of errors and uncertainties in laterally loaded pile analysis. Therefore, these models are often unable to fully characterize the three-dimensional behavior of the pile-soil interaction (Terzaghi, 1955; Meyer et al., 1979).

The method of analysis of laterally loaded piles proposed by Matlock and Reese (1956), based on the horizontal subgrade reaction coefficient, considers that the piles behave as semi-infinite flexible beams embedded in an elastic medium. The criteria to determinate if the piles behave as a flexible element (long piles) are divergent. However, in this paper, piles with maximum depth coefficient ( $Z_{max}$ ) equal or superior to 5, are considered as flexible, as proposed by Matlock and Reese (1956).

This paper analyzes a case of laterally loaded piles embedded in stiff clay above the water table using the method proposed by Matlock and Reese (1956). Further, the influence of the pile diameter variation on the lateral deflection and bending moment diagram is analyzed.

## 2 DIMENSIONAL ANALYSIS

The analysis of deformations and applied loads in laterally loaded piles is based on the evaluation of the reaction modulus ( $E_s$ ), defined as the soil resistance per unit of length at a point along the pile divided by the deflection of the pile at the same point.

A simplification proposed by Matlock and Reese (1956), applicable for stiff clays, considers that the reaction modulus ( $E_s$ ) varies linearly with depth. This linearity is also explored by Reese and Impe (2011), proving to bring satisfactory results for laterally loaded piles analysis. Thus, the reaction modulus ( $E_s$ ) as a function of depth ( $x$ ), can be written as:

$$E_s = k_h \cdot x \quad (1)$$

The term  $k_h$  ( $\text{kN/m}^3$ ) represents the horizontal subgrade reaction coefficient and, in the case of stiff clays, it is considered to be constant with depth (Gudmundsdottir, 1981).

Table 1 relates the value of  $k_h$  proposed by Terzaghi (1955) for stiff to hard clays obtained by tests with horizontal plates of 0,30 x 0,30m with the unconfined compressive strength ( $q_u$ ) of the soil.

Table 1 – Values of  $k_h$  in  $\text{kN/m}^3$  (Terzaghi, 1955)

Clay	Stiff	Very stiff	Hard
$q_u$ ( $\text{kN/M}^2$ )	100 – 200	200 – 400	> 400
Range of $k_h$	160 – 320	320 – 640	> 640
Proposed values	240	480	960

Reese and Impe (2011) point out the fact that for long piles, the length loses significance in the analysis of displacements since the most relevant deformations occur near the ground surface. For that reason, it is convenient to introduce the relative stiffness factor ( $T$ ), that expresses the relationship between the flexural stiffness of the pile ( $E_p I_p$ ) and the stiffness of the soil.

For the case of a shear force  $P_t$  and a moment  $M_t$  applied at the top of the pile, the calculations of lateral deflections ( $y$ ) consider the relative stiffness factor ( $T$ ) and other terms (see Fig. 1).

$$y = y \cdot (x, T, L, E_s, E_p I_p, P_t, M_t) \quad (2)$$

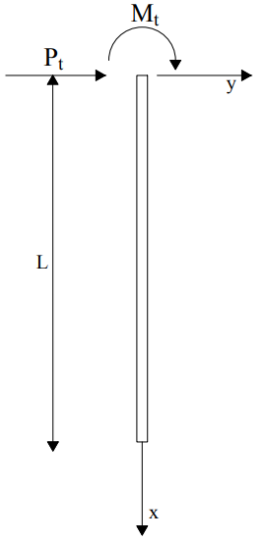


Figure 1. Arrangement for dimensional analysis

The analysis of laterally loaded piles proposed by Reese and Impe (2011) assuming the linear behavior of the pile, considers that the lateral deflections are small compared to the pile length. In this way it is possible to superpose the effects caused by application of the shear force  $P_t$  and the moment  $M_t$ . Assuming that  $y_A$  and  $y_B$  are the lateral deformations caused by  $P_t$  and  $M_t$  respectively, the total lateral deflection is

$$y = y_A + y_B \quad (3)$$

The ratios of  $y_A/P_t$  e  $y_B/M_t$  seek generalized solutions for the linearly-behaving pile. For the isolated application of  $P_t$  at top of the pile, the solutions may be expressed as

$$\frac{y}{P_t} = f_A \cdot (x, T, L, E_s, E_p I_p) \quad (4)$$

For an isolated application of the moment  $M_t$

$$\frac{y}{M_t} = f_B \cdot (x, T, L, E_s, E_p I_p) \quad (5)$$

Equations 4 and 5 represent two functions of the same variables. In both cases, there are six terms in units of length or force, which together form four non-dimensional coefficients. As

proposed by Matlock and Reese (1956), these coefficients related to the shear force  $P_t$  and the moment  $M_t$  are

$$\text{Depth coefficient, } Z = \frac{x}{T} \quad (6)$$

$$\text{Maximum depth coefficient, } Z_{max} = \frac{x}{L} \quad (7)$$

$$\text{Soil reaction, } f_z = \frac{E_s \cdot T^4}{E_p I_p} \quad (8)$$

$$\text{Deflection coefficient, } A_y = \frac{y_A \cdot E_p I_p}{P_t \cdot T^3} \quad (9)$$

$$\text{Deflection coefficient, } B_y = \frac{y_B \cdot E_p I_p}{M_t \cdot T^2} \quad (10)$$

The lateral deformations caused by the shear force  $P_t$  and the moment  $M_t$  along the pile length are obtained combining equations 3, 9 and 10.

$$y = \left[ \frac{P_t \cdot T^3}{E_p I_p} \right] \cdot A_y + \left[ \frac{M_t \cdot T^2}{E_p I_p} \right] \cdot B_y \quad (11)$$

Reese and Impe (2011) also suggest solutions to determine shear stress and bending moment along the pile length.

$$\text{Shear, } V = P_t \cdot A_v + \left[ \frac{M_t}{T} \right] \cdot B_v \quad (12)$$

$$\text{Moment, } M = P_t \cdot T \cdot A_m + M_t \cdot B_m \quad (13)$$

For the calculations of lateral deflections and applied loads for long piles ( $Z_{max} \geq 5$ ) with free head, table 2 shows the values for the coefficients  $A_y$  and  $A_m$  proposed by Matlock and Reese (1956) as a function of depth coefficient ( $Z$ ). The coefficients used to determine deformations or applied forces that are not studied in this paper (shear stress, for example) were omitted.

Table 2 – Coefficients for long piles (Matlock and Reese, 1956)

$Z = x/T$	$A_y$	$A_m$
0,0	2,435	0,000
0,1	2,273	0,100

0,2	2,112	0,198
0,3	1,952	0,291
0,4	1,796	0,379
0,5	1,644	0,459
0,6	1,496	0,532
0,7	1,353	0,595
0,8	1,216	0,649
0,9	1,086	0,693
1,0	0,962	0,727
1,2	0,738	0,767
1,4	0,544	0,772
1,6	0,381	0,746
1,8	0,247	0,696
2,0	0,142	0,628
3,0	-0,075	0,225
4,0	-0,050	0,000
5,0	-0,009	-0,033

In situations where the reaction modulus ( $E_s$ ) is considered to vary linearly with depth, the relative stiffness factor ( $T$ ) used to calculate the depth coefficient ( $Z$ ), presented in table 2, can be expressed according to Matlock and Reese (1956) as

$$T = \left[ \frac{E_p I_p}{k_h} \right]^{1/5} \quad (13)$$

The whole purpose of  $p$ - $y$  curves, proposed initially by McClelland and Focht (1956), is to study the variation of the reaction modulus ( $E_s$ ) with depth. The graph in figure 2, considered as linear for this case, shows that variation.

In this case, the slope of the straight line which passes through the origin, mathematically represents the horizontal subgrade reaction ( $k_h$ ) of the soil.

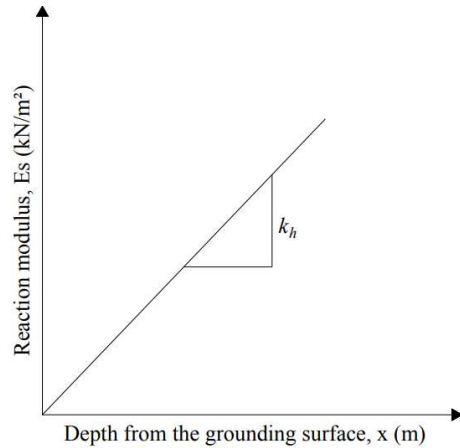


Figure 2. Simplified variation of reaction modulus

Hence, the analysis of the influence of pile diameter variation on the lateral deflection and applied loads for laterally loaded piles is based on the values of horizontal subgrade reaction coefficient for stiff clays proposed by Terzaghi (1955), according to table 1.

### 3 PROPOSED MODEL

#### 3.1 Geometry of the model

In the analysis of lateral deflections of the laterally loaded pile embedded in stiff clay above the water table and in obtaining the respective applied loads diagrams, the method proposed by Matlock and Reese (1956) was used considering the reaction modulus varying linearly with depth. The coefficients A and B, present in equations 9 to 13 were calculated by linear interpolation of the values shown in table 2.

The geometry of the problem is shown in figure 3. In this paper, the analysis considered a short-term static loading.

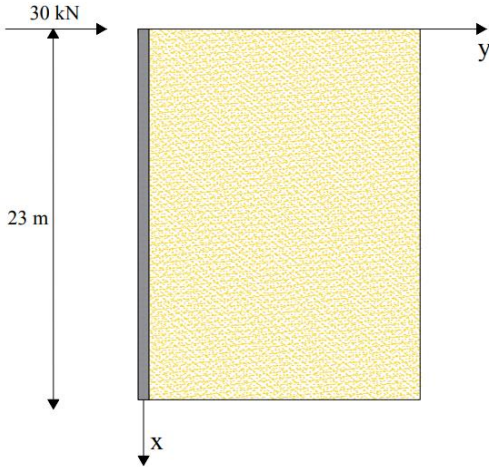


Figure 3. Geometry of the model

### 3.2 Material properties

#### 3.2.1 Soil parameters

The soil used to analyze lateral deflections and applied loads on the pile is a stiff clay above water table. For the horizontal subgrade reaction coefficient ( $k_h$ ), the value proposed by Terzaghi for stiff clays of 240 kN/m<sup>3</sup> was considered. Table 3 shows the soil parameters considered in this paper.

Table 3. Soil parameters

Unit weight, $\gamma$ (kN/M <sup>3</sup> )	Compressive strength, $q_u$ (kPa)	Subgrade reaction, $k_h$ (kN/M <sup>3</sup> )
19	100	240

#### 3.2.2 Pile properties

The pile is modeled as a structural element made up of reinforced concrete with length equal to 23 meters, Poisson's ratio of 0,2, unit weight of 23 kN/m<sup>3</sup> and compressive strength ( $f_{ck}$ ) of 25 MPa.

In this paper, the pile diameter varies according to table 4 with the aim to evaluate the influence on lateral deflections and applied loads.

Table 4. Pile properties

Diameter, $D$ (cm)	Modulus of elasticity, $E_p$ (MPa)	Moment of inertia, $I_p$ (m <sup>4</sup> )
50	23800	$3,07 \cdot 10^{-3}$
55	23800	$4,50 \cdot 10^{-3}$
70	23800	$1,18 \cdot 10^{-2}$
80	23800	$2,01 \cdot 10^{-2}$

## 4 EFFECT OF PILE DIAMETER ON LATERAL DEFLECTIONS

The results of lateral deflections of the pile for the different diameters analyzed in this paper are shown in figures 4 and 5. For convenience, only the maximum lateral deformation at the top of each pile is being highlighted.

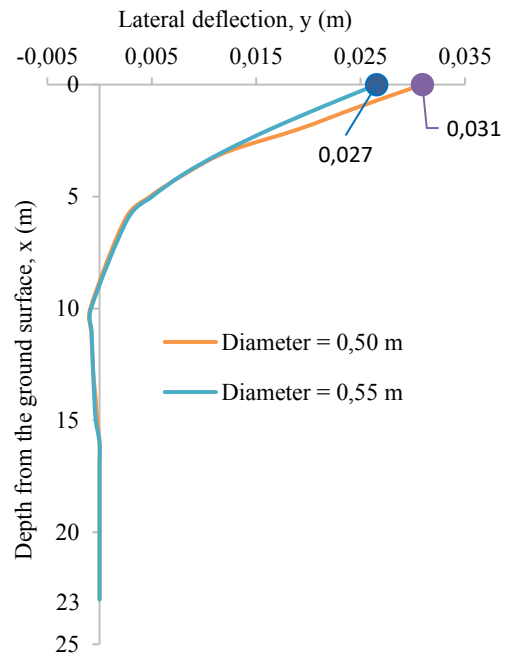


Figure 4. Lateral deflection x depth diagram for  $d = 0,50m$  and  $0,55m$

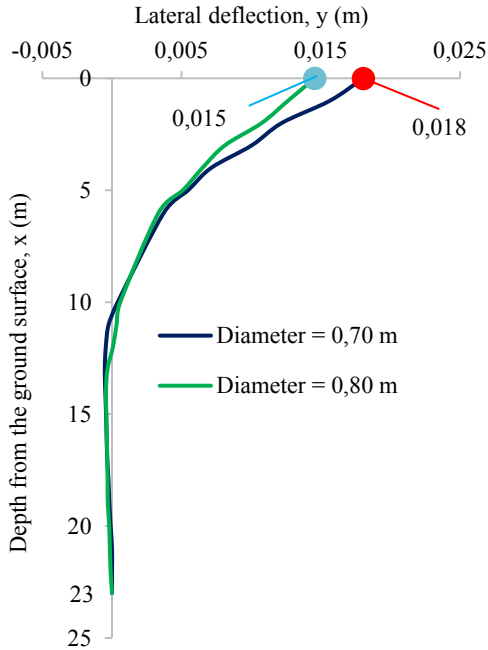


Figure 5. Lateral deflection  $x$  depth diagram for  $d = 0,70m$  and  $0,80m$

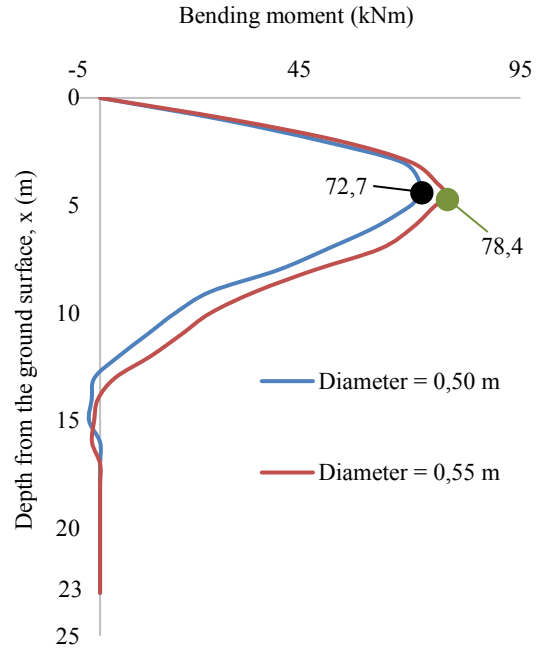


Figure 6. Bending moment diagram for  $d = 0,50m$  and  $0,55m$

## 5 EFFECT OF PILE DIAMETER ON BENDING MOMENT DIAGRAM

In order to analyze the influence of the diameter variation on the bending moment diagram, figures 6 and 7 show bending moment  $x$  depth curves for the different diameters analyzed in this paper. The highlight points present the maximum bending moment value for each pile.

It is interesting to note that, as occurs with many reinforced concrete structures, elements with greater flexural stiffness tend to absorb more stress, and therefore, in figures 6 and 7, piles with larger diameters present higher bending moment values when compared to smaller piles.

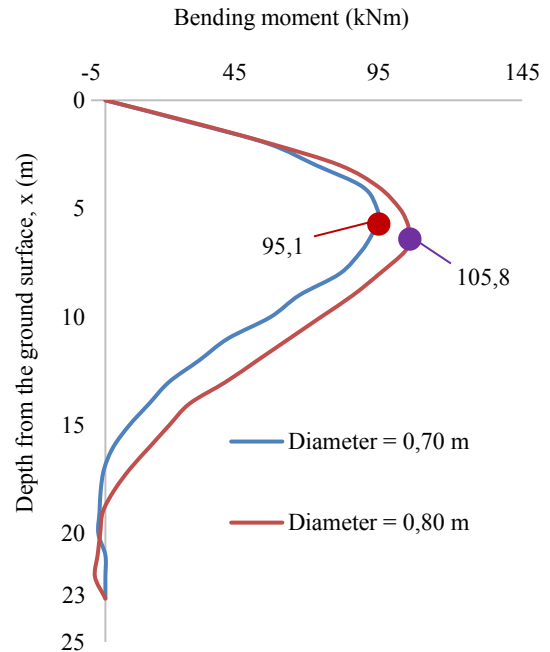


Figure 7. Bending moment diagram for  $d = 0,70m$  and  $0,80m$

## 6 CONCLUSIONS

The numerical analysis is used to analyze the influence of pile diameter variation on the lateral deflections and bending moment diagram. Even though the values of horizontal subgrade reaction coefficient ( $k_h$ ) proposed by Terzaghi (1955) are dependent on the pile diameter and need to be corrected by multiplying them by the factor  $b/B$  (where  $b = 0,30\text{m}$  and  $B$  is the pile diameter), as mentioned by Velloso and Lopes (2010), the same horizontal subgrade reaction coefficient ( $k_h$ ) was considered for all piles with the aim to analyze separately the effect of the diameter variation on laterally loaded piles.

Figures 4 and 5 show, as expected, that the more significant deformations occur near the ground surface, reaching maximum value at the top of each pile (in this case, for depth  $x = 0$ ).

In table 5, it is possible to note that an increase in the pile diameter from 0,50m to 0,55m, that is 10% increase, resulted in reduction of the deformation at the top of the pile from 0,031m to 0,027m, which means a reduction of approximately 13%.

In the same way, an increase of 40% in the initial pile diameter, 0,50m to 0,70m, resulted in a reduction close to 42% in the deflection at the top of the pile (0,031m to 0,018m). Finally, an increase of 60% in the pile diameter, 0,50m to 0,80m, exhibited a reduction of almost 50% in the initial lateral deformation at the top of the pile.

Table 5. Maximum lateral deflections

Pile diameter (m)	Deflection at the top (m)
0,50	0,031
0,55	0,027
0,70	0,018
0,80	0,015

An interesting point is that, according to Reese and Impe (2011), long piles are characterized by having at least two points of zero deflection along its length. Clearly, this can be seen in figures 4 and 5 by noticing the two points where the diagram intercepts the vertical axis.

Equation 13 indicates that the maximum bending moment for long piles occurs when the coefficient  $A_m$  also reaches its maximum value. From table 2, we notice that this situation happens when the depth coefficient ( $Z$ ) is equal to 1,4.

For the different pile diameters analyzed in this paper, table 6 shows the maximum bending moment along the length of each pile and the point below the ground surface where it happens.

Table 6. Maximum bending moment along the pile length

Pile diameter, D (m)	Maximum bending moment (kNm)	Depth from the ground surface, x (m)
0,50	72,7	4,4
0,55	78,4	4,7
0,70	95,1	5,7
0,80	105,8	6,4

According to table 6, an increase of 10% in the pile diameter resulted in an increase of 8% in the maximum value of bending moment. Similarly, an increase of 40% in the initial pile diameter causes an increase of approximately 31% in the initial bending moment. The last pile diameter variation shows that it is possible to increase almost 45% of maximum bending moment along the pile length by increasing the diameter 60%.

We can see in figures 6 and 7, that the bending moment diagram for smaller piles approaches zero closer to the ground surface than piles with larger diameters, which means that the required longitudinal reinforcements are longer in piles with larger diameters.

In spite of being laborious, the analysis of laterally loaded piles using the method proposed by Matlock and Resse (1956) brings good results. Even though the bending moment along the pile increases as the pile diameter also increases, this practice is a very effective way to reduce laterally loaded pile deflections in case of ultimate limit state and serviceability limit state analysis.

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