Determination of unsaturated soil property functions for engineering practice

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ABSTRACT: The Jennings Memorial lecture starts with a recollection of notes I made in 1965 when I first met Professor Jennings. I have clear recollections of that meeting and have noted some of the significant comments made at that time; comments that have impacted my career and given me focus for research into unsaturated soil mechanics. My research and engineering career has been built around many of the ideas discussed more than five decades ago.

Unsaturated soil mechanics is now recognized as an important value-added component to routine geotechnical engineering practice. The key to its implementation has centered around the measurement and use of unsaturated soil property functions that are primarily estimated based on the soil-water characteristic curve, SWCC. In recent years research has shown that there are further benefits to be accrued by obtaining the shrinkage curve, SC, for the soil. Numerous estimation procedures have emerged that have been found to be acceptable in engineering practice. Estimation procedures for the determination of unsaturated soil property function have been proposed for all applications commonly encountered in engineering practice (e.g., seepage, shear strength and volume change). This paper synthesises general methodologies that have resulted in increased usage of unsaturated soil mechanics in geotechnical engineering practice.

1 PREAMBLE

In 1962 I graduated from the University of Saskatchewan, Canada, with a Bachelor's degree in civil engineering. My first job was with the Division of Building Research, National Research Council of Canada, Saskatoon. Upon my first day of employment my supervisor tried to explain the nature of the most serious geotechnical engineering problem faced by geotechnical engineers in the prairies of Saskatchewan, Canada. He then proceeded to place a large stack of research papers on my desk related to expansive soil problems in various countries of the world. My first observation was that many of the papers were from South Africa. I must admit that when I thought about the country of South Africa, expansive soils were not the first thought that came to my mind. There were names such as Jerry Jennings, Tony Williams, Geoffrey Blight, George Donaldson, etc. As I read about the problems associated with expansive soils I immediately realized that I needed more education and consequently I enrolled at the University of Alberta and completed my Master of Science degree in 1964. My thesis involved, "The Measurement of the Soil-Water Characteristic Curve for Regina Clay". And the rest of my life can be summed up as, "measuring and trying to understand the soil-water characteristic curve".

In 1965 I was working at a consulting engineering firm in Alberta, Canada and I was selected to attend the First International Conference on Expansive Soils held at Texas A & M, Texas. I believe that I was the youngest person at the conference; that was 54 years ago. It was at that conference that I met Jerry E. Jennings. We met and talked. The thing I remember most was the fact that he had ample time to talk to a young, eager-to-learn lad. I can remember his lecture on the Double Oedometer method for testing expansive soils. In particular I remember the question period that followed. During the discussion period someone stood up and criticized the testing procedure because effective stresses and total stresses had been plotted on the same axis of the graph. Professor Jennings responded, "I am not that concerned about whether I have mixed total and effective stresses because I know that my methodology gives me that correct answers for the prediction of heave". Professor Jennings was truly a highly respected pioneer and originator of research into the behaviour of expansive and unsaturated soils.

2 INTRODUCTION

Research in unsaturated soil mechanics has witnessed rapid growth over the past couple decades. The primary focus area appears to have been related to the study of practical methodologies and protocols that are acceptable for use in geotechnical engineering practice. Problems encountered in engineering practice often involve the ground surface infiltration of water into the soil, its movement within the soil above the phreatic line and actual evaporation or the removal of water to the atmosphere. The two soil property functions required for computer simulation of this process are the hydraulic conductivity (or coefficient of permeability) and water storage relationships. Water flow and water storage are no longer governed by constant soil parameters but rather are nonlinear soil property function rendering the computer simulation highly nonlinear and computationally demanding.

There are a wide range of unsaturated soils problems that also require the quantification of shear strength and volume change properties. In all cases the unsaturated soil properties take the form of functions that are essentially constant up to the air-entry value of the soil and thereafter become nonlinear relationships (Fredlund 2006). The application of unsaturated soil mechanics and the proposed estimation procedures for unsaturated soils have been found to be increasingly acceptable in terms of the added-value brought to the project. All estimation procedures for unsaturated soil property functions, USPFs, appear to be make use of a measured (or in some cases an estimated) soil-water characteristic curve, SWCC. This paper assumes that the SWCC for the soil has been measured. The SWCC has been referred to as the key to the implementation of unsaturated soil mechanics in geotechnical engineering practice (Fredlund 2015). In addition, this paper draws attention to the synergistic effect of measuring (or estimating) the shrinkage curve, SC, for the soil.

The term, "soil-water characteristic curve" (and water retention curve), has been used in diverse manners without a clear and precise definition as to its meaning (Fredlund 2017). One of the reasons for the lack of precision in the definition of the SWCC is the result of numerous related disciplines that use similar terminologies. Soil physics and other agronomy-related disciplines that have used the term prior to its usage in geotechnical engineering. Research studies in agriculture-related disciplines have provided valuable guidance in establishing testing protocols and data interpretation principles for the application of unsaturated soil mechanics in geotechnical engineering. The discipline of soil physics has made extensive usage of the relationship between the amount of water in the soil and soil suction since the early 1900s. Analytical models were developed for the prediction of water availability for plant growth (Klute 1965).

With the rising interest in applying unsaturated soil mechanics in geotechnical engineering, there have been attempts to directly transfer and apply the research findings in soil physics without paying sufficient attention to the basic assumptions related to the application of the SWCC in soil physics. Of noteworthy importance was the assumption that overall volume changes when changing soil suction (i.e. measuring the drying SWCC), are of secondary interest and can be assumed to be negligible. Geotechnical engineers have been involved with a wide range of soil types with both high and low volume changes associated with changes in soil suction. Geotechnical engineers want to develop estimated unsaturated soil property functions that are adequate for analysing commonly encountered unsaturated soils problems.

The primary objective of the paper is to provide a state-of-progress on the protocols that appear to be emerging for the implementation of unsaturated soil mechanics in routine geotechnical engineering practice. The scope focuses on the three classic areas of soil mechanics; namely, i.) flow of water through unsaturated soils, ii.) the shear strength of unsaturated soils and iii.) to a lesser degree volume changes in unsaturated soils. The emerging protocols are based on estimation procedures based on two basic and commonly available unsaturated soil laboratory test; namely, i.) the soil-water characteristic curve, SWCC, and ii.) the shrinkage curve, SC, test. The paper illustrates the synergistic relationship which exists between the main (drying) soil-water characteristic curve and the shrinkage curve. The ease with which these unsaturated soil relationships can be measured in the laboratory has strongly influenced the procedures that have become the state-of-practice in geotechnical engineering.

3 UNSATURATED SOIL MECHANICS TERMINOLOGY

It may seem strange to suggest that at this stage there needs to be a definition given for the term, "unsaturated soil", within the context of geotechnical engineering. However, it is important to define what is meant by an unsaturated soil because more than one definition exists in the research literature. The definition for an unsaturated soil is closely related to the procedures that are subsequently proposed for the measurement of the SWCC and the definition of unsaturated soil property functions, USPFs.

The United States Geological Survey (Robinson & Spieker 1978), defined the vadose zone as "that part of the earth between the land surface and the water table (i.e. atmospheric pressure)". In geotechnical engineering, the vadose zone is more commonly referred to as the "unsaturated soil zone" even though the soil zone immediately above the phreatic surface (i.e. the capillary zone) has voids that may be essentially filled with water. The soil in the capillary zone may have some occluded air bubbles and as such the pore fluid has increased compressibility. Stated another way, the "unsaturated soil zone" is defined by the stress state in the soil (i.e. negative pore-water pressures) rather than on the amount of air or water existing in the voids. This definition for an "unsaturated soil" is important since the determination of the soil-water water characteristic curve, SWCC, always starts from a "zero suction" state regardless of the actual degree of saturation. The unsaturated soil zone is consequently divided into three sub zones; namely, i.) capillary zone immediately above the phreatic surface, ii.) two-phase zone with continuous water and air fluid phases, and iii.) dry zone where the water phase becomes discontinuous. The demarcation between these sub zones is based on changes in the rate of degree of saturation change in the soil in response to changes in the negative pore-water pressure (Fredlund 2015).

3.1 *Transition between the Laboratory and the Field Conditions*

The relationship between the pore-water stress state in the field and the laboratory measurement of the soil-water characteristic curve can be visualized in terms of the equilibrium stress conditions as shown in Figure 1 (Fredlund 2015). Let us assume that the water table was previously at ground surface and then was slowly lowered to a designated depth.

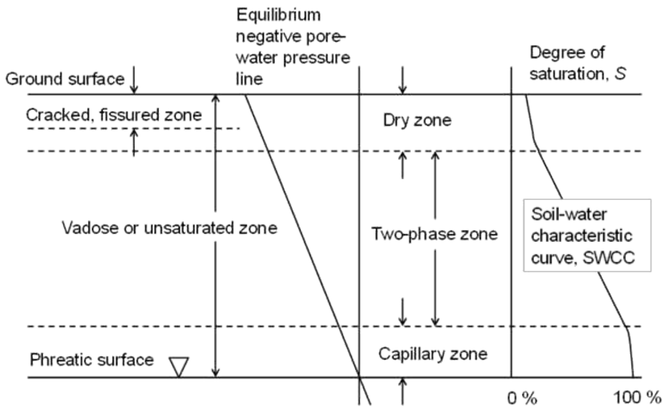

Figure 1. Definition of the subdivisions of the unsaturated soil zone

The negative pore-water pressures correspond to hydrostatic equilibrium conditions. Let us suppose that a borehole was drilled in the field and water content, solids and bulk density measurements were taken, allowing the determination of the degree of saturation. The degree of saturation immediately above the water table would be found to be near saturation and a distinct change occurs at the top of the capillary zone. If the soil deposit is sufficiently thick, a second transition would be found between the "two-phase" zone and the "dry" zone. This second break is referred to as "residual conditions" (i.e. residual water content and residual suction.

The above description provides an indication of the relationship between the field soil conditions (including its stress state), and an important laboratory test that has become the key to the implementation of unsaturated soil mechanics; namely, the soil-water characteristic curve, SWCC (Fredlund 2002). Geotechnical engineers have been able to greatly benefit from past experience and research undertaken in soil physics, soil science and agronomy-related disciplines. It has also been important for geotechnical engineers to consider the end goals of geotechnical engineering practice. The goal of the geotechnical engineer is to perform a relatively inexpensive test on the undisturbed soil sample that could provide him with information related to the physical behaviour of the entire soil deposit. This goal is achieved in the laboratory, to a large extent by measuring the amount of water retained in the soil as drying occurs through the application of suction. The measured relationship can be referred to as the (main) drying soil-water characteristic curve.

The ground surface is continually subjected to a moisture flux that is changing in accordance with weather conditions, subsequently forming a complex moisture flux type of boundary condition. Figure 2 shows the components that combine to give rise to net infiltration (or percolation) at the ground surface. The relative magnitudes of the upward moisture flux (i.e., evaporation and evapotranspiration) and the downward water flux (i.e., precipitation) perturb the hydrostatic equilibrium (negative) pore-water pressure profile giving rise to a trumpet shape for soil suction variations over time.

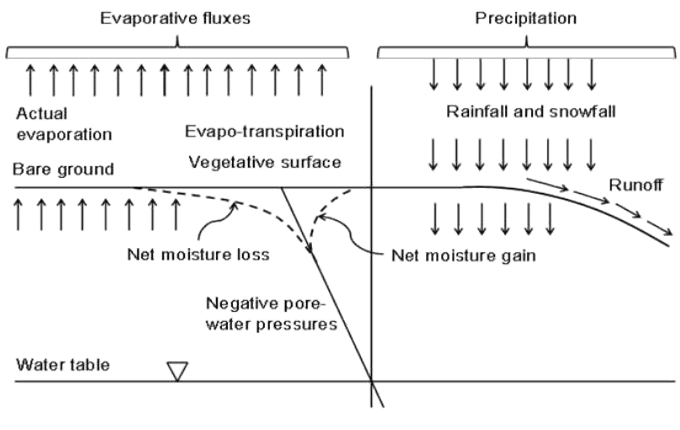

Figure 2. Moisture flux components associated with the calculation of net moisture flux at ground surface

Figure 2 shows how negative pore-water pressures in the vadose zone are altered by the imposed weather conditions. Negative pore-water pressures can also vary due to fluctuations in the groundwater table. Common to numerous geotechnical engineering problems (e.g. soil cover designs) is the assessment of the water balance at the ground surface as a function of time. The ground surface water balance can be assessed based on weather station and soils information data. Published results associated with field case histories suggest that, in general, the water balance components near ground surface can be adequately predicted for geotechnical engineering purposes (Wilson et al. 1994, Tran et al. 2014, Fredlund et al. 2012).

An unsaturated soil can have degrees of saturation ranging from 0 to 100%. The wide variation has led to the need to define soil properties in terms of nonlinear "unsaturated soil property functions, USPFs". The soil property functions are mathematical equations that render the engineering analysis (e.g. numerical modelling) the solution of one or more nonlinear partial differential equations.

There are three basic pillars for the practice of unsaturated soil mechanics; namely, i.) engineering protocols, ii.) laboratory testing procedures and iii.) numerical modeling techniques (Fig. 3). Laboratory testing procedures need to either directly or indirectly provide information on the physical soil properties while numerical modelling techniques simulate physical processes.

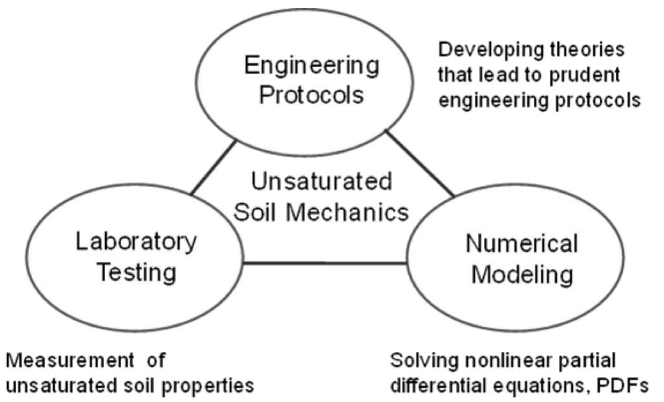

Figure 3. Basic pillars related to the implementation of unsaturated soil mechanics

3.2 *Contributions from Soil Physics and Geotechnical Engineering*

The study of unsaturated soil behaviour has historically emerged on two fronts; one within soil physics (and related agricultural disciplines), and the other within geotechnical engineering. Some of the early contributors in soil physics are as follows: Haines (1927), Richards (1931), Edelefsen & Anderson (1943), Childs & Collis-George (1950), Klute (1965), Burdine (1952), Gardner (1961), Brooks & Corey (1964), van Genuchten (1980), and Mualem (1976). Some of the early research contributors in geotechnical engineering can be listed as follows: Croney (1952), Jennings & Knight (1957), Bishop, Alpan, Blight & Donald (1960), Aitchison (1961), Jennings (1969), Barden (1965), Lytton & Woodburn (1973), Fredlund & Morgenstern (1977), and Escario (1980). The above-mentioned list is by no means complete; however, the list of references illustrates the different time periods over which basic unsaturated soils research has been conducted.

Extensive research studies took place within soil physics in the early 1900s. These studies mainly focused on moisture movement through soils in the vadose zone. Little attention was given to overall volume change of the soil as soil suction was changed. Formulations generally assumed that the soil structure was rigid. The water storage capacity of near-ground-surface soils was of interest from the standpoint of plant growth. The amount of water storage in the soil was measured in terms of volumetric water content, θ_w, and presented as a function of the negative pore-water pressure (or suction) in the soil.

The soil-water characteristic curve, SWCC, first emerged in the context of water movement modelling from the early studies in soil physics (Klute 1965). In 1986, Klute identified a series of "findings" related to the SWCC that form important reference points for both soil physics and geotechnical engineering. Some of these "finding" are paraphrased below.

1. The relationship between soil water content and soil suction (i.e., herein referred to as the soil-water characteristic curve, SWCC, but also referred to as the water retention curve, WRC, in the research literature) relates a "capacity factor" (i.e., the amount of water in the soil), and "the energy state", (i.e., suction stress state) for the soil-water.
2. The SWCC "is a fundamental part of the characterization of the hydraulic properties of a soil".
3. The "energy per unit volume", is equivalent to force per unit area or pressure (i.e., soil suction).
4. Water content can "be expressed on a weight, volume, or degree of saturation basis". "For analysis of water flow in soil profiles, the volume basis is most useful". While the volume basis for water content has been used in soil physics, other designations for the amount of water need to be given consideration for geotechnical engineering.
5. The SWCC is "primarily dependent upon the particle-size distribution of the soil and the structure ---" (Croney et al. 1958).
6. The SWCC is hysteretic (i.e., water content at a given suction on the wetting curve is less than that along the drying curve), (Haines 1927, Topp & Miller 1966).
7. The branches of the SWCC are defined as follows: i.) the initial drying curve starting at a degree of saturation of 100%, ii.) the main wetting curve measured after the soil has been dried to

near residual water content conditions, iii.) the main drying curve which may have 10 to 20% entrapped air due to incomplete saturation upon wetting, and iv.) there are an infinite set of scanning curves inside the drying and wetting bounding curves. Figure 4 shows a typical set of (initial and main) drying and wetting SWCCs. The compilation of SWCCs defines the boundaries for the relationships between the amount of water in the soil and soil suction.

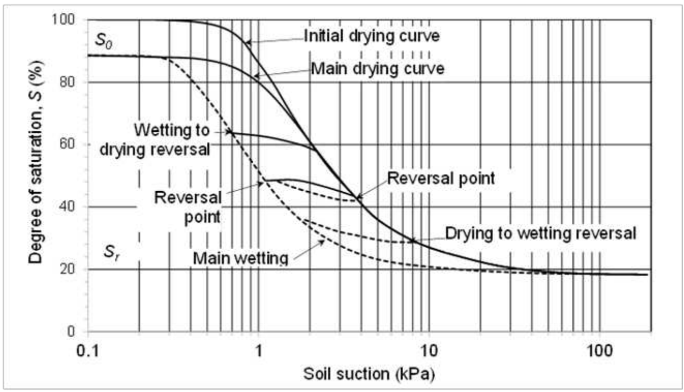

Figure 4. Hysteresis with respect to drying and wetting curves of degree of saturation versus soil suction for an unsaturated soil (modified from Klute 1986)

8. The SWCC for "rigid structure soils show constant water content up to the air-entry value" for the soil.
9. If the soil structure deforms (i.e., shrinking or swelling), the water content can decrease as soil suction increases without reaching the entry of air, (i.e., no change in degree of saturation).
10. In some cases, only the drying SWCC is required for modeling water flow. In other cases, only the wetting curve is required. Modeling flow across the ground surface requires the incorporation of hysteretic behavior (Mualem 1976).
11. The SWCC is a constitutive material property that relates a stress state (e.g., matric suction and total suction) to the amount of water in a soil.
12. Water flow occurs in response to gradients of total hydraulic head (i.e., pore-water pressure head plus elevation head).
13. SWCCs should be measured on "undisturbed core samples". In some cases, remolded and disturbed material may be all that is available for the measurement of the SWCC.

An arbitrary division of 1500 kPa was suggested for the boundary between the "low suction range" and the "high suction range". The arbitrary division was selected mainly on the basis of the highest air-entry ceramic disk that can be manufactured and utilized in pressure plate devices. Consequently, soil suction has been defined in terms of matric suction from zero to 1500 kPa, and measured or imposed total suction in the range from 1500 to 1,000,000 kPa.

The arbitrary division between the low and high suction ranges results in two different components of soil suction being used when measuring the SWCC. Inconsistency in the use of two suction components does not appear to create significant application difficulties in either soil physics or geotechnical engineering (Fredlund 2015).

Geotechnical engineers were interested in shear strength and volume change behaviour in addition to water flow through unsaturated soils. Several research conferences were held in the 1960s and there was an attempt to transfer research formulations related to physical processes studied in soil physics into the emerging field of unsaturated soil mechanics. The proceedings of the 1961 London, England conference was titled, "Pore Pressure and Suction in Soils". In 1965 a symposium-in-print titled "Moisture Equilibria and Moisture Changes in Soils beneath Covered Areas" was edited by G. Aitchison (Aitchison 1965). The initiative for a series of conferences focused on better understanding the behaviour of expansive and unsaturated soils was spear-headed by Professors Spenser Buchanan (Texas, USA) and Jerry Jennings (South Africa). The First International Conference on Expansive Soils at Texas A & M, TX, in 1965 and the Second International Conference on Expansive Soils was also held at Texas A & M, TX, in 1969. These conferences mainly focused on moisture movement and swelling clay problems encountered in geotechnical engineering. A series of research studies were undertaken at Imperial College, London, in the 1950s mainly under the supervision of Professor Allan W. Bishop.

A number of difficulties became apparent as mathematical formulations were transferred from soil physics into unsaturated soil mechanics. Geotechnical engineers were accustomed to using soil mechanics' principles for water flow through soils. Hydraulic head was used as the driving potential for saturated soils and the desire was to maintain a similar formulation for water flow above the phreatic surface.

Geotechnical engineers also viewed water flow problems in terms of steady state and transient type analyses. As a result, the hydraulic properties for unsaturated soils were viewed in terms of two independent soil property functions; namely, i.) the water permeability function, kw, and ii.) the water storage function, m2w. These two material property functions served different roles when considering solutions of interest in geotechnical engineering. Each of the hydraulic property functions involved different mathematical operations for their assessment even though both properties were closely related to the soil-water characteristic curve, SWCC.

One of the practical engineering problems facing geotechnical engineers was the prediction of heave in swelling soils. The need to predict total heave in swelling soils provided the primary impetus for a series of international research conferences from 1964 to 1992. The conferences were directed towards better understanding expansive soil behaviour. Partial differential equations governing moisture movement were re-derived in a form more fitting for geotechnical engineering applications. Studies in soil physics had given little consideration to shear strength and volume change problems but these were of significant interest in geotechnical engineering.

3.3 *Terminology for Various SWCC Relationships*

There are a number of volume-mass versus soil suction relations that need to be defined (Fredlund et al. 2018). Figure 5 illustrates the labels for the family of gravimetric water content SWCCs. It is suggested that the following designation be used for soil-water characteristic curves describing the amount of water in the soil. The “pre SWCC” variable (i.e. *w, S,* or θ), designates the measure of the amount of water in the soil while the “post SWCC” variable (i.e. *i* for initial, *d* for drying, or *w* for wetting), referring to the bounding curve under consideration.

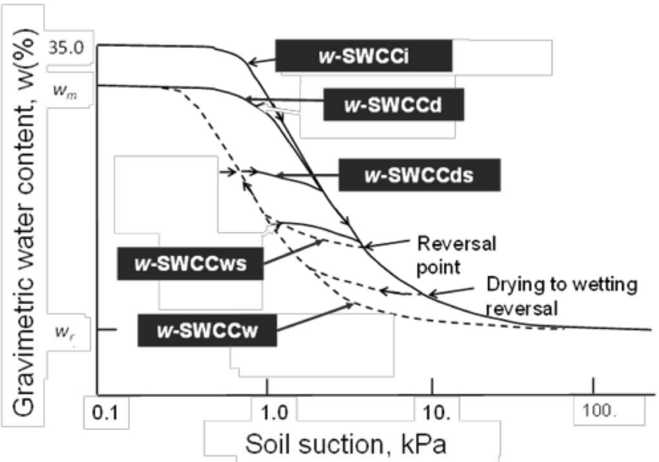

Figure 5. Suggested labelling for the gravimetric water content versus soil suction family of curves

Each of the family of SWCCs have independent roles to play in understanding unsaturated soil property functions. For example, it is the *S*-SWCCs that can be integrated to compute the permeability function for an unsaturated soil. It is also the *S*-SWCC drying curve that is used to determine the “true” air-entry value of a soil (Fredlund 2017). The *θ*-SWCCs can readily be differentiated to get the water storage function for an unsaturated soil. The *w*-SWCC is most easily measured in the laboratory and forms the basis for the calculation of other volume-mass functions when combined with shrinkage curve data. Overall volume change characteristics can be written in terms of changes in void ratio (i.e. *e*-CC*i*, *e*-CC*d*, and *e*-CC*w*).

3.4 *Philosophy of Estimation of Unsaturated Soil Property Functions*

Research into the behaviour of unsaturated soil focused mainly on improving our understanding of the constitutive laws controlling physical behaviour. Routine geotechnical engineering practice had pursued a strong science basis for the application of saturated soils. This involved obtaining undisturbed soil samples from field boreholes and performing specific physical tests in the laboratory. The test results produced the soil properties that can be inserted into constitutive soil models.

It seemed intuitive that a similar application model would be used for solving soil mechanics problems involving unsaturated soils. However, this was not to be the case because of the excessive costs associated with transferring such a model into the unsaturated soil realm. For example, the cost of measuring the coefficient of permeability function for an unsaturated soil would likely be in the order of 10 times the cost of measuring the saturated coefficient of permeability for the soil. The economics of such an applications model was simply untenable.

Over time, research studies in both soil physics (and agriculture) and geotechnical engineering began to gravitate towards the possibility of using “estimation models” for the determination of unsaturated soil property functions. This realization first became apparent in agricultural applications where the primary concern was the availability of water storage for plant growth. All “estimation models” appeared to be related to applying mathematical tools to the soil-water characteristic curve. In fact, the soil-water characteristic curve, SWCC, quickly became the key to the implementation of unsaturated soil mechanics into geotechnical engineering and other disciplines.

The water storage function, m2w, versus soil suction could be estimated through differentiation of the volumetric water content SWCC. “Estimation procedures” have also emerged for the estimation of the unsaturated soil permeability functions (Fredlund et al. 1994). All procedures appeared to be based on a type of integration procedure applied along the SWCC (note: further details are provided later). Estimations for shear strength functions were proposed that involved the use of the SWCC (Vanapalli et al. 1996). As well, other volume-mass unsaturated soil property functions have also been proposed (Fredlund et al. 2012).

The remainder of this paper focuses on presenting some of the most recent and most reliable estimation procedures for determining unsaturated soil property functions. In all cases, the estimations are based on the SWCC. The only exception will be the refinement of the estimation procedure through use of an additional laboratory test; namely, the measurement of the shrinkage curve, SC, for the soil. The desire through ongoing research is to gradually improve the accuracy of our ability to estimate unsaturated soil property functions through use of cost-effective estimation procedures.

4 EQUATIONS DEFINING UNSATURATED SOIL RELATIONSHIPS

There are two independent stress state variables that can be used to describe unsaturated soil behaviour; i.) net total stress (along with shear stress) variables with components in three orthogonal directions, and ii.) isotropic matric and total suction (or soil suction) (Fredlund 2016). The complexities of the stress state of an unsaturated soil are such that it is possible for a wide variety of stress paths to be followed in geotechnical engineering problems. However, to make the analysis of problems amenable, the soil suction variable has largely been considered in a manner independent of the total stress state. While this may not be as accurate as desired, it has considerable added-valued within the context of geotechnical engineering.

There are two possible volume-mass properties that can change in response to a change in stress state. These are: i.) a change in overall volume (i.e., void ratio change, (de), and ii.) a change in degree of saturation, (dS). Equation [1] shows the basic volume-mass relationship.

$$Se = wG_s \tag{1}$$

where S = degree of saturation; e = void ratio; w = gravimetric water content; and G_s = specific gravity of soil solids. Incremental differentiation of Eq. [1] shows that a change in gravimetric water content can occur as a result of either a change in void ratio or a change in degree of saturation (Fredlund & Rahardjo 1993). Consequently, it is imperative to define two constitutive relations when defining all volume-mass relations.

4.1 *Measuring the Laboratory Unsaturated Soil Tests*

Figure 6 illustrates how a pressure plate soil specimen, (i.e. w-SWCC), and a shrinkage curve soil specimen can be trimmed from a common undisturbed soil sample. The SWCC specimen is generally about 70 mm in diameter and 30 mm thick. The SC specimen can be much smaller with a diameter of 30 mm and a thickness of 10 mm. The smaller size is used to ensure the specimen does not crack. The soil specimens are confined in a metal ring and can be prepared in a slurry state, a compacted state or an undisturbed state. The initial conditioned state should be the same for the SC test and the independently run SWCC test.

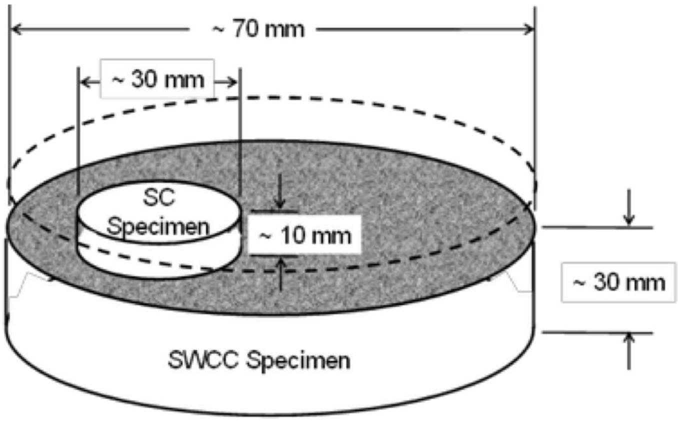

Figure 6. Use of similar initial volume-mass conditions for the w-SWCC and SC tests

Figure 7 shows a pressure plate apparatus that can be used to obtain gravimetric water content versus suction data corresponding to the lower matric suction range (i.e. suctions less than 1500 kPa). Equilibrium between the soil specimen and each applied suction is usually achieved within approximately one day. A typical test can be completed in one to two weeks depending on the soil type.

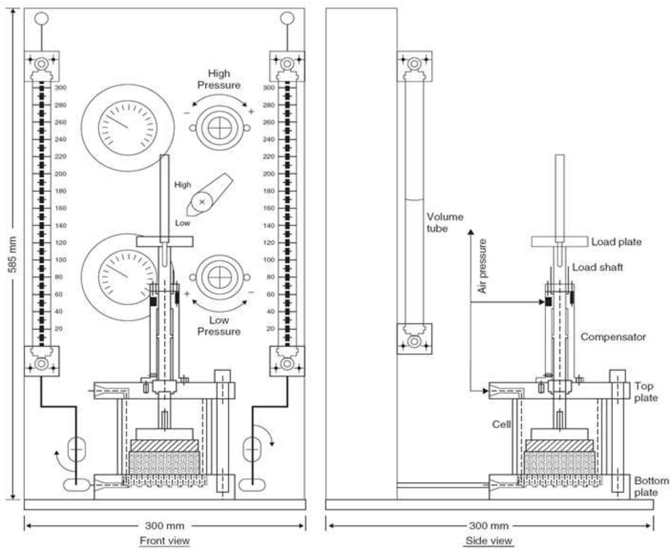

Figure 7. Front and side views of Pressure Plate cell (Courtesy of GCTS, AZ)

Figure 8 shows pressure plate test results measured on three similar tests on silt soil that had essentially zero volume change as soil suction was increased. The initial drying curve, (w-SWCC$_i$), the main drying curve, (w-SWCC$_d$), and the main wetting curve, (w-SWCC$_w$) were measured on all three specimens. Several observations can be made; namely, i.) the results are reproducible, ii.) the initial drying curve, (w-SWCC$_i$), and the main drying curves, (w-SWCC$_d$), produce similar air-entry and residual suction values, and iii.) the drying and wetting SWCCs were essentially congruent (i.e., parallel) on a semi-log plot. It should also be mentioned that each set of w-SWCC tests took more than one month to perform. It is the considerable length of time required to perform the complete characterization of the SWCC that has led to a favoured testing protocol that simply involves the measurement of the drying soil-water characteristic curve in engineering practice.

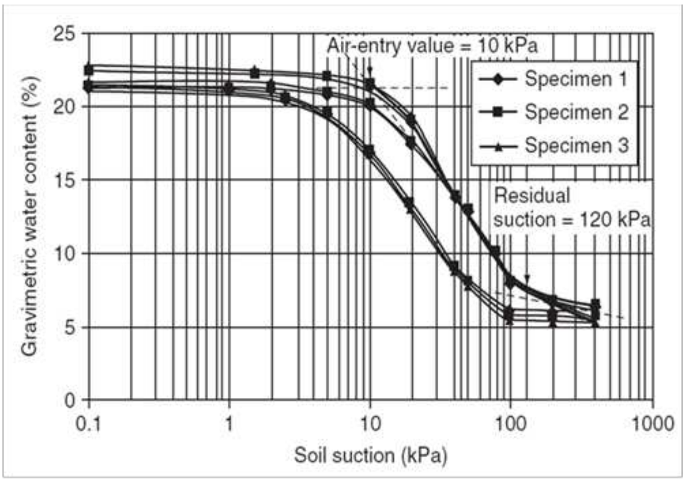

Figure 8. Drying and wetting SWCCs measured on a silt soil using the Pressure Plate apparatus (Pham, 2002)

A typical set of data has been generated for an artificial clayey silt soil to show the relationship between the various SWCC and explain the interpretation of the data. Figure 9 shows the measured gravimetric water contents plotted versus soil suction, *w*-SWCC, for an artificial clayey silt. The initial water content was 31.5 %. Data points for the suction range below 1500 kPa were measured using a pressure plate apparatus. Data points at higher suctions measured the equilibrium water content in small soil specimens allowed to come to equilibrium in desiccators kept at fixed relative humidity.

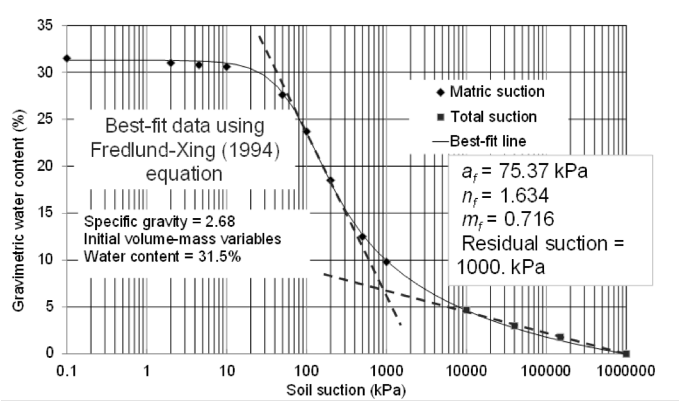

Figure 9. Soil suction data with best-fit Fredlund-Xing (1994) SWCC function for the artificial clayey silt

There was a gradual downward trending of the *w*-SWCC plot as matric suction was applied. The water content versus suction tends towards a straight-line relationship when suctions in excess of about 40 kPa were applied. The relationship showed a second bend at suction in excess of 1000 kPa (i.e. the approximate residual suction) as suctions goes towards 10^6 kPa. The sigmoidal shape of the relationship allows for a best-fit of all the data from a suction of 0.1 kPa to 1,000,000 kPa by using the empirical equation proposed by Fredlund-Xing (1994). It is later shown that the first curvature does not represent the "true" air-entry value of the soil when the specimen undergoes volume change as suction is increased. The best-fit of the data yields the following parameters; a_f = 75.37 kPa, n_f = 1.634, and m_f = 0.716. Residual suction is estimated to be around 1000 kPa.

Shrinkage curves, SCs, can readily be measured in the laboratory on a small specimen trimmed within a metal ring. Mass and volume measurements are recorded once or twice daily as the soil specimen dries to surrounding atmospheric conditions (Fig. 10).

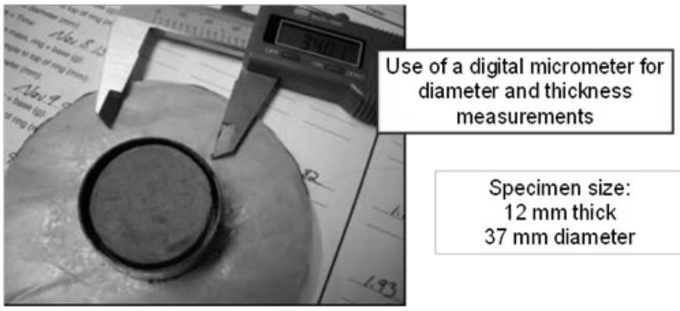

Figure 10. Measurement of the "Shrinkage Curve", SC using micrometer calipers

The challenge has been to find a reliable means of measuring the volume of the soil specimen each time the mass is measured (Wong et al. 2018). The volume of shrinkage curve soil specimens is commonly measured using micrometer calipers.

Figure 11 shows the measured shrinkage curve data for the artificial clayey silt soil. The fitting parameters for the M. Fredlund (2000) shrinkage curve equation are; a_{sh} = 0.40; b_{sh} = 0.146 and the c_{sh} = 3.0. The specific gravity of the soil is 2.68. Each of the fitting parameters has physical significance which will later be explained. It is also possible to estimate the shrinkage curve for many geotechnical engineering problems (Fredlund & Zhang 2017).

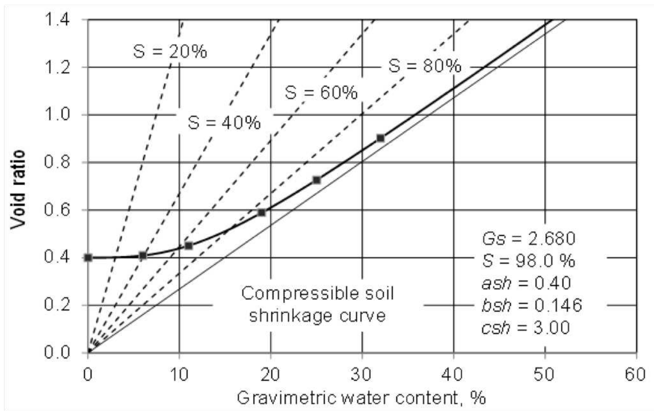

Figure 11. Typical shrinkage curve for a compressible soil.

5 GENERAL EQUATION FOR THE SWCC

Let us now consider a typical drying soil-water characteristic curve, such as is shown in Figure 9. Data for the drying SWCC generally takes on the form of a single sigmoidal mathematical function on a semi-log graph. There are numerous equations that have been proposed to best-fit to experimental *w*-SWCC data (van Genuchten 1980, Fredlund & Xing 1994, Pham & Fredlund 2011). The proposed methodology for interpretation applies to any equation that can best-fit the data in the low suction range while ending at zero water content at a suction of one million kPa. The Fredlund & Xing (1994) equation meets these requirements and will be used for illustration purposes. If a soil specimen undergoes excessive volume change upon drying it might be difficult to fit the entire data range with a single sigmodal equation. A bimodal form of the w-SWCC equation can also be used if necessary (Gitirana Jr. & Fredlund 2005, Satyanaga et al. 2013, Zhang & Chen 2005).

The Fredlund & Xing (1994) equation has four fitting parameters including residual suction. Residual suction only needs to be approximate and can be estimated using the empirical construction procedure suggested by Vanapalli et al. (1998). The fitting variables associated with the SWCC equation can be obtained using a regression analysis. The fitting of the gravimetric water content SWCC (*w*-SWCC) should not be confused with subsequent regression on the calculated degree of saturation SWCC (*S*-SWCC) data. The fitting parameters for *w*-SWCC will have a slightly different physical meaning from those assigned to the degree of saturation SWCC, (*S*-SWCC) if the soil undergoes volume change as soil suction is increased. The Fredlund & Xing (1994) SWCC equation can be written as follows.

$$w(\psi) = \frac{w_s C(\psi)}{\{1\text{n}[\exp(1)+(\psi/a_f)^{n_f}]\}^{m_f}} \tag{2}$$

where $w(\psi)$ = water content at any soil suction, ψ, a_f = fitting parameter related to the suction near the inflection point of the *w*-SWCC, n_f = fitting parameter related to the maximum rate of gravimetric water content change, m_f = fitting parameter related to the curvature near residual gravimetric water content conditions, ψ_r = suction near residual conditions of the soil, and $C(\psi)$ = correction factor directing the *w*-SWCC towards a suction of 10^6 kPa at zero water content, written as;

$$C(\psi) = 1 - \frac{1\text{n}(1+\psi/\psi_r)}{1\text{n}(1+10^6/\psi_r)} \tag{3}$$

A close fit can generally be achieved with laboratory data when using the Fredlund-Xing (1994) equation; however, this is not always the case. The data may show a gradual downward trend even under relatively low applied suction values. This bend on the *w*-SWCC may not indicate the approach of the air-entry value (or desaturation). The downward trend indicates the true air-entry value when there is no volume change as soil suction is increased. The "true" air-entry value needs to be determined from the degree of saturation SWCC (*S*-SWCC). It should be noted that Equations 2 and 3 can also subsequently be used to best-fit the calculated degree of saturation versus suction data.

6 GENERAL EQUATION FOR THE SHRINKAGE CURVE, SC

Measured changes in gravimetric water content might be the result of volume changes (i.e. void ratio changes), or degree of saturation change (Fredlund et al. 2002). By combining the *w*-SWCC with a shrinkage curve, SC, it is possible to assess two independent suction-related processes; namely, volume change, (e-CC_d) and degree of saturation change, (S-$SWCC_d$). The shrinkage curve has an important role to play when analysing unsaturated soil behaviour for a compressible soil.

The shrinkage curve for a soil has received minimal attention in soil mechanics. The reason for limited attention appears to be related to the end goal for the application of the SWCCs in agriculture-related disciplines which was the quantification of water storage in the soil (Klute 1985). The estimation of hydraulic conductivity was of secondary interest. The amount of water in a soil was quantified in terms of volumetric water content and the soil was assumed to undergo negligible volume change as soil suction was increased. The assumption of "no volume change" made for easy interpretation of SWCC laboratory data (Fredlund & Zhang 2017).

6.1 *Shrinkage curve equation for gravimetric water content versus void ratio*

Leong & Wijaya (2015) summarized several equations that have been proposed for the fitting of complex multi-modal shrinkage curves; the types sometimes encountered when drying from a variety of initial conditions. However, when matching the stress path of the *w*-SWCC test, the M. Fredlund (2000) hyperbole equation can best-fits measured shrinkage curve data.

$$e(w) = a_{sh}[\frac{w}{b_{sh}}^{c_{sh}} + 1]^{1/c_{sh}} \tag{4}$$

where a_{sh} = minimum void ratio upon complete drying, b_{sh} = variable related to the slope of the drying curve calculated as: $b_{sh} = (a_{sh} \times S_o)/G_s$, and c_{sh} = variable related to the sharpness of curvature as the soil desaturates, and S_o = conditioned degree of saturation. The emphasis is on characterizing the drying SC along a similar stress path to that used for the measurement of the *w*-SWCC. The initial degree of saturation will be near 100% (e.g. S = 98%).

The a_{sh} variable is relatively easy to determine since it is the minimum void ratio that a soil can attain upon complete drying. The a_{sh} variable can be determined while measuring the entire shrinkage curve or by simply measuring the minimum void ratio corresponding to an oven-dried soil specimen. Figure 12 shows a series of shrinkage curves for soils with various void ratios upon complete drying (i.e., a_{sh} ranging from 0.4 to 1.0). The c_{sh} was set to 3.0 and the b_{sh} variable is calculated from the a_{sh} variable, the specific gravity (G_s = 2.65) and the initial degree of saturation.

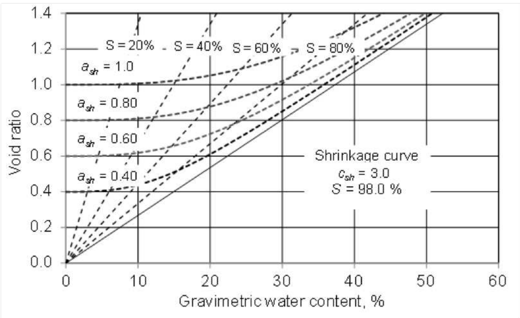

Figure 12. Effect of varying the ash fitting parameter of the shrinkage curve

The a_{sh} variable can also be estimated from the shrinkage limit, *SL*, of the soil. Casagrande (1932) showed that the shrinkage limit, *SL*, of a slurry or remolded soil could be estimated from the liquid limit and plasticity index of a soil. The shrinkage limit of a soil has been shown to lie either above or below the A-Line on the Casagrande plasticity chart. The A-Line on the plasticity chart corresponds to a straight line that is mathematically defined as follows.

$$PI_A = 0.75LL_s - 15 \quad (5)$$

where PI_A = plasticity index on the A-Line, and LL_s = measured liquid limit of the selected soil. The shrinkage limit, SL_S, of the soil can then be written in terms of the plasticity index of the soil, PI_S.

$$SL_S = 20 + (PI_S - PI_A) \quad (6)$$

Holtz & Kovacs (1981) suggest that the estimation of the shrinkage limit of a slurry or remolded soil appears to be of similar accuracy to those measured in a laboratory. The ability to accurately estimate the shrinkage limit of a soil from the other plasticity classification values means that one of the primary variables associated with the characterization of the volume-mass behavior of a soil can be determined independent of the measurement of the entire shrinkage curve. The shrinkage limit of the soil, SL_S, is equal to the amount of water required to fill the voids when the soil is dried to its minimum void ratio.

$$a_{sh} = SL_S G_S \quad (7)$$

where the initial degree of saturation of the slurry soil is assumed to be 100%. Consequently, the a_{sh} parameter for the shrinkage curve can be calculated from the soil plasticity values.

The second variable required for the hyperbolic characterization of the shrinkage curve equation is b_{sh} (Fredlund et al. 2002). The b_{sh} variable is calculated from the a_{sh} variable, the specific gravity, G_s, and the initial degree of saturation, S_o, as follows:

$$b_{sh} = a_{sh} S_0 / G_S \quad (8)$$

The remaining fitting parameter for the shrinkage curve is the c_{sh} variable when using the M. Fredlund (2000) equation. The c_{sh} variable defines the curvature of the shrinkage curve when moving between the near saturated state and the completely dried state. Figure 13 shows a family of shrinkage curves where the c_{sh} variable is arbitrarily varied while the a_{sh} variable is set at 0.6 and the specific gravity is 2.70. The initial degree of saturation was assumed to be 98%. The results show that there are practical limits to the c_{sh} variable. For example, a c_{sh} value of 15 or higher will essentially produce a horizontal line from the initial void ratio, a_{sh} to the initial saturation line (~ 100%). On the other hand, a c_{sh} value of about 1.5 produces a gradual curve that immediately starts to curve from the completely dry void ratio and gradually tends towards saturation.

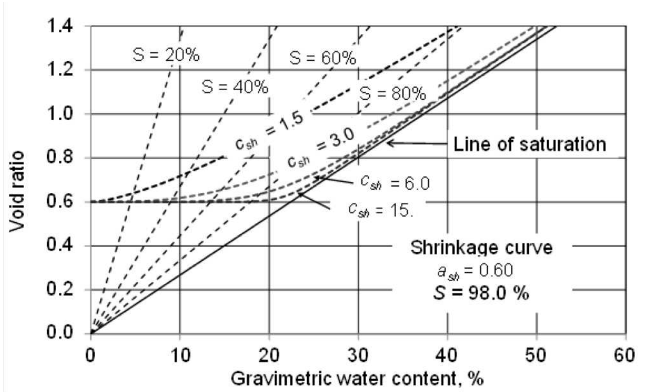

Figure 13. Effect of varying c_{sh} on the shrinkage curve

The common range of c_{sh} values for soils lies between 3 and 15. Soils with a c_{sh} value of 15 are considered low compressibility soils while soils with a c_{sh} value of 1.5 would have much higher compressibility (or undergo considerable volume change) as soil suction is increased. It would appear to be necessary to make several volume-mass measurements on a soil specimen as it dries in order to quantify the most likely value for the c_{sh} variable. A typical value of c_{sh} for a moderately compressible silty soil would be about 5. Research literature shows a number of studies that have been undertaken on the shrinkage curve behaviour of soils (Ho 1988, Marinho 1994, 2017, Tripathy et al. 2002). A further fitting study of shrinkage curve test results was recently undertaken by Wong et al.

(2018). The c_{sh} fitting parameter was found to vary with the plasticity of the soil and a close correlation was found with the plasticity ratio, PL/LL as shown in Figure 14. Each dot represents the best-fit c_{sh} value. A total of 27 soils were analysed. The best-fit equation for c_{sh} was found to be as follows.

$$c_{sh} = 11.7 + e^{3.1^{PL/LL}} \tag{9}$$

where e = is the base of the natural logarithm.

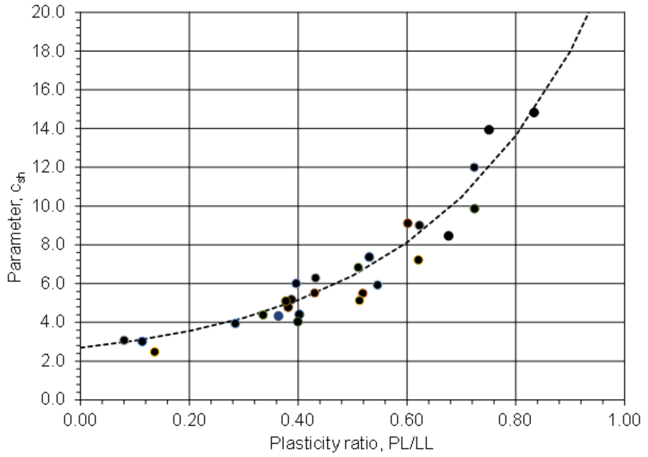

Figure 14. Relationship of c_{sh} to the plasticity ratio (i.e., PL/LL) (after Wong et al., 2018)

6.2 *Computation of other Volume-Mass SWCCs*

The measured *w*-SWCC and SC (i.e. *e* versus *w*) data can now be combined to allow the calculation of other volume-mass versus soil suction relationships for the artificial clayey silt.

6.2.1 *Void ratio versus soil suction relationship*

The first graph plotted is the void ratio versus gravimetric water content curve (Fig. 15). The plot shows that the soil starts to change volume when soil suction exceeds about 10 kPa. Once the applied suction exceeds about 500 kPa there is essentially no further change in volume as soil suction is increased. The maximum volume change is about 20 %.

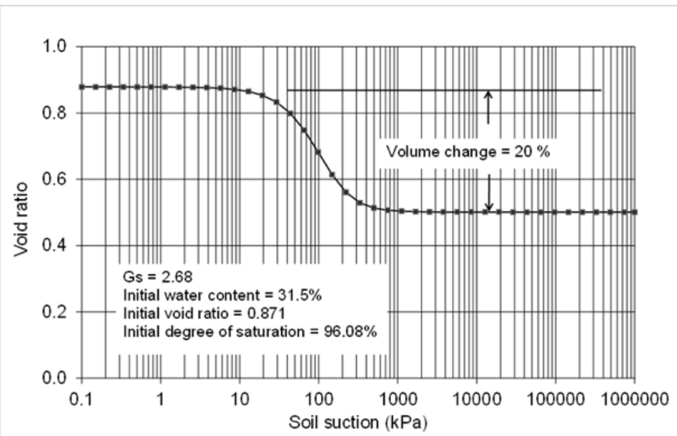

Figure 15. Void ratio versus soil suction showing volume change.

6.2.2 *Volumetric water content versus soil suction relationship*

Volumetric water content, θ, data points can be calculated using the following equation.

$$\theta(w) = \frac{w(\psi)G_S}{1+e_0} \tag{10}$$

where $\theta(w)$ = volumetric water content as a function of gravimetric water content, $w(\psi)$ = gravimetric water content as a function of soil suction and e_o = initial void ratio.

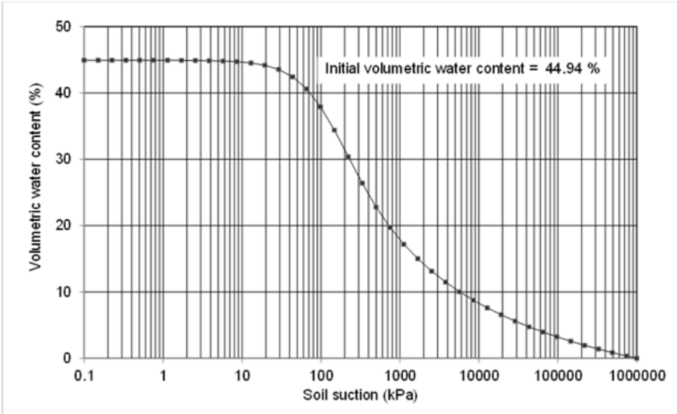

Figure 16. Volumetric water content, θ, versus soil suction

6.2.3 *Degree of saturation versus soil suction relationship*

The degree of saturation can be written as a function of gravimetric water content and void ratio.

$$S(w) = \frac{w(\psi)G_S}{e(w)} \tag{11}$$

where: $w(\psi)$ = gravimetric water content written as a function of soil suction (i.e., the *w*-SWCC), and $e(w)$ = void ratio written as a function of gravimetric water content (i.e. the shrinkage curve). Figure 17 shows the calculated data points for the degree of saturation versus suction relationship.

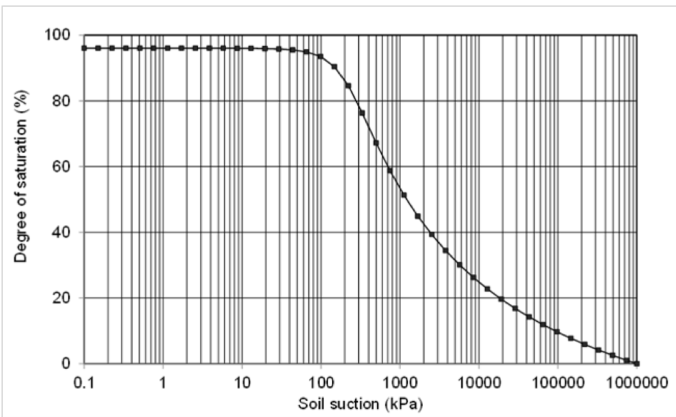

Figure 17. Data points for the degree of saturation SWCC calculated from *w*-SWCC and the SC.

6.2.4 *Determination of the Air-Entry Value, AEV, for a Soil*

Figure 18 shows how the calculated degree of saturation data points versus soil suction for the drying SWCC can be used to determine the “true” air-entry value, AEV, for the soil. The results show that there is quite a distinct air-entry value for the artificial clayey silt of 147 kPa. The degree of saturation also provides a more accurate value for residual conditions.

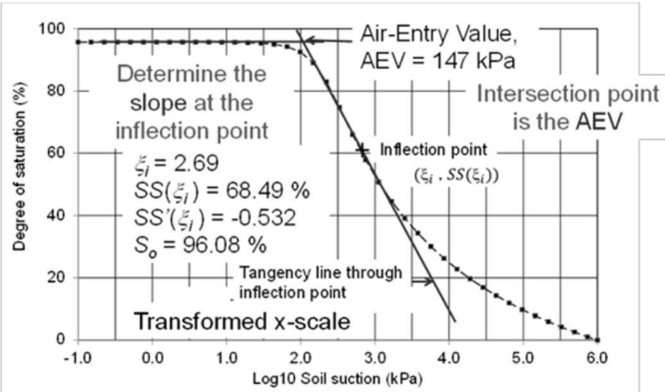

Figure 18. Definition of terms used on the substitution equation for the calculation of the “true” air-entry value.

All volume-mass relationships have been shown for the drying SWCC for initially saturated silty clay soil. These relationships can be generated from data obtained from two laboratory tests that are quite common in most soil physics and soil science testing laboratories as well as in geotechnical testing laboratories.

Changes in the volume of the soil specimens, as soil suction is increased, can significantly affect the interpretation of soil-water characteristic curve information. It is important to use analytical protocols that properly account for the independent effects of volume change when interpreting the *w*-SWCCs.

7 UNSATURATED SOIL PROPERTY FUNCTIONS FOR SEEPAGE PROBLEMS

The SWCC is most commonly used to calculate the hydraulic properties required when modelling flow through unsaturated soils. The relative permeability function is generated through use of one or more integration methodologies applied to the *S*-SWCC. The Fredlund et al. (1994) integration procedure will be used for illustration purposes. The integration procedure results in the calculation of a series of coefficient of permeability values that can be plotted against soil suction. The Van Genuchten (1980) (Mualem) and van Genuchten (1980) (Burdine) procedures, as well as other integration-based procedures can also be used.

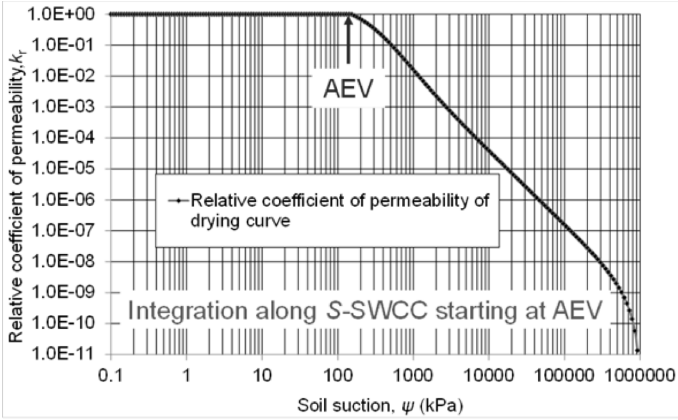

Figure 19. Relative permeability function for the artificial clayey silt as soil suction is increased beyond “true” air-entry value

The term “relative” permeability means that the unsaturated soil portion of the permeability function can be referenced to a value of 1.0. The “relative” permeability function can then be scaled vertically on the graph to apply to the same soil with any initial saturated coefficient of permeability as shown in Figure 20.

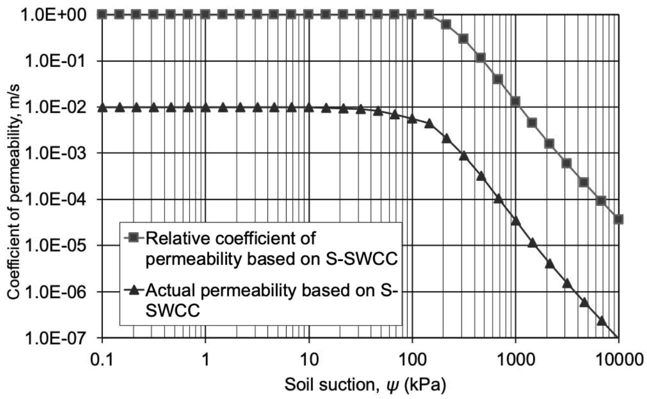

Figure 20. Scaling between the relative and actual permeability functions

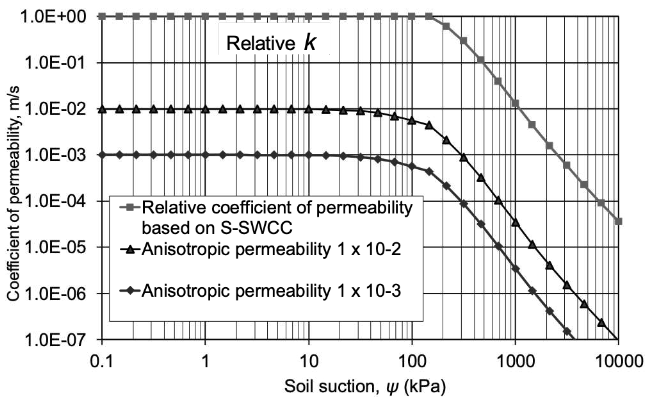

Figure 21. Scaling of the permeability functions for an anisotropic soil

It is also suggested that a single S-SWCC can also be scaled vertically for a soil that is anisotropic in nature (Freeze & Cherry 1980). Figure 21 illustrates how a single “relative” permeability function might be scaled to represent how the permeability functions might be scaled for a soil with a major saturated coefficient of permeability of $1x10^{-2}$ m/s and a minor coefficient of permeability of $1x10^{-3}$ m/s.

Changes in the coefficient of permeability can also occur from changes in the void ratio (or volume change) as shown in Figure 22. However, changes in permeability that are due to void ratio change are generally quite small in comparison to changes that occur when the degree of saturation is reduced. The coefficient of permeability versus soil suction can be calculated as an independent seepage related function.

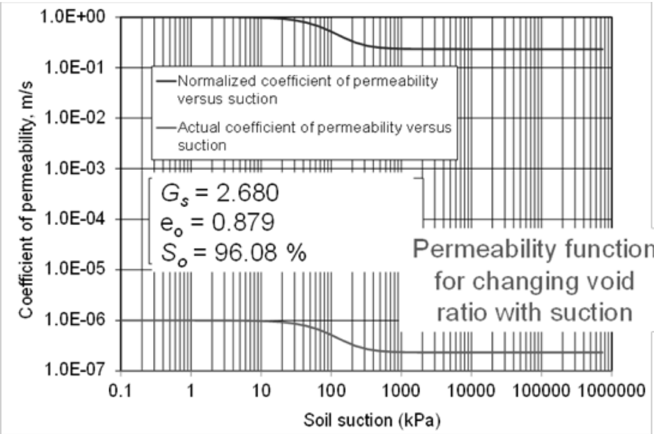

Figure 22. Changes in the permeability function in response to changes in void ratio.

The effect of void ratio changes required the incorporation of an independent physical relationship such as the (e3/(1 + e)) rule used by Taylor (1948) where e is void ratio.

A water storage function constitutes a second function that is required when modelling unsteady or transient flow through unsaturated soils. A water storage function can be calculated as the slope (or first derivative) of the volumetric water content versus soil suction curve (θ-SWCC). Figure 23 shows the water storage function for the clayey silt soil used for illustration purposes. Water storage is shown to be small at both low suction (but it must always be greater than zero), and at high suctions. The largest water storage occurs at the inflection point along the θ-SWCC.

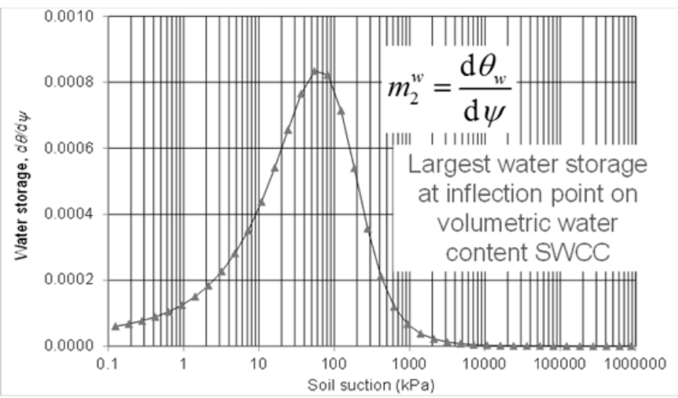

Figure 23. Water storage function for the artificial clayey silt soil.

8 UNSATURATED SOIL PROPERTY FUNCTIONS FOR SHEAR STRENGTH PROBLEMS

There are numerous shear strength equations or functions that have been proposed for an unsaturated soil. The proposed equations can be subdivided into: i.) equations that can be "fitted" to a data set, and ii.) equations that can be used to "estimate" the shear strength function. It appears that essentially all the "estimation procedures" claim to be based on the soil-water characteristic curve, SWCC. Comparisons of estimated shear strength functions to measured laboratory data are not all that reassuring since considerable scatter is often observed between measured and estimated shear strength functions (Fredlund et al. 2012).

The objective is to identify the critical conditions that must be satisfied in order to claim that an "estimation methodology" truly adheres to the conditions defined by the SWCC. The fundamental elements of physics contributed by a measured SWCC are first described and then these conditions are used to propose a new "estimation methodology" for the shear strength function.

8.1 *Fundamental Elements of Physics that Must be Adhered to*

1. A soil behaves as a saturated soil from the time a negative pore-water pressure is applied to the soil until the air-entry value, AEV, of the soil is reached.
2. The "true" air-entry value of the soil must be defined from the degree of saturation SWCC, S-SWCC.
3. Once the residual suction for the soil is reached, the rate of change of shear strength with respect to soil suction is zero. This condition (i.e. zero change in strength beyond the residual suction), has been commonly observed for low to medium plastic soils (Nishimura & Fredlund 2000).
4. The range of soil suction applications in geotechnical engineering seldom goes beyond the residual suction of the soil. Donald (1957) observed an actual decrease in the shear strength of sands at suction beyond the residual suction. However, it is generally agreed that the change in strength for sands beyond residual suction is small. Clayey soils generally show an increase in strength beyond the residual suction. Even when the suctions are well beyond residual conditions, they may not be relied upon for geotechnical engineering applications.
5. The slope of the shear strength function must be equal to the tangent of the effective angle of internal friction up to the AEV. This condition has been adhered to for most proposed shear strength functions.

6. The slope of the shear strength function must be zero at residual shear strength. This condition has not been adhered to by most previously proposed shear strength functions.
7. The transition between the suctions defined by the AEV and residual suction should scale in accordance with the logarithm of soil suction since the S-SWCC relationship is approximately linear between these two points on the S-SWCC. The Bao et al. (1998) function scales in accordance with the logarithm of soil suction but it appears that it is the "slope" of the shear strength function that scales in accordance with the logarithm of soil suction.

The proposed mathematical function must meet the following basic conditions; i.) the slope must be equal to the tangent of the effective angle of internal friction at the air-entry value for the soil, and ii.) the slope must go to zero as the residual suction is approached.

8.2 *Example using a Modified Bao et al (1998) Shear Strength Parameter*

Bao et al. (1998) proposed that the increase in strength of a soil should be scaled downward at soil suctions beyond the air-entry value. The soil parameter, ζ was introduced to scale down the shear strength between the air-entry value and residual suction on a log suction basis. Figure 24 shows that the scaled down soil parameter, ζ meets the required conditions near the air-entry value but does not satisfy conditions near residual suction. It is suggested that the scaled down soil parameter, ζ be used to designate the slope of the shears strength function rather than the actual shear strength.

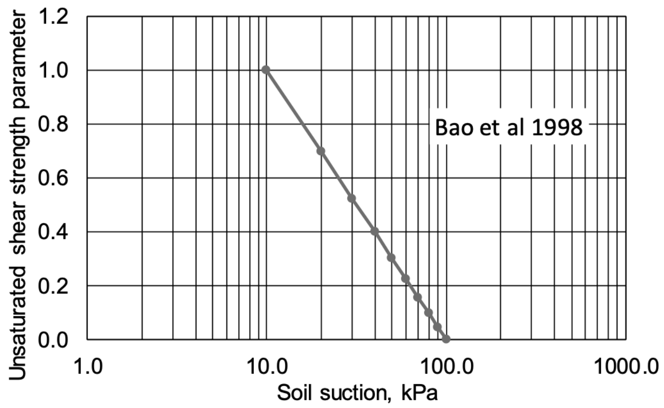

Figure 24. Proposed shear strength parameter by Bao et al., (1998) for an unsaturated soil

The boundary conditions for the shear strength envelope become the slope of the shear strength function at the air-entry value and at residual conditions. The "slope" of the shear strength function can be scaled between these two extremes using an incremental tangent method as shown in Figure 25.

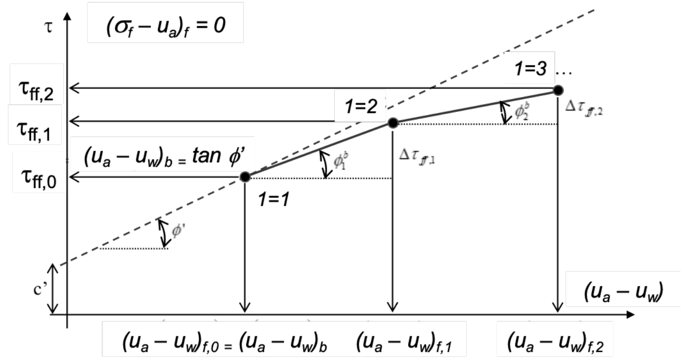

Figure 25. Marching forward process using the incremental tangent method

Figure 26 shows a comparison of the unsaturated shear strength envelope generated when using the Bao et al. (1998) equation to designate shear strength at a designated suction as opposed to using the Bao et al. (loc cit.) function to designate the slope of the shear strength envelope.

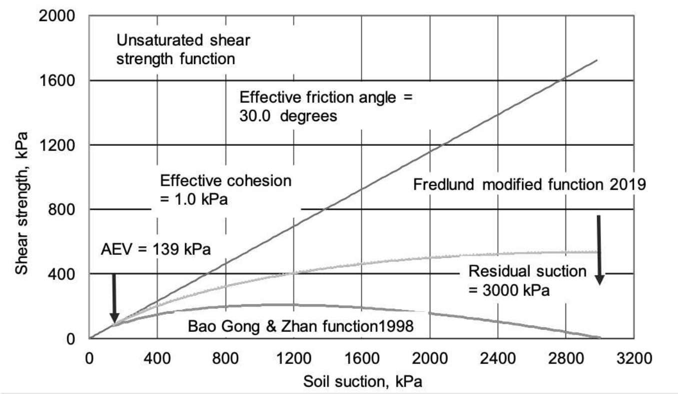

Figure 26. Comparison of the Bao et al., (1998) and function satisfying boundary conditions

Figure 27 compares the unsaturated soil shear strength envelope for two different soils; one with a residual suction of 600 kPa and another where residual suction conditions occur at 100 kPa. The air-entry value in both cases was 10 kPa. The graph clearly shows the important effect of residual conditions on the shear strength function.

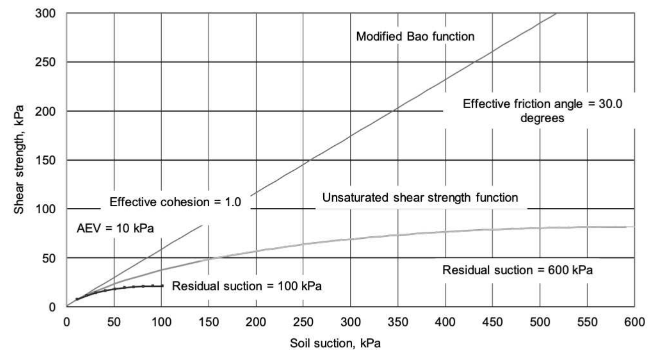

Figure 27. Effect of residual suction on the shape of the shear strength function

9 UNSATURATED SOIL PROPERTY FUNCTIONS FOR VOLUME CHANGE PROBLEMS

Historically one-dimensional volume change (or heave) has been analysed based on the interpretation of K_o oedometer test results. For the sake of this paper, let us give consideration to the use of the shrinkage curve, SC, to provide a general estimate of potential heave for an expansive soil. The SC provides a relationship between void ratio and gravimetric water content. Gravimetric water content is the most common soil test performed during a soils investigation. Let us examine how this information can be used to estimate potential heave. Let us also assume that there is negligible hysteresis between the drying and wetting shrinkage curve relations.

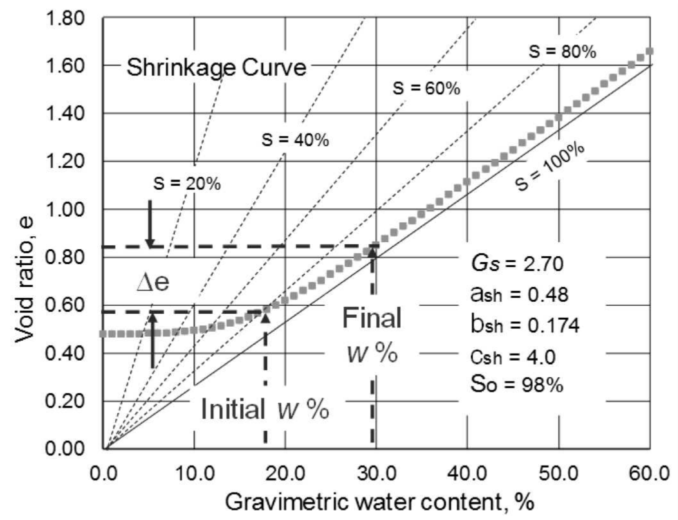

Figure 28. Shrinkage curve for the prediction of heave example

The data for the illustration of this application is fabricated. Figure 28 shows a shrinkage curve along with the fitting parameters for the M. Fredlund (2000) equation. The application of the shrinkage curve for the prediction of heave can be solved using a Spreadsheet. Figure 29 shows a profile of gravimetric water content versus depth. The solution requires that an estimation be made of the likely maximum value for the final water contents upon wetting. Various values could be assumed. Small "shrinkage limit" type specimens could be allowed to imbibe water with the final water content used for the wetted soil condition.

Figure 29 shows the calculated amounts of heave assuming various possible wetted conditions. It is also possible to assume that the wetted condition goes to a designated depth such as the depth of seasonal wetting. Many assumptions have been made in order to carry out this analysis. The intent is not to justify the use of the SC for the prediction of actual heave. Rather, the intent is to provide information that can be used as a simple guide for geotechnical engineering decisions.

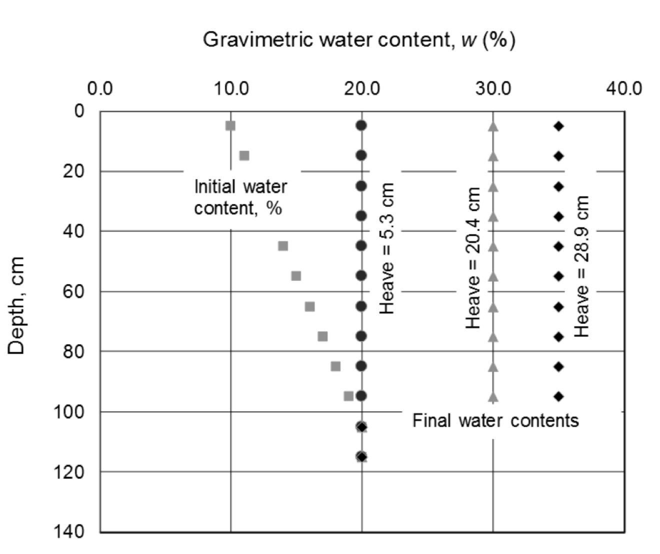

Figure 29. Prediction of heave calculations for 3 different assumed final water content conditions

10 CONCLUSIONS AND SUGGESTIONS

The main conclusions at the end of this paper can be brief.

1. "Estimation procedures" based on the gravimetric water content soil-water characteristic curve, *w*-SWCC, are acceptable for unsaturated soil property functions for all the main types of analyses involving unsaturated soils.
2. The use of estimated USPFs may not be ideal but they constitute a great step forward in our ability to model unsaturated soil behaviour.
3. Refinements can be made to the "estimations procedures" through combining the gravimetric water content SWCC (w-SWCC) and the shrinkage curve, SC.
4. The most important variables to assess for an unsaturated soil is its "air-entry value" and the residual conditions. These variables should be determined from the drying degree of saturation SWCC.

It is recommended that "estimation procedures" be used for the determination of unsaturated soil property functions, USPFs. Under these conditions, the saturated and unsaturated portions of a soil profile can simultaneously be modelled using partial differential solvers or computer software package that are programmed to accept a variety of unsaturated soil property functions.

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