Investigation of the probability density functions for suction potential of soils

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ABSTRACT: Statistical analysis appears to be the approach most likely to deliver designs with a realistic estimate of probability against failure. First Order Reliability Method (FORM), one of the simplest statistical methods, is theoretically and practically very attractive. It relies on estimates of mean and coefficient of variation for soil properties. The current procedure of limit state design, effectively a very simplified version of FORM, obtains its soils characteristic values from the same statistical considerations. Due to the time required for sufficient tests on a specific soil to give a similar degree of reliability to that required for probability against failure, estimates of soil probability density functions (PDFs) are typically obtained by estimation, or by data base mining combined with an assumed probability density function. By using small-scale suction tests it has been possible to derive experimental PDFs for suction potential of a number of individual soils. Some of these PDFs are markedly different to the normally assumed curves. Since many soils properties are related to soil suction it is possible that PDFs for other soil properties may also not be well described by current assumptions. Possible ramifications for design of soil structures is considered.

1 INTRODUCTION

Geotechnical engineering deals with materials more variable than those used in most branches of engineering. Variability of properties result in uncertainties. Uncertainties are best dealt with by statistical methods. Reliability based design has been developed over many years to be a very powerful tool to enable a reasonable estimate of a design's probability against failure (Phoon et al. 2003, Dithinde et al 2011, Ching & Phoon 2012, Simpson 2012). Reliability based design procedures tend to be mathematically intensive, and not popular with many practicing engineers. Even simplified procedures, like the first order reliability method, may require more time than many engineers would like to spend on analysis. The standard method currently used in many countries, including South Africa (Limit States Design) could be viewed as an ultimate simplification, but nevertheless it still relies on the same statistical parameters as more rigorous and complex RBD analysis. Any kind of reliability based design method requires probability density functions (PDFs) for action and resistance parameters involved in a design.

2 SANS 10160 REQUIREMENTS

The characteristic value of a geotechnical parameter is defined as the value so determined that the probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%. For this a probability density function for the parameter in question is required. In principle one considers a PDF for a parameter governing resistance as in Fig.1. For estimating the characteristic value one would take point A, below which not more than 5% of the area of the curve remains. Estimating the characteristic values for an action would involve the value above which not more than 5% of the area of the curve remains.

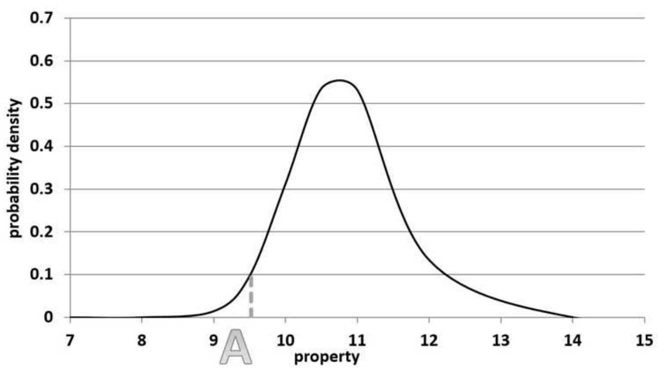

Figure 1. Characteristic value chosen so that not more than 5% of values governing resistance will be lower

Unfortunately tests on many samples are required to give a meaningful PDF, so it is common practice to make assumptions. One very common assumption is that the PDF is log-normal. Table 5 of chapter 1-2 of the Background report to SANS 10160 (Retief & Dunaiski 2009) indicates a log-normal distribution. This choice seems reasonable since one might well expect a distribution not too dissimilar to a normal distribution. But a normal distribution can run to negative values on the property axis. Soils are not likely to have negative values for properties relevant to engineering, and negative values could have serious consequences for some analytical procedures. The log-normal distribution cannot have negative values, it is reasonably similar in general form to the normal distribution, it is convenient for analysis, and therefore seems to be an obvious choice. To give a confidence level comparable to the 5% above or below required for limit states design, a normal type of distribution needs 636 test results (Phoon 2013). For most soil tests this number of specimens would require a large quantity of soil, and a very long time to complete the testing process. To get over this problem it is common practice to either estimate values, or to employ database mining techniques. Databases of published results for tests on various types of soil are examined. A coefficient of variation (COV) is estimated from the samples discovered in the database. A mean may be estimated from a few tests on the soil in question. A PDF can be generated from those values. This procedure is not always very convincing. For example, Phoon & Ching (2013) give a range (in their Table 1) of mean for PI of clay and silt of (10-40) and guideline estimate of COV for PI as {(3-12%)/mean}. It seems reasonable to expect that one could gain an estimate of mean by averaging a small number of test results. But it is not clear how to choose a reasonable COV. The range {(3-12%)/mean} for a PI at the centre of their stated range (25), would allow the COV to be anywhere between 12 and 48. Retief & Dunaiski (2009) in Table 5 of Chapter1- 2 specify a choice of 10%, 15%, 20% and 25% (a substantially smaller range than suggested by Phoon & Ching). There appears to be no recommended procedure for deciding which of these values to use. Chapter 5 of SANS 10160, which deals specifically with Geotechnical aspects notes that an unreasonably large number of tests would be required to establish reasonable estimates of the probability density function and that one may use "*a cautions estimate of the value affecting the occurrence of the limit state under consideration*." But there is scant guidance on how to come to such a cautious estimate.

3 SMALL SCALE SUCTION TESTS

The soil parameter generally considered most valuable (if not essential) in unsaturated soil mechanics is the soil water retention curve (SWRC). This traces the variation of water content with negative pore pressure in soil. By combining the SWRC with standard soil properties it is possible to extend Geotechnical analysis from saturated to unsaturated conditions and deal with the full range of soils behavior from completely saturated to very dry. Soil suction appears to be a unifying factor across a large range of soil properties. For an active clay it can take a considerable time to measure the SWRC over the full range of suctions (from effectively zero to 1 000 000 kPa). Tests on a range of clayey soils from engineering projects in Central South Africa have suggested that many if not most SWRCs for Central South African clayey soils have a similar form, and a single strategic value can give a good estimate of a large part of the SWRC. This can allow an estimate to be made quite quickly with a small sample (Stott & Theron 2017). This in turn allows many samples to be tested so that an indication of variability can be measured for a large number of samples in a reasonable time. Variability is widely considered to be a function of packing, with tight packing giving higher suction and loose packing giving lower suction. X-ray diffraction (XRD) tests on samples of a particularly high variability soil show particularly high variability in mineral composition. Both variability in packing and in mineral content would be expected to give corresponding variability in a wide range of soil properties. Many soils have been tested to assess variability (up to 100 samples per soil). Several soils have been tested using sufficient samples to give reasonable confidence that the resulting PDSs are valid. For a normal distribution 636 tests may be needed to give acceptable confidence, but some of the distributions are far from normal and up to 800 tests were performed to give reasonable certainty that no obvious change in the form of the distribution was still taking place. Table 1 gives brief descriptions 10 of the soils concerned.

Table 1. Soils tested

Name	Description	Retention	CoV
Heidedal	Dark brown sandy clay	10.7	15
BK3270	Black transported clay	10.8	6
Kloof Lodge	Brown clayey sand	7.9	11
Southbridge	Reddish brown clay	12.2	21
Steelpoort 1	Red alluvium	17	26
Steelpoort 2	Brown clay	24	10
NMC 1	Dark brown clay	11	15
NMC 2	Red banded grey clay	11.8	5
Witherow S2	Dark olive clay	11.3	17
Witherow S24	Reddish orange silty clay	7.6	11

4 MEASURED PROBABILITY DENSITY FUNCTIONS

Figure 2 shows pdfs for the 10 soils listed in Table 1. Plasticities range from low to very high. One feature which is evident for all of the PDFs is that they fall away from the peak values to the upper limit rapidly, usually much more rapidly than to the lower limit. The lognormal is therefore not likely to be a better approximation to any of the soils tested than a normal distribution, since it falls away from the maximum more slowly than a normal distribution at the high-value side, and more quickly at the low-value side.

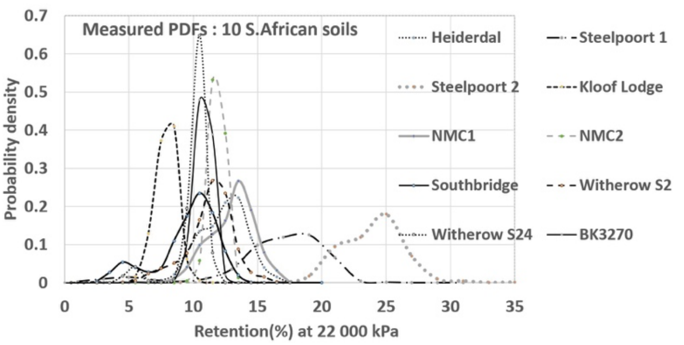

Figure 2. Probability density curves for 10 soils (600 to 800 samples tested per soil).

The curves in Figure 2 tend to group themselves into three types: approximately symmetrical, bimodal and bulging. These three groupings are shown in Figures 3, 4, and 5.

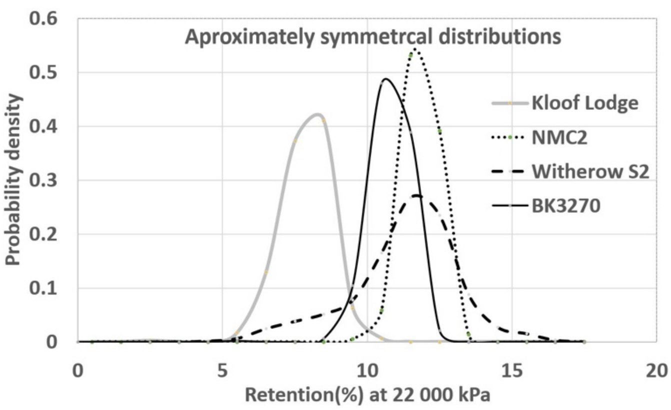

Figure 3. Approximately symmetrical distributions

Of the soils tested to date, all of those with approximately symmetrical distribution have generally low variability. They have shown no obvious characteristics which would have allowed prediction before testing that they would show low variability.

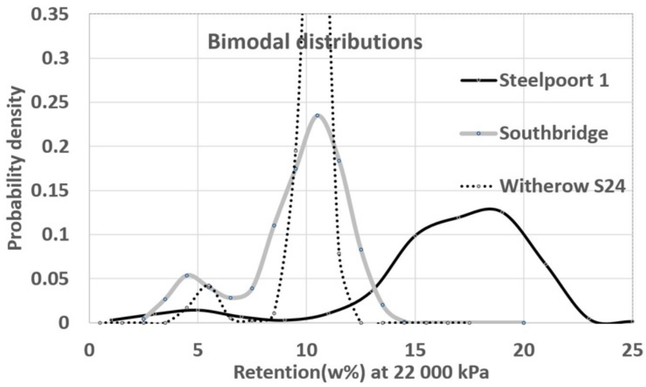

Figure 4. Bimodal distributions

Bimodal distributions may be somewhat surprising, but they do not appear to be uncommon. Although the initial suspicion was that such distributions may come from mixing of soils while sampling, this does not appear to be the case. Close-up photographs of samples with the highest retentions appear to be almost indistinguishable from samples with the lowest retentions. X-ray diffraction results from 10 samples of Steelpoort1 (the soil with the highest COV in Figure 2), although visually identical gave strikingly different diffraction patterns. On the other hand, 10 samples of Southbridge (the soil with the second highest COV in Figure 2) showed only moderate differences in X-ray diffraction patterns. The use of the recommendations of SANS 10160 would appear to be questionable for such soils. A lognormal distribution is such a poor approximation to some of the distributions shown that it appears that a lognormal assumption might lead to unsafe design in at least some cases. If the measured PDF were available then it should be possible to arrive at a "cautious estimate", but without it such an estimate might remain elusive.

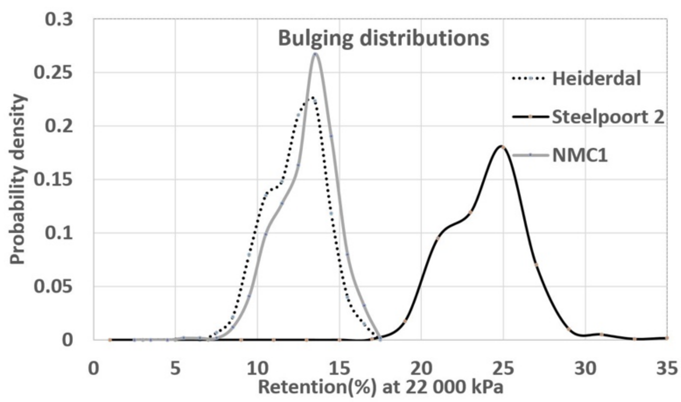

Figure 5. Bulging distributions

This pattern shows a "bulge" on the low-value side which, with a finer bin size, might be actually revealed to be a bimodal pattern. Several "partial PDFs" (from typically about 100 samples) show this pattern and the bulge has so far always been observed to be on the low-value side. For small sample numbers the distribution often shows a bimodal pattern which gradually softens to the bulging form as the number of tests increases. A normal distribution curve can be fitted to give a fair approximation to all of this type of curve so far measured.

4.1 *Estimating PDFs from a smaller number of tests*

Soils which prove to have a small COV tend to be unimodal with PDF close to normal and only about 30 tests are needed for the PDF to begin to settle into a normal distribution pattern. After about 200 tests very little change usually occurs. This is not the case with bimodal distributions. The relative heights of the peaks tend to fluctuate and sometimes the pattern only settles into a stable configuration after four or five hundred tests. In one case more than 800 samples were tested to make sure that the pattern was stable. The bulging pattern is intermediate requiring more tests than simple pattern but less than bimodal pattern soils before settling into a stable configuration. It is difficult to predict whether any particular soil will prove to have low or high variability. Some of the most variable soils found to-date appear to be uniform, some soils which are visually not uniform – e.g. mottled or blotched - have low variability. Examination of photographs of samples with the highest and lowest retentions from a given batch have not given an indication of why the difference should exist.

4.2 *Comparison with other tests*

Most soil tests are too tedious and time-consuming to contemplate 600 tests. A program is underway, however, to establish how many tests may be needed to give a possible correlation with suction test results. Both the Casagrande cup and the fall cone give results directly related to shear strength. Tests comparing results from both sets of apparatus suggest that there may be a strong correlation with suction values. If this is confirmed then it should be possible to identify soils which need many tests to give reliable values for design, and gain an idea of how many tests are needed with various types of test to give a reliable link to PDFs found by suction tests.

4.3 *Implications for design*

SANS 10160 requires values of properties to be chosen so that there is less than 5% probability of a more unfavourable value occurring. Without a PDF for the property one is reduced to making a "cautious estimate". Perhaps one could then simply take the maximum suggested COV (25%) from the table referred to above (Retief & Dunaiski 2009). This would possibly give uneconomic over-design in some cases, and it is not immediately obvious whether this would give a safe design in all cases.

Figures 6, 7 and 8 give examples of each of the patterns noted above. These may or may not be typical of the majority of soils encountered in South Africa but should at least give an indication of how cautious one needs to be in the application of SANS 10160.

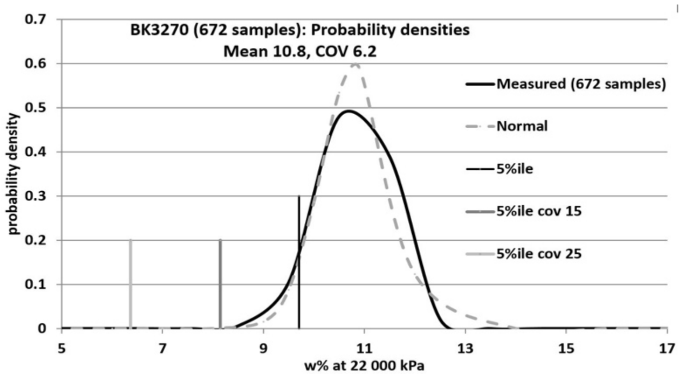

Figure 6. PDF and possible choices of values of properties for design: approximately symmetrical PDF

Figure 6 shows that the measured probability density is not very far from a normal distribution. The 5th percentile vertical line (black) shows the maximum characteristic value (9.7) acceptable to the code. The 5th percentile for COV 15 shows the maximum allowable value (8.1) which results from assuming a COV of 15 (dark grey vertical line). The 5th percentile for COV 25 (light grey vertical line) shows the maximum allowable value (6.3) if the maximum recommended COV of 25 is assumed. Both assumptions, of COV 15 and COV 25, would appear to be unnecessarily conservative. This is true of the other approximately symmetrical distributions also.

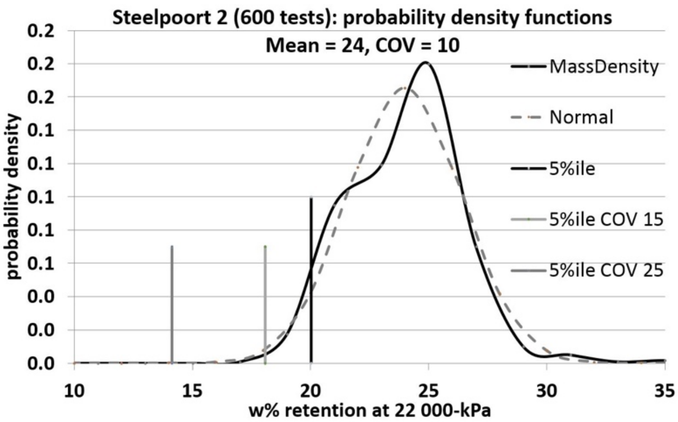

Figure 7. PDF and possible choices of values of properties for design: bulging PDF

The bulging distribution shown in Figure 7 is also reasonably well approximated by a normal curve. The characteristic value which satisfies the code's requirements (marked by the vertical, black 5-percentile line) is 20.3. The value for an assumed COV 15 is 18.1 and for COV 25 it is14.1.

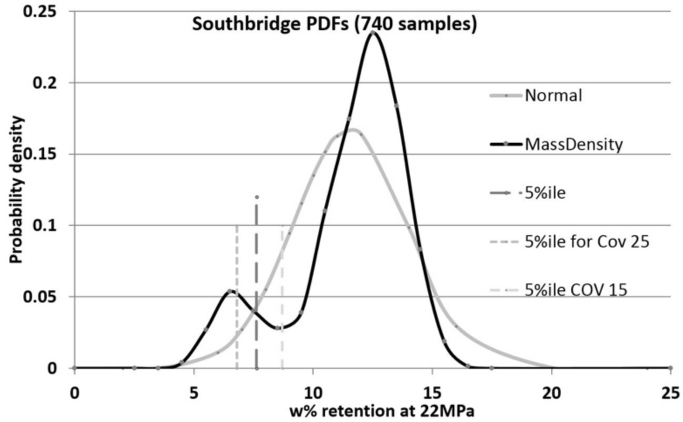

Figure 8. PDF and possible choices of values for use in design: bimodal distribution.

The measured PDF shown in Figure 8 is far from a normal distribution (and even farther from lognormal). This is likely to be a common feature of bimodal distributions. The measured COV is 20 and the assumption of COV 15 is obviously unsound. But even the use of the actual COV (20) is unsound. The allowable percentage of 5 % represents the area under the actual curve below the maximum allowable value. That is equal to the area below the normal curve to the left of the vertical, black 5th percentile line. But the actual area below the measured curve to the left of the 5th percentile is at least twice as large as that below the normal curve. Even the 5th percentile of the COV 25 assumption has a larger area below the actual curve than that below the normal curve at the measured 5th percentile and therefore does not meet the requirement of giving no more than 5 % chance of a value being below the characteristic value.

The most unusual PDF found to date is that from the Steelpoort 1 soil, which comes from North-Central South Africa. It is illustrated in Figure 9. The COV is a little over 25, the largest value proposed for consideration by Retief & Dunaiski. Assumption of anything smaller would probably therefore be unsound. Again the 5th percentile for the normal distribution with the same mean and COV appears to be a questionable choice for a characteristic value. The area below the measured curve is apparently greater than that below the normal curve, and the values of the parameter may approach very close to zero.

The fact that this soil may manifest an extraordinary probability density function for properties other than just retention against applied suction is not difficult to illustrate. Because structures built on this soil have given trouble, samples have been sent to several certified soils laboratories. Eight liquid limits from six laboratories have been used to produce a rudimentary PDF. Obviously with only eight test results one would not expect a reliable PDF, but as can be seen in Figure 10 there is a clear indication that this soil is not likely to be well described by a normal or lognormal distribution.

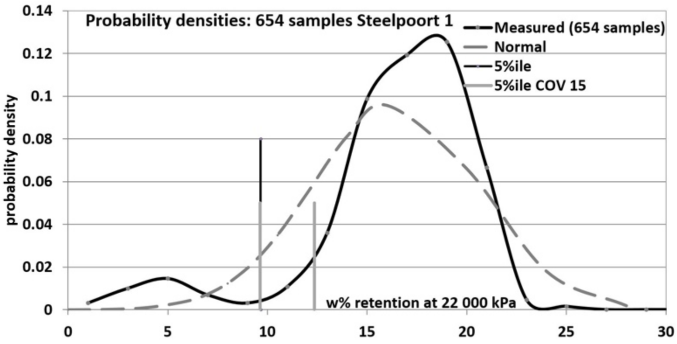

Figure 9. PDF and possible design values for Steelpoort 1

For comparison a rudimentary PDF for the soil dealt with in Figure 6 is shown in Figure 11. Nine Liquid limit results were used to construct the PDF.

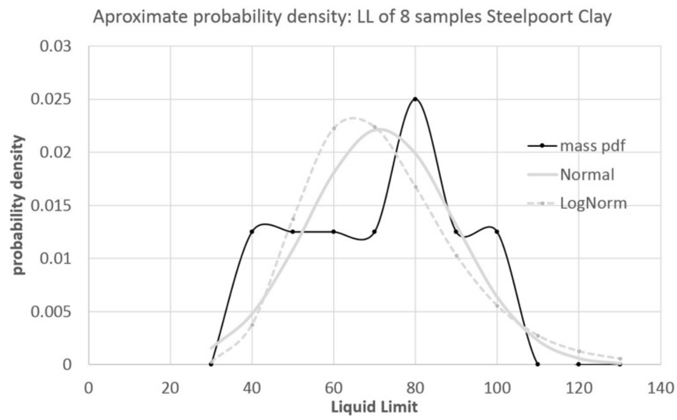

Figure 10. Rudimentary PDF constructed from 8 Liquid Limits from 6 different commercial laboratories: Steelpoort 1

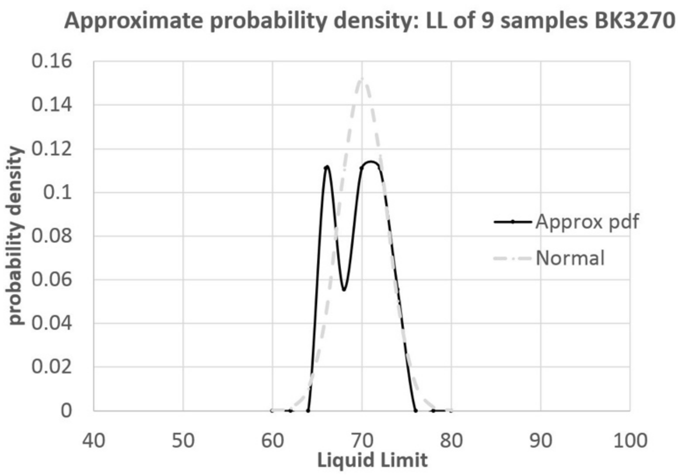

Figure 11. Rudimentary PDF constructed from a total of 9 Liquid Limits from 4 different laboratories: BK3270

5 CONCLUSIONS

Measured PDFs for water retention against suction do not appear to correspond well with the lognormal distributions widely assumed and adopted by SANS 10160. There are theoretical reasons, and preliminary experimental indications, that some other soil properties may have similarities with water retention values, and consequently with their PDFs. The soils tested to date show such wide variation in the form of PDFs that without some indication of the actual COV of a particular soil one may either significantly overdesign or under-design while apparently making cautious choices. Attempts are being made to make the estimation of valid PDFs for engineering soils less costly and time consuming. Until more quick and reliable estimations of PDF become available it would appear that assumption coupled with reliance on tests results from just a small number of samples may carry significant risk.

6 REFERENCES

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