Plane strain viscoplastic modelling in vacuum consolidation

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ABSTRACT: The mechanism involved in ground improvements with vacuum assisted prefabricated vertical drains (PVDs) for road embankments is essentially 3-dimensional (3-D) but could reasonably well approximated as axisymmetric. In this context, axisymmetric unit cell modelling generally provides a very good representation for Finite Element (FE) modelling. However, it carries significant limitations in terms of the scope of the analysis. Hence, to get an understanding of the overall deformational behaviour of the foundation soil being improved, a full scale Plane Strain (PS) FE model would be necessary. In this paper, conversion of axisymmetric unit cell to an equivalent PS model is carried in the context of a vacuum consolidation project. Foundation soft soil is modelled using an elastic-viscoplastic (EVP) model which accounts for the time dependent behaviour of soft clay. Results of this PS conversion is compared with the axisymmetric FE solution. Stability of the embankment is also analysed using maximum lateral displacements.

1 INTRODUCTION

The axisymmetric unit cell would be more representative model to idealise the physical phenomena occurring around a PVD. The size of the FE mesh associated with such a model is rather small ensuring the computational efficiency. However, this convenience in unit cell modelling comes with a significant cost to a researcher or a design engineer due to its inherent limitations. One option would be to create a 3D model of the interested area. This is a very expensive option in terms of computational resources and time, since such analysis requires excessive amount of 3D elements. Also, research has shown that without extensive set of foundation soil data, 3D modelling of embankment like structures will not yield significantly accurate results. Even though some accuracy can be obtained, it generally outweighs the computationally intensive effort and cost. In this context, two-dimensional (2D) Plane Strain (PS) model can be considered as a proper balance between the costs and benefits.

There are several methods to convert axisymmetric unit cell to an equivalent PS model. These are commonly known as matching procedures. Hird et al. (1992) proposed three matching techniques called permeability matching, geometry matching and combined matching. These methods require the PVDs to be modelled as done in unit cell modelling. Either the permeably, the spacing between them or both should be varied. Lin et al. (2000) proposed a simpler method to convert axisymmetric condition to PS. Rezania et al. (2017) showed that the Lin et al. (2000) method has a significant limitation since the method requires the actual drain spacing (in the field) to be close to the PS drain spacing for the results to be accurate. When PS drain spacing becomes larger than the actual spacing in the field, settlements are heavily overestimated. Chai et al. (2001) proposed an even simpler method to approximate the settlements by calculating an equivalent permeability for the entire soil mass considering the effect of PVDs as well. In this approach, explicit modelling of each PVD was not necessary.

Extremely limited analysis has been done in PS modelling, considering the long-term deformational behaviours in vacuum consolidation. In this paper, the matching technique proposed by Hird et al. (1992) is used along with an EVP model in the context of vacuum consolidation. The performance of the Ballina test embankment (Kelly & Wong 2009) at SP11 location is modelled using the PS model. Then the PS FE analysis results are compared with that of the equivalent axisymmetric case. Vertical deformations are compared with maximum lateral deformations and embankment stability is also discussed.

2 PLANE STRAIN MATCHING PROCEDURE

2.1 *Combined matching procedure*

In this paper, out of the three matching approaches proposed by Hird et al. (1992), combined matching was selected to gain some control over the FE mesh. Combined matching permits to change the PS unit cell width with appropriate changes to the permeability of the foundation soil.

According to Hird et al. (1992), combined matching approach can be summarised as follows: Neglecting well resistance, for the average degree of consolidation ($\overline{U}$) to be matched at every time and every depth it requires,

$$\overline{U}_{hpl} = \overline{U}_{hax} \tag{1}$$

where subscript *hpl* and *hax* denote PS and axisymmetric conditions respectively. From Hansbo (1981),

$$\overline{U}_h = 1 - \exp\left(-8T_h/\mu\right) \tag{2}$$

where T_h is the time factor for radial drainage such that, $T_h = C_h/4R^2$ and neglecting well resistance $\mu = \ln(n/s) + (k/k_s)\ln(s) - 3/4$. In the latter expression $n = R/r_w$ and $s = r_s/r_w$, where C_h is the coefficient of consolidation, k is the permeability of the intact (undisturbed) zone in the unit cell, k_s is the permeability of the smear zone. R, r_w and r_s are equivalent radius of the unit cell, radius of the well and radius of the smear zone respectively.

Hird et al. (1992) proposed the following equation for combined matching:

$$\frac{k_{pl}}{k_{ax}} = \frac{2B^2}{3R^2\left[\ln\left(\frac{n}{s}\right) + \left(\frac{k}{k_s}\right)\ln(s) - \left(\frac{3}{4}\right)\right]} \tag{3}$$

where $2B$ is the drain spacing in PS and R is the radius of the unit cell in axisymmetric condition. From the Equation 3, geometric matching can be obtained by substituting $k_{pl} = k_{ax}$, whereas permeability matching can be obtained by substituting $B = R$.

Once the parameters are determined a ratio between PS permeability (k_{pl}) and axisymmetric permeability (k_{ax}) can be determined as,

$$k_{pl} = \eta k_{ax} \tag{4}$$

where η is the conversion ratio.

2.2 *Modelling the smear zone*

There are few options to model the smear zone in PS condition. First and the most obvious method would be to explicitly dedicate elements with reduced permeability to represent the smear zone. Small elements which require to be modelled adjacent to the PVD centreline make the mesh denser and certain FE programs require to define new material types, which is not convenient. Instead of the above approach, an equivalent permeability can be calculated considering both smear and intact zone parameters. It can be shown that,

$$k^* = \left(\frac{\mu_0}{\mu}\right)k \tag{5}$$

where k^* is equivalent permeability of the soil with the effect of smear zone and μ_0 is without the effect of smear zone and it can be written as,

$$\mu_0 = \ln\left(\frac{R}{r_w}\right) - \frac{3}{4} \tag{6}$$

combining Equation (4) and (5),

$$k^*_{pl} = \left(\frac{\mu_0}{\mu}\right)\eta k_{ax} \tag{7}$$

where k^*_{pl} is the equivalent PS permeability with the effect of smear zone.

2.3 *EVP model*

It is not intended to describe the EVP model in this paper due to the limited scope and length. The model is developed based on the Perzyna (1963) formulation. Kumarage & Gnanendran (2019) introduced time dependant boundary conditions and enabled the model to be used to model vacuum consolidation. Detailed description of the EVP model has been presented elsewhere in Kumarage & Gnanendran (2019).

2.4 *PS conversion of Ballina Embankment*

Ballina embankment is the first vacuum consolidation project in Australia which was constructed during the Pacific highway project near the town Ballina in NSW. Application of vacuum, embankment construction etc. have been reported by several researches (Kelly & Wong 2009, Indraratna, Rujikiatkamjorn, Kelly & Buys 2012).

Respective ratios calculated by different matching approaches for this field case are displayed in Table 1. These ratios were calculated based on the input parameters adopted for the case (displayed in the Table 2).

Table 1. Ratios by different matching approaches

Matching method	Resultant Ratio	Value
Geometry Matching	B/R	$B/R = 2.495$
Permeability Matching	k_{pl}/k_{ax}	$k_{pl}/k_{ax} = 0.161$
Combined Matching	k_{pl}/k_{ax} (for a given B value)	When $B = 1$; $k_{pl}/k_{ax} = \eta = 0.506$

Table 2. Input parameters for PS matching (modified from Kumarage & Gnanendran 2019)

Parameter	Value
R	0.5642 m
n	33.19
s	4
k/k_s	2

From the above data μ_0/μ = 0.67 was calculated. Having calculated η = 0.506 from combined matching (Table 1), k^*_{pl}/k_{ax} ratio was calculated as per the Equation (7). Hence the correlation of PS permeability to axisymmetric permeability was determined as $k^*_{pl} = 0.34\ k_{ax}$ which was used for the analysis.

2.4.1 *Material properties*

Determination of critical state soil parameters and creep characteristics are well described in Kumarage & Gnanendran (2019) and is not repeated here. Summary of material properties are presented in Table 3. Foundation soil was modelled with the EVP model mentioned earlier. Permeability of the foundation soil layers in Table 3 were converted to respective PS permeability by the previously discussed conversion method.

Fill materials were modelled as an elasto-plastic Mohr-Coulomb continuum. This was done intentionally to reduce the modelling complexity. It was experienced during the analysis that modelling fill material as Biot type coupled cam clay or equivalent model can lead to numerical instability especially in vacuum consolidation. This happens primarily in the adjacent layers to the ground surface where vacuum suction prevails. Since elasto-plastic Mohr-Coulomb type model has only two degrees of freedom per node in the mesh, analysis become simpler without any significant effect on the accuracy of the results.

Unlike in unit cell analysis, in PS modelling stress on the foundation soil were not modelled as a traction. Instead, actual filling was done by first generating the FE mesh with the embankment, then disabling the embankment fill material at the start of the analysis and re-enabling them in the correct sequence to represent the embankment construction. The FE mesh adopted for the analysis is illustrated in Figure 1.

3 RESULTS AND SYNTHESIS

3.1 *Settlement and excess pore water pressure*

Figure 2 is a comparison between axisymmetric and PS FE results against field data. Axisymmetric FE results were adopted from Kumarage & Gnanendran (2019).

When Hird et al. (1995) matching techniques are used, the average excess pore water pressure (EPWP) in PS model is generally less than of its axisymmetric case. From Figure 2b it is clear that this result holds true for vacuum consolidation as well. It can also be observed that the magnitude of the deviation appears to be higher in vacuum consolidation. The deviations in Hird et al. (1995) predictions (without vacuum) are around 7 - 10 %. In this case the maximum differences are around 20%. However, when vacuum is switched off the axisymmetric and PS predictions get closer as time elapse.

In terms of settlements, it can be observed that PS model under predicts settlements than the axisymmetric model during the 150 - 250 days period. From 250 - 900 days predictions from both models are almost equal. From 1000 days onwards PS model over predicts settlements. This could be due to the faster EPWP dissipation in the model resulting more effective stress being transferred to the foundation soil.

3.2 *Lateral displacements & embankment stability*

Lateral displacements are an important indication of the embankment stability. It is generally expected that the embankments with vacuum assisted PVDs would be more stable during their construction, since vacuum exerts some inward lateral pressure and helps to reduce the outward lateral displacements.

Plotting lateral displacements against vertical settlement is one way of assessing the stability of the embankment. A function can be suggested to represent such curve as,

$$y_h = f(S) \tag{8}$$

where y_h is the maximum lateral displacement and S is the vertical settlement at the embankment centreline. If the above curve approximated to a straight line with a slop of m such that,

$$m = \frac{\Delta y_h}{\Delta S} \tag{9}$$

Tavenas & Leroueil (1980) suggested that when m reaches 1.0 it would reflect undrained distortion of the clay foundation, while ratio of 0.15 to 0.2 indicates a low risk of instability.

Table 3. Material properties (modified from Kumarage & Gnanendran 2019)

	Depth (m)	λ	κ	e_0	γ_{sat} (kN/m^3)	k_h (10^{-10} m/s)	OCR
Fill material	c = 5.0 kPa, $\phi = 35.0$, $\gamma = 19.0$ kN/m^3, ν = 0.3, K = 750, β = 0.5						
Soft Soil	0.0-0.5	0.57	0.057	2.75	14.0	10.0	2.0
	0.5-4.0	0.57	0.057	2.75	14.0	10.0	1.8
	4.0-15.0	0.57	0.057	2.74	14.5	10.0	1.7
	15.0-25.0	0.48	0.048	2.09	15.0	3.3	1.1

*K and β are material data for stress dependant stiffness characterisation (Janbu 1963). OCR is the overconsolidation ratio

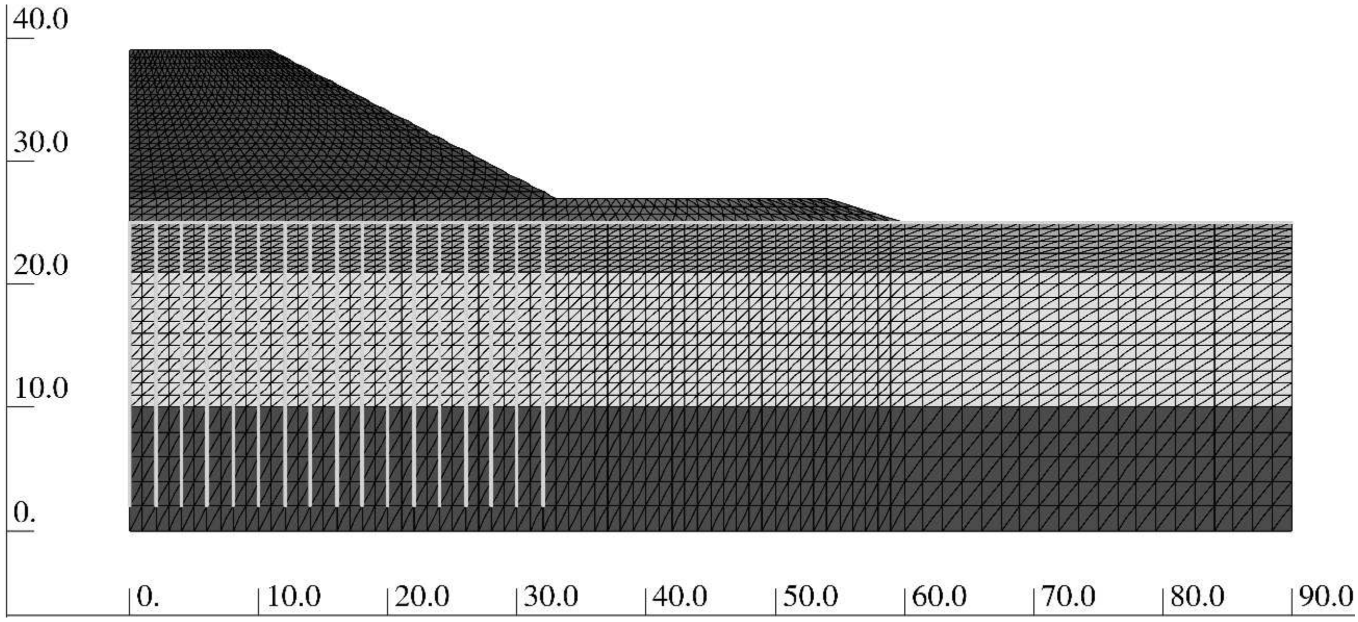

Figure 1. FE mesh for the PS analysis

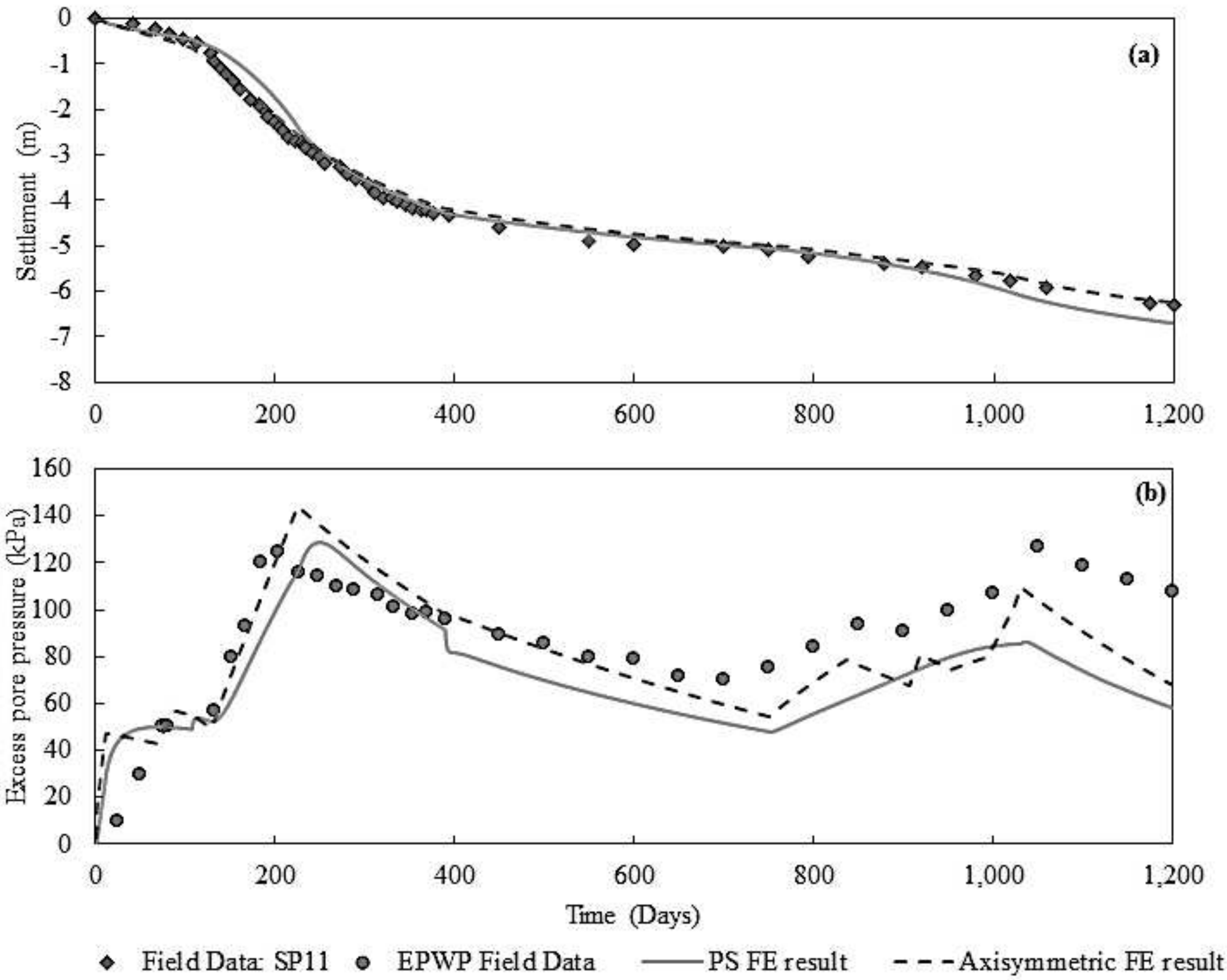

Figure 2. Comparison of axisymmetric and PS predictions of settlements and EPWP

Figure 3 illustrates the maximum lateral displacement against the embankment centreline settlement in two locations (I6 and I4). In I6 location a straight line was drawn according to the to field data (L-1 in Figure 3). As per the straight line drawn m = 0.18. However, since FE results have overestimated the lateral displacements $m = 0.22$ was observed for the trend line drawn for FE results at I6. Both field data and FE results have not indicated a potential embankment failure. Such analysis is not required for the I4 location since the ratio is much lower indicating good stability and it shows a reasonably good agreement between field data and FE results as well.

4 SUMMARY AND CONCLUDING REMARKS

PS conversion of the unit cell axisymmetric model for vacuum-assisted PVDs was discussed in this paper. Soft soil was modelled with an EVP model with time dependant boundary conditions and embankment fill material was modelled as a Mohr-Coulomb continuum. Both settlement and EPWP had a good agreement with field data although the amount of deviation was higher in EPWP than what Hird (1992) illustrated. Maximum lateral displacements were compared in two inclinometer locations with vertical deformations to access the embankment stability. Both field data and FE results indicated a stable embankment. These results confirmed the Hird (1992) method together with EVP model can successfully predict embankment deformation behaviours in PS conditions.

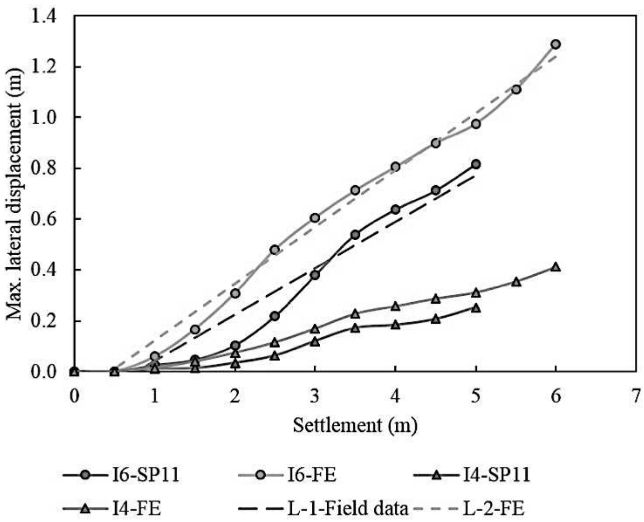

Figure 3. Maximum lateral displacement vs vertical settlements

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6 REFERENCES

Chai, J.-C. Shen, S.-L. Miura, N. & Bergado, D. T. 2001. Simple Method of Modeling PVD-Improved Subsoil. *Journal of Geotechnical and Geoenvironmental Engineering.* 127(11): 965-972.

Hansbo, S. 1981. *Consolidation of Fine-grained Soils by Prefabricated Drains Proceedings of the International Conference on Soil Mechanics and Foundation Engineering* (Vol. 3). Stockholm.

Hird, C.C. Pyrah, I.C. & Russel, D. 1992. Finite element modelling of vertical drains beneath embankments on soft ground. *Géotechnique.* 42(3): 499-511.

Hird, C.C., Pyrah, L.C. Russell, D. & Cinicioglu, F. 1995. Modelling the effect of vertical drains in two-dimensional finite element analysis of embankments on soft ground. *Canadian Geotechnical Journal.* 32(1992): 795-807.

Indraratna, B. Rujikiatkamjorn, C. Kelly, R. & Buys, H. 2012. Soft soil foundation improved by vacuum and surcharge loading. *Proceedings of the Institution of Civil Engineers - Ground Improvement.* 165(2): 87-96.

Janbu, N. 1963. Soil Compressibility as Determined by Oedometer and Triaxial Tests. *European Conference on Soil Mechanics & Foundation Engineering.* 1: 19-25.

Kelly, R.B. & Wong, P.K. 2009. An embankment constructed using vacuum consolidation. *Australian Geomechanics.* 44(2): 55-64.

Kumarage, P.I. & Gnanendran, C.T. 2019. Long-term performance predictions in ground improvements with vacuum assisted Prefabricated Vertical Drains. *Geotextiles and Geomembranes.* 47(2): 95-103.

Lin, D. G. Kim, H.K. & Balasubramaniam, A.S. 2000. Numerical modeling of prefabricated vertical drain. *Geotechnical Engineering.* 31(2): 109-125.

Perzyna, P. 1963. Constitutive equations for rate-sensitive plastic materials. *Quarterly of Applied Mathematics* 20: 321-331.

Rezania, M. Bagheri, M. Mousavi Nezhad, M. & Sivasithamparam, N. 2017. Creep analysis of an earth embankment on soft soil deposit with and without PVD improvement. *Geotextiles and Geomembranes.* 45(5): 537-547.

Tavenas, F. & Leroueil S. 1980. The behaviour of embankments on clay foundations. *Canadian Geotechnical Journal.* 17(2): 236-260.