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Circular shallow foundations with V-M loading: A course in geotechnical centrifuge testing

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ABSTRACT: Shallow foundation design is a classical and fundamental geotechnical engineering problem. However, beyond very basic geometries and loading conditions, even this apparently simple problem proves difficult to solve analytically. Instead, various empirical modifications to the Prandtl solution are used. The easy to use of an in-flight robotic actuator and the failure mechanism complexities for non-trivial design cases make a shallow foundation investigation an excellent candidate for geotechnical centrifuge education. This paper presents the results of two centrifuge tests carried out on the Turner Beam Centrifuge at the Schofield Centre, University of Cambridge, as part of a course to teach graduate students how to perform a geotechnical centrifuge test. Six shallow circular foundations were loaded vertically and with moments, on a uniform, cohesionless Hostun HN31 sand layer. The three smaller foundations were tested at a higher "g" level compared to the three larger foundations, though all six foundations share the same prototype dimensions. The load, displacement and rotation were captured during loading, and comparisons were made between the two flights to confirm the "modelling of models". The results from the centrifuge tests have a dual function of acting as an exercise in data processing and also as a comparison to predictions based on established literature such as Meyerhof (1953) and Butterfield and Gottardi (1994).

Keywords: centrifuge testing, shallow foundations, eccentric load, modelling of models, education

1 INTRODUCTION

Shallow foundations under combined vertical and moment loading, often with an eccentric vertical load, can be a difficult problem to describe analytically, particularly given geometric and material complexities. Instead, several empirical modifications to the Prandtl solution (Prandtl, 1921) exist to account for lift-off due to moment loading in bearing capacity calculations (Meyerhof, 1953). An alternate empirical 3D failure envelope was also developed by Butterfield and Gottardi (1994).

Whilst complex to analytically describe, this is a simple problem to model experimentally and is thus instructive in introducing students to geotechnical centrifuge modelling. As part of a graduate course at the Schofield Centre, University of Cambridge graduate students use the Turner beam centrifuge to study the response of rigid, shallow foundations to Vertical (V) and Moment (M) loading. Different combinations of V and M loadings were considered by changing the eccentricity of the vertical load from the center of the circular foundation.

The main learning objectives of this exercise were;

1. Preparation of a uniform, dry sand bed in a standard model container.
2. Use of a 2D actuator for application of vertical and moment loading.
3. Obtain centrifuge test data in the form of loading on the foundation, settlement of the foundation and the rotation of the foundation using MEMS accelerometers.
4. Carry out necessary signal processing on the data to obtain plots of load-displacement and moment-rotation.
5. Interpret the centrifuge results and compare them to the solutions from the literature.

Six circular model foundations, three small and three large diameter rigid foundations, were tested with vertical and eccentric loading. Comparisons were made both within and between the two sets of circular foundations especially to demonstrate the principle of "modeling of models".

2 EXPERIMENTAL METHODOLOGY

2.1 Model design

Six circular model foundations were used, three small (S1,S2,S3) and three large (L1,L2,L3). Each foundation was a metal disc of 10 mm thickness, with diameters of 63.68 mm and 82.83 mm respectively. All model foundations may be considered to be 'rigid' for the loading regimes considered in this paper. The small foundations were tested at 52*g*, the larger ones at 40*g*. Using centrifuge length scaling (Schofield, 1980), all foundations have a diameter of 3.312 m at the prototype scale. From the principle of "modeling of models", one should expect similar results between the small and large discs models as the prototype they represent are identical.

The model container was a tub with diameter of 850 mm and a depth of 397 mm, this provided a rigid boundary condition. Figure 1 shows the foundation layout and the loading lines. Positions are approximate and designed to avoid failure mechanism overlap, with consideration to the alignment of the load lines for each flight. This meant most space was given to the centrally loaded foundations where the largest deformation mechanisms were expected from logarithmic spirals (standard drained mechanism).

2.2 Model preparation

The sand deposit was created using the automatic sand pourer, described in detail in Madabhushi *et al.* (2006). Calibrations for the Hostun HN31 sand used are detailed by Chian *et al.* (2010). The system is comparable to a 3D printer, and uniformly air pluviate sand over a 1 m^2 area up to a height of 0.5 m.

The relative density of the resulting sand was 51%, this value is affected by the drop height, nozzle diameter and sieve present, nozzle travel speed and nozzle path. The pourer is capable of producing repeatable samples with relative densities of 40 to 90%.

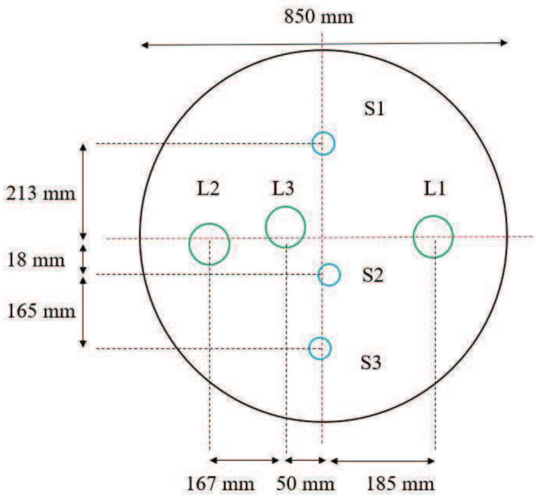

Fig. 1. Test layout (dimensions in model scale)

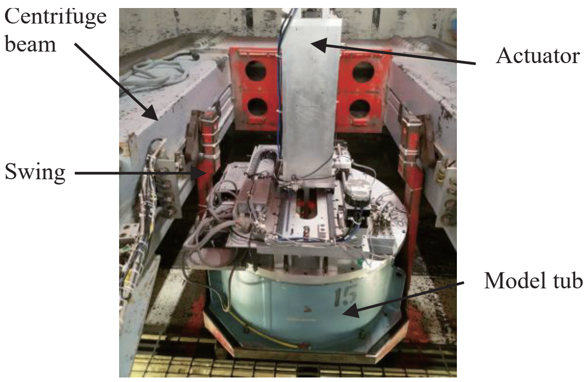

Fig. 2. Model tub with actuator on the centrifuge.

2.3 Load application

Foundations S1 and L1 were loading vertically in the middle, S2 and L2 were loaded with an eccentricity of 1/3 the foundation radius (0.552 m) and S3 and L3 were loaded with an eccentricity of 2/3 (1.104 m), (Table 1).

Loading was done via a blade attached to an actuator operated using a stepper motor per axis of travel. The complete setup is shown in Figure 2 before the start of the test, with the camera pointed radially outwards from the center of the centrifuge. The 2D actuator position and velocity were controlled using motor count and revolutions per minute (RPM) respectively via “Clearpath" software. As the actuator is able to translate to different locations in-flight, three distinct loads could be applied to three separate model foundations in each flight, resulting in only two flights, one at each *g* level. The model foundations were placed to allow three loading events without the need to rotate the blade.

Table 1. Foundation properties and loading conditions.

Reference	Mass (*g*)	Diameter, *D* (*mm*)	*g*-level	Eccentricity
S1	87.21	63.68	52	0
S2	86.79	63.68	52	*D*/6
S3	86.88	63.68	52	*D*/3
L1	468.4	82.83	40	0
L2	465.3	82.83	40	*D*/6
L3	461.4	82.83	40	*D*/3

2.4 Instrumentation

Vertical displacement of the loading blade was recorded by a draw wire potentiometer and a load cell attached to the blade captured the axial load applied. Lastly two MEMS were glued to each disc, one on the top, one on the side, these record the tilting of each foundation.

3 TEST RESULTS

3.1 L1 model foundation – 40*g* test

An example of the loading event on the L1 model foundation is shown in Figure 3, following manual offsets, calibration and filtering using MATLAB (eighth order lowpass Butterworth filter). In this figure the load, settlement and rotation time histories are presented. A peak load of 3 kN was applied to cause a settlement of 3 mm. Although the foundation was loaded axially with zero nominal eccentricity, a small rotation of about 0.5º was recorded by the MEMS as seen in Figure 3. As this test was conducted at 40g, the vertical stress and strain are 0.557 MPa and 3.621% respectively.

In all of the tests described in this paper, a nominal settlement equal to 5% of the diameter was chosen as a representative vertical strain. This is commonly used to determine the bearing capacity of foundations in granular soils, (Tomlinson, 1986).

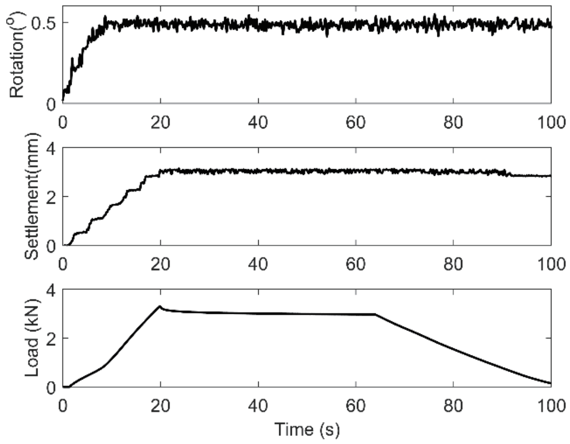

Fig. 3. Loading event of L1 foundation at model scale.

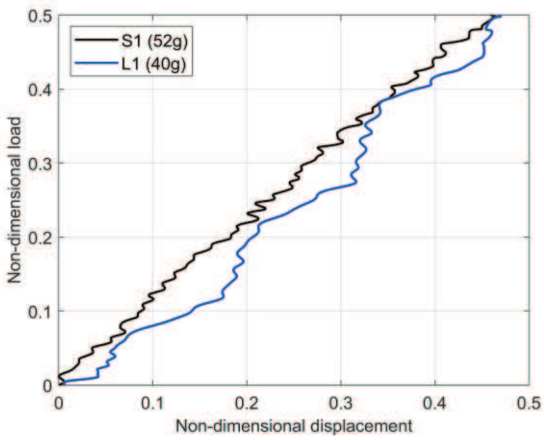

Fig. 4. Non-dimensional load-displacement plot for vertically loaded foundations, using the load and settlement equal to 5% of the foundation diameter.

3.2 Modelling of models

As the centrifuge tests were carried out at 40*g* and 52*g* respectively, both the model foundations scale to the same prototype as explained earlier. It is possible to plot the vertical loading events on foundations L1 and S1 at these *g* levels. In Figure 4 the non-dimensional load and displacement are overlaid. The non-dimensional load in both tests was calculated by dividing the current load with the load that produces a vertical settlement equal to 5% of the foundation diameter. Similarly the non-dimensional displacement is obtained by normalising the current displacement with the 5% of the foundation diameter.

The results confirm similar soil stiffness's were mobilized in both tests. This is in line with the principle of modelling of models i.e. testing foundations at different g levels still yields the same soil stiffness. The slight variation in the L1 (40*g*) line is possible due to the potentiometer wire not extending smoothly as the load was applied.

4 EMPIRICAL SOLUTIONS

4.1 Meyerhof approach

To estimate the bearing capacity using the Meyerhof approach, it is first necessary to calculate the peak friction angle ϕ_{peak}, using Bolton's dilatancy index (Bolton, 1986) and an empirical relationship for triaxial conditions (White *et al.*, 1986).

$$I_R = I_D(Q - \ln(p')) - 1 \quad (1)$$

$$p' = \lambda q \quad (2)$$

$$\phi_{peak} = \phi_{crit} + 3I_R \quad (3)$$

where the natural logarithm of the crushing stress, Q, is taken to be 10 for Hostun HN31 sand, and the critical friction angle, ϕ_{crit}, is 32° (Chian et al, 2010). This gives a peak friction angle of 35°. Next the effective area A' is calculated from foundation geometry and loading eccentricity as follows;

$$A' = \frac{D^2\theta}{4} - 2e\sqrt{\frac{D^2}{4} - e^2} \quad (4)$$

$$\theta = 2cos^{-1}(\frac{2e}{D}) \quad (5)$$

Using the above equations the bearing capacity can be estimated as;

$$\frac{V_{ult}}{A'} = q_f = (1 + sin\phi)N_q\sigma'_{vo} + 0.7N_\gamma\frac{\gamma' B}{2} \quad (6)$$

The bearing capacities were calculated using the above equations for all the model foundations and are presented in Table 2 using the bearing capacity factors N_q and N_γ.

4.2 Butterfield & Gottardi approach

Following Butterfield and Gottardi (1994) the vertical, horizontal and moment loading on a foundation can be simulated as a 3D failure envelope in the V-H-M space as:

$$[\frac{H/V_{ult}}{t_h}]^2 + [\frac{M/BV_{ult}}{t_m}]^2 + [\frac{2C(M/BV_{ult})\ (H/V_{ult})}{t_h t_m}]^2 = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2 \quad (7)$$

In equation 7, C is defined as;

$$C = \tan(\frac{2p(t_h - t_m)(t_h + t_m)}{2t_h t_m}) \quad (8)$$

where t_m, t_h, and p are approximately 0.4, 0.5, 15°.

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In above equations, the horizontal loading H is zero, as there was no horizontal loading applied in the model tests conducted here. Using this and Eq. 6, the V_{ult} and hence the bearing capacity can be calculated.

Table 2 summarises the bearing capacities obtained from all the experiments and from empirical solutions. Again, here we see on par values between S1 and L1 despite the fact the loading blade slipped on some of the foundations (Figure 4). The calculated bearing capacities for each load case assumes that the bearing capacity is reached when the vertical settlement is 5% of the foundation diameter. The load at 5% diameter settlement, either measured or interpolated, is then divided by the foundation area (assuming the full area of 8.615 m^2).

Table 2. Normalised foundation bearing capacities, V/V_{ult}.

Eccentricity (m)	S	L	Meyerhof	Butterfield
0	1.00	0.90	1.00	1.00
0.552	0.55	0.63	0.44	0.59
1.104	0.19	0.27	0.10	0.18

5 CONCLUSIONS

The main objective of this paper was to introduce centrifuge modelling to young researchers. It must be emphasised that this exercise was more pedagogical than research into bearing capacity of shallow foundations. However, eccentrically loaded shallow foundations are of interest from a geotechnical practice point of view. The problem that was selected was the case of *V-M* loading on a shallow, rigid, circular foundation on sandy soil. The researchers experienced making the centrifuge models, calibrating the required instruments and conducting the centrifuge tests at two different *g* levels. The post-processing of the data was carried out using necessary signal processing such as filtering of the data. Using the centrifuge test data the following conclusions are drawn.

1. The “modelling of models” in centrifuge testing was demonstrated and very similar foundation stiffnesses were obtained using different centrifuge models that represent the same prototype.
2. The data obtained showed good agreement with empirical solutions that are commonly available in the literature.

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