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Reliability design framework for geosynthetics reinforced retaining walls

P. Rimoldi & P. Pezzano
Officine Maccaferri, Bologna, Italy

E. Zannoni
Maccaferri Africa, Johannesburg, South Africa

ABSTRACT: Geosynthetic reinforced soil walls (GRSW) have been widely used as retaining structures. In the traditional design of geosynthetic reinforced soil walls, Allowable Stress Design (ASD) is used to address the uncertainties. However, it cannot explicitly consider the uncertainties in a systematic way in the design process; especially geotechnical uncertainties which are typically project specific. The present paper introduces a framework for the reliability design of GRSW to explicitly address uncertainties in the design process and account for the actual safety and reliability level of a given design. The basics of probabilistic analysis and design for the internal and external stability limit states of GRSW are explained, and reliability analyses of GRSW are put in a rational framework, The framework concepts are general and can be applied to any GRSW for which the stability can be expressed by limit state functions, even if the present paper addresses only vertical walls. A closed-form solution is presented to calculate margins of safety in terms of probability of failure through first order reliability method, results from which can provide a useful decision-making tool for preliminary design of GRSW based on target reliability levels.

1 RELIABILITY APPROACH TO GRSW DESIGN

According to Zannoni (2016), the current design philosophy for GRSW is based on limit state analyses applying partial factors. This design approach enables the study of the behaviour of the structure at different state (usually ultimate limit states and serviceability limit states) and it allows to apply reduction factors to the single parameters rather than one factor of safety for all (working stress approach).

The scope of any design philosophy is to ensure that the design is safe, but how much is it safe? This is a hidden issue as a temporary structure should not be characterized by the same margin of safety that a bridge abutment requires.

The idea behind structure category is to apply a further partial factor to the overall design which increase the reliability of the design, thus from a reliability index of 3 (US Army Corps of Engineers 1997) for a temporary retaining wall, multiplying by the partial factor of 1.25 the reliability index increases to 4. In terms of reliability approach, reliability is linked to probability, therefore a reliability index of 3 generally means a probability of failure of 1x10-3.

Bathurst (2018) argues that the internal stability design for MSE walls in the UK is based on a partial factor approach in which factors are applied to soil and reinforcement material properties and to load contributions in different combinations to ensure safe designs. Geotechnical foundation design codes in North America adopt a load and resistance factor design (LRFD) approach which has been used by structural engineers for decades in Canada and the USA. In this approach, load terms are multiplied by load factors (magnitude of one or more) and the resistances are multiplied by a single resistance factor (with a magnitude of one or less).

The intent of a properly calibrated limit state design equation expressed in a LRFD framework is to ensure that a target maximum probability of failure will not be exceeded. However, the load and resistance factors that appear in LRFD codes have been selected largely by fitting to factors of safety used in allowable (working) stress design (ASD) past practice. Whether a designer uses a partial factor approach as in the UK or a LRFD approach as in North America, the margin of safety expressed probabilistically is unknown. This leads to the conundrum of a limit state being satisfactory when viewed from a factor of safety point of view but unsatisfactory from a probability of failure perspective. This point is demonstrated for the case of the rupture and pull-out limit states for the geosynthetic reinforcement layers in the MSE wall in Figure 1a (Bathurst 2018). The nominal tensile load Q_n can be calculated using one of a number of load models found in the literature and design codes. Similarly, the nominal resistance R_n can be calculated for tensile rupture and pull-out limit states using equations found in design guidelines. In conventional allowable stress design, the ratio of nominal resistance to nominal load defines the factor of safety; the resistance term is adjusted so that the factor of safety satisfies a minimum acceptable value. However, both nominal resistance and nominal load sides have uncertainty as visualized by the idealized frequency distributions in Figure 1b (Bathurst 2018). Notionally, the area of the overlap of the lower tail of the resistance distribution on the right with the upper tail of the load distribution on the left indicates a non-zero probability of failure. Clearly, two different combinations of load and resistance distributions can have different probabilities of failure (P_f) while the average factor of safety remains the same.

An alternative parameter to quantify margins of safety in geotechnical engineering is the reliability index β. The relationship between probability of failure and reliability index is:

$$P_f = 1 - F(\beta) \quad (1)$$

where F is the standard normal cumulative distribution function.

Bathurst (2018) argues that values of β = 2.33 and 3.09 correspond to probabilities of failure of 1/100 and 1/1000, respectively. The smaller β value is recommended as the target minimum reliability index for internal limits states design and LRFD calibration for GRSW. This value may appear small but GRSW walls are highly strength-redundant systems. In other words, if one reinforcement layer fails, other layers can compensate and thus system failure is unlikely. If a reinforcement layer is designed to just satisfy a target reliability of β = 2.33, the corresponding factor of safety can be as high as 1.70.

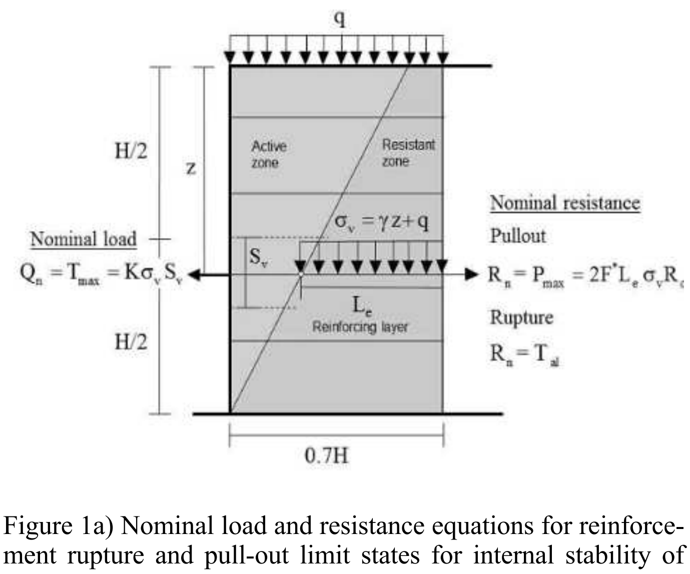

Figure 1a) Nominal load and resistance equations for reinforcement rupture and pull-out limit states for internal stability of GRSW (modified from Bathurst 2018)

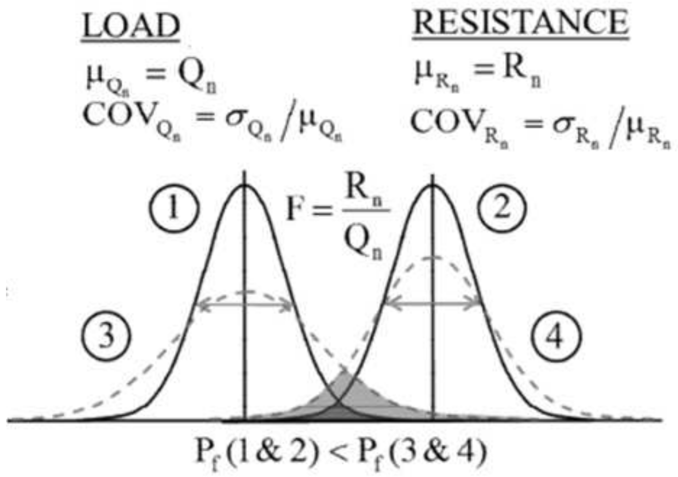

Figure 1b) Factor of safety and probability of failure concepts (modified from Bathurst 2018)

2 PROBABILISTIC MODEL OF GEOSYNTHETIC REINFORCED SOIL WALLS

There are mainly three types of stability requirements for the deterministic analysis of GRSW, including the external stability, the internal stability, and the global stability. External stability concerns about the stability of the entire reinforced soil wall body, including the checking for the possible failure modes such as sliding, overturning, and bearing capacity failures. The internal stability concerns about the stability of the reinforced materials, including the checking for the possible failure modes due to pull-out and tensile failure. In the present paper only limit states for internal and external stability are considered. For considering global stability, additional equations would be required, which are beyond the scope of the present paper.

The limit state functions which can be derived to evaluate the performance of GRSW against each failure mode for external and internal stability are represented by the equations providing the Factor of Safety FS, given by the ratio of resisting and active forces or moments for each of the considered failure modes. Here these limit functions are set for the design of GRSW in uniform granular soils with zero effective cohesion, but they can be easily extended to the case of cohesive soils.

The framework for defining the limit state functions is hereinafter presented, based on the resisting forces and driving forces for each failure modes. For sake of brevity only geogrids will be referenced, but the same concepts apply to any geosynthetic reinforcement.

2.1 *Sliding Failure*

For GRSW the critical sliding usually occurs along the base geosynthetic reinforcement, when the friction between the fill and the geosynthetic is not enough to compensate for the external load, which causes the retaining wall to slide. The limit state function for the sliding failure is:

$$FS_{ds} = \frac{L \cdot f_{ds} \cdot tan\,\phi_f \cdot (\gamma_R \cdot H + q)}{(0.5 \cdot \gamma_S \cdot H^2 + q \cdot H) \cdot tan^2(45 - \phi_s/2)} \quad (2)$$

where:
φ_f, φ_s = friction angle of foundation soil and back soil, respectively (deg)
γ_R, γ_s = unit weight of reinforced soil and back soil, respectively (kN/m^3)
f_{ds} = direct shear factor (-)
H = height of wall (m)
L = length of reinforcement (m)
q = uniformly distributed surcharge (kPa)

Note that tan φ_f is used in Eq. (2) assuming the friction angle of the foundation soil, φ_f, is lower than the friction angle of the reinforced soil, φ_R.

2.2 *Overturning Failure*

Overturning occurs when the soil thrust behind the GRSW body is great enough to offset the retaining wall by rotation around the wall toe. The limit state function for overturning is given by the ratio of resisting and active overturning moments around the toe:

$$FS_{ot} = \frac{0.5 \cdot L \cdot (\gamma_R \cdot L \cdot H + q \cdot L)}{(1/6 \cdot \gamma_S \cdot H^3 + 0.5 \cdot q \cdot H^2) \cdot tan^2(45 - \phi_s/2)} \quad (3)$$

2.3 *Bearing Capacity Failure*

Bearing capacity failure occurs when the subgrade soil beneath the reinforced soil wall fails under shear due to overloading or insufficiently constructed subgrades. Using the Terzaghi formula for general shear failure of foundations, the limit state function for bearing capacity is given by the ratio of the bearing capacity of foundation soil and the vertical stress on the foundation:

$$FS_{bc} = \frac{0.5 \cdot \gamma_f \cdot L \cdot N_\gamma}{\gamma_R \cdot H + q} \quad (4)$$

with:

$$N_\gamma = 2 \cdot tan\left[e^{\pi \cdot tan\,\phi_f} \cdot tan^2(45 + \phi_f/2) + 1\right] \quad (5)$$

where:
φ_f = friction angle of foundation soil (deg)
γ_f = unit weight of foundation soil (kN/m^3)
N_γ = bearing capacity factor (-)

Note that the Terzaghi formula can be replaced by more complex formulas (Meyerhoff, Brinch-Hansen, etc.) in the limit state function (4), while the framework remains valid.

2.4 *Pull-out Failure*

Pull-out failure occurs when the geogrid does not have sufficient length to resist the soil thrust in the influence area of each layer, thus causing failure by pull-out.

The critical condition usually occurs for the top geogrid, which has the lowest vertical stress producing the pull-out shear stresses needed for anchorage of the geogrid itself in the fill behind the potential failure surface. The anchorage length L_a is defined as the total geogrid length L minus the length L_e from the face to the point where the potential failure surface intersects the geogrid. Assuming that the geogrid is an extensible reinforcement, the failure surface coincides with the Rankine failure surface, identified by a line passing from the toe and inclined of the angle ϑ. The limit state function for pull-out is therefore given by the ratio of the pull-out resisting force developed along the anchorage length of the top geogrid and the soil thrust in the influence area of the top geogrid:

$$FS_{po} = \frac{F_{po}}{F_{a-po} + F_{q-po}} \quad (6)$$

with:

$$F_{po} = 2 \cdot \tau_{po} \cdot L_a = 2 \cdot \tau_{po} \cdot (L - L_e) = 2 \cdot f_{po} \cdot (\gamma_R \cdot z + q) \cdot (L - (H - z))/\,tan\,\vartheta \quad (7)$$

$$F_{a-po} = 0.5 \cdot \gamma_R \cdot tan^2(45 - \phi_R/2) \cdot (z + S/2)^2 \quad (8)$$

$$F_{q-po} = q \cdot (z + S/2) \cdot tan^2(45 - \phi_R/2) \quad (9)$$

$$\vartheta = 45 + \phi_R/2 \quad (10)$$

$$z = H - (N_{GG} - 1) \cdot S \quad (11)$$

where:
F_{po} = pull-out resisting force (kN/m)
F_{a-po} = active horizontal force produced by the self weight of the back soil for pull-out (kN/m)
F_{q-po} = active horizontal force produced by the surcharge for pull-out (kN/m)
τ_{po} = pull-out shear stresses on both sides of the geogrid (kPa)
f_{po} = pull-out factor (-)
φ_R = friction angle of reinforced soil (deg)
L = total length of the geogrid (m)
L_a = anchorage length of the geogrid (m)
L_e = length of geogrid between the face and the failure surface (m)
ϑ = inclination of the failure surface on the horizontal (deg)

z = depth of the geogrid from the top of wall (m)
N_{GG} = total number of geogrid layers (-)
S = uniform vertical spacing of geogrid (m)

Note that the variable surcharge q is both favourable for safety, when calculating F_q, and unfavourable, when calculating F_{po}: hence, strictly speaking, FS_{po} should be calculated with both applying and not applying q. Neverthelss, the situation with q applied is usually the critical one.

2.5 *Tensile Failure*

Tensile occurs when the tensile strength of the geogrids is not enough to withstand the forces applied by the thrust of the soil. For GRSW the critical condition usually occurs for the first geogrid above the toe, which has to withstand the highest horizontal stresses multiplied by its influence area. The limit state function for tensile failure is therefore given by the ratio of the design strength T_D of this geogrid and the active horizontal force produced by the back soil and the surcharge on the influence area of the geogrid:

$$FS_{tf} = \frac{T_D}{F_{a-ts}+F_{q-ts}} \quad (12)$$

with:

$$T_D = \frac{T_{ult}}{RF_{cr} \cdot RF_{id} \cdot RF_c \cdot RF_b} \quad (13)$$

$$F_{a-ts} = 0.5 \cdot tan^2(45 - \phi_R/2) \cdot [(H - S/2)^2 - (H - 3/2 \cdot S)^2] \quad (14)$$

$$F_{q-ts} = q \cdot S \cdot tan^2(45 - \phi_R/2) \quad (15)$$

where:
T_D = design tensile strength of the geogrid (kN/m)
$F_{a\text{-}tf}$ = active horizontal force produced by the self-weight of the reinforced soil for tensile failure (kN/m)
$F_{q\text{-}tf}$ = active horizontal force produced by the surcharge for tensile failure (kN/m)
RF_{cr} = Reduction Factor for tensile creep of geogrids (-)
RF_{id} = Reduction Factor for installation damage of geogrids (-)
RF_c = Reduction Factor for chemical damage of geogrids (-)
RF_b = Reduction Factor for biological damage of geogrids (-)

Note that RF_b can be assumed always equal to 1.0 for geosynthetic reinforcement; hence this parameter can be considered as a deterministic value, while the other RF_s can have a variability in respect of their nominal value and therefore these are considered as stochastic values. Here all uncertainty in the true magnitude of load and resistance terms for the limit states introduced above is solely due to the estimation of the friction angle (φ) and unit weight (γ) of soils, and of the design tensile strength of geogrids T_D; Nevertheless, all parameters in the above defined limit state functions could be considered as stochastic variables.

For this paper the first order reliability method by Low and Tang (2007) will be used for the computation to calculate the probability of failure of each limit state.

Unlike naturally deposited soils, the fill soil in a GRSW is an engineered material and therefore the variabilities in soil unit weight γ_R, however the friction angle φ_R are can be as high as 10% according a low coefficient of variation based on Table 3 For the back soil and the foundation soil the variabilities in soil unit weights (γ_s, γ_f) and friction angles (φ_s, φ_f) can be very large, as shown in Tables 3. The direct shear factor f_{ds} and the pull-out factor f_{po} are usually obtained from direct shear and pull-out laboratory tests on the specific geosynthetic and standard and/or site-specific soil. Their value can be set with no variability or with an associated variability, in which case also f_{ds} and f_{po} become stochastic parameters in Monte Carlo simulations. In any case the maximum value of f_{ds} and f_{po} shall be 1.0, since the interface cannot afford higher friction angle than the adjacent soil.

All the stochastic parameters are assumed to vary, according to the set variabilities, with higher or lower values than the nominal value, except for the friction angle of foundation soil φf, which can assume only lower values than the nominal value (otherwise the resulting extremely large variability of Nγ can make the standard deviation of the Factor of Safety for bearing capacity FSbc larger than its average value).

The probability distribution of all parameters is assumed to be the normal distribution.

For GRSW the length L and the ultimate tensile strength of geogrids Tult are the output of the design as usually the vertical spacing is given by the facing or the geometry and it is not stochastic.

Moreover, the above framework is based on the "traditional" method of calculating the limit state equations with the nominal values of parameters; yet, the framework remains valid even when semi-probabilistic methods are used (like LFRD in USA and EuroCode 7 in Europe) by applying amplification factors to loads and reduction factors to resistances: in this case it is enough to substitute the values of q, φ, γ, etc., with the factorized values, modifying the expected FSs as required by the specific code which is addressed.

3 SET-UP USING FIRST ORDER RELIABILITY METHOD

First order reliability method as introduce by Low & Tang (1997) requires a performance function *g(x)* depending on the distribution correlated parameters and it is based on the formula for the reliability index beta:

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T \boldsymbol{R}^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \qquad (16)$$

where R is the correlation matrix, σ_i the standard deviations, μ_i the mean values, x_i the random variables. The point denoted by the x_i values, which minimizes Equation 16 and satisfies $x \in F$ (belonging to the domain failure), is the design point (Fig. 2). In order to solve Equation 16, Low & Tang (2007) have developed an Excel spreadsheet which aims using Excel's solver to minimize β.

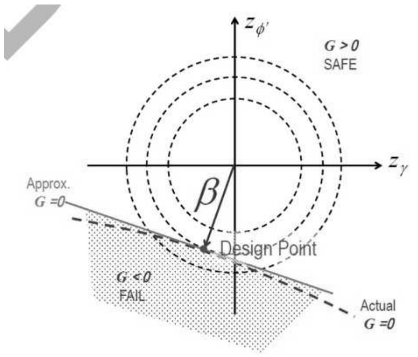

Figure 2: Illustration of the first order reliability method

4 EXAMPLE

In order to illustrate the robustness of the framework, a 10 m high wall is considered. For sake of simplicity, only cohensionless soils are considered in this example. The reinforcement is assumed to be bonded polyester geogrids, for which typical values of RFs, f_{ds}, and $f_{po,}$ are assumed (BBA 2010). The vertical spacing between the reinforcements is set as 0.6 m. Surcharge is a uniform load applied along the top horizontal surface of the wall with a set value of 20 kPa. Using an Allowable Stress Design approach, the following FoS are reported in Table 1.

Table 1. ASD results

Analysis	FoS
Sliding (over reinforcement)	2.53
Overturning	4.78
Bearing Capacity	6.77
Tensile failure (min)	2.19
Pull-out (min)	5.56

Using the SANS 8006 design code for soil reinforcement structures:

Table 2. SANS 8006 results

Analysis	Comb	FoS
Sliding (over reinforcement)	A	1.56
Overturning	B	3.19
Bearing Capacity	A	4.09
Tensile failure (min)	A	1.56
Pull-out (min)	B	1.02

Using the new framework on reliability-based design, considering the following information:

Table 3. Input data for example calculation

Variable	Mean	CV	Distribution
T	90 kN/m	1%	Normal
φ	32°	20%	Log-Normal
γ	18 kN/m^3	5%	Normal
L	7m	10%	Normal

The following probability of failures are summarised in Table 4:

Table 4. RBD results

Analysis	P(f)	β
Sliding (over reinforcement)	0.82%	2.4
Overturning	0.00%	4.06
Bearing Capacity	0.00%	6.25
Tensile failure (min)	4.4%	1.70
Pull-out (min)	7.88%	1.41

From the above, it results the new framework developed for reliability design method is consistent with ASD and limit state analysis as design codes have been developed taking into account a general variation of geotechnical properties as used above.

Interesting to note, while the ASD and limit state method would not vary the factor of safety, the RBD method is able to highlight that increasing the coefficient of variation of friction angle to 30% the probability if failure would increase as reported in table 5 above values of P(f) higher than in table 3.

Table 5. increment of friction angle CV to 30%

Analysis	P(f)
Sliding (over reinforcement)	23.44%
Overturning	0.01%
Bearing Capacity	5.00%
Tensile failure (min)	14.91%
Pull-out (min)	17.30%

5 CONCLUSIONS

Nowadays the benefit on the use of geosynthetics in soil reinforcement applications is well known.
Geosynthetics provide high tensile strength inside the reinforced soil body, thus increasing not only the Factors of Safety against failure but more importantly reduces the probability of failure due to the low coefficient of variation.

Soil parameters can be characterised by coefficients of variation up to 40 %, when poor, or more often limited, geotechnical investigations provide insufficient or not reliable information.

Notwithstanding the use of geosynthetics, which afford very low coefficient of variation for their technical properties, being manufactured under rigorous quality control testing, allows to achieve the required low probability of failure even when the variability of soil parameters is very high, at a reduce cost or time required for high quality geotechnical investigations.

In fact, considering the cost for a thorough geotechnical investigation, especially in remote areas with lack of resources, machineries and adverse conditions, geosynthetics are able to provide a safe and sound design, even with a high variability of geotechnical properties, ensuring an adequately low probability of failure at a lower capital expense.

Uncertainties in the geotechnical and loading parameters have significant effects on the design of geosynthetic reinforced soil walls (GRSW).

This paper is an attempt to describe a rational reliability theory-based approach for the analysis and preliminary design of GRSW.

A reliability analyses framework is introduced for external and internal stability of GRSW, considering the variability of several parameters required in the design process.

An example application is carried out for a GRSW with granular soils to demonstrate the potential of the proposed reliability framework. Results show that the probability of failure decreases with improved reliability of geotechnical data, which in turn requires higher costs for geotechnical investigations.

The proposed framework can be very useful for engineers to make a more informed design decision based on target reliability requirements, considering the costs of geotechnical investigations vs the cost of the GRSW.

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