Investigation of M-V interaction domain for small-scale simplified root system architectures

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ABSTRACT: Assessing the stability of trees against transversal loads requires the definition of a reliable limit condition. This latter is usually estimated for practical application from the interpretation of non-destructive loading tests on trees, along a single loading path. Toppling of root plates is however a complex phenomenon, whose activation depends (i) on the specific loading path (i.e. the combination of vertical, V, and horizontal, H, forces, and overturning moment, M) that can arise e.g. during a windstorm, and (ii) on additional second order geometrical effects, associated to possible large rotations of the tree. In this perspective, the limit condition should then be defined by a complete interaction domain in the generalized loading space, rather than by a single limit value for the toppling action, and the failure criterion must be formulated in a large rotation scheme. To this goal, some original small-scale experimental tests investigating the shape of the interaction domain for simplified root plate geometries in the M-V plane, also accounting for upward load components, have been conducted. An original load-controlled experimental setup is conceived, in order to allow for the development of large rotations in the model and to capture the onset of the unstable toppling phenomenon. Different simplified root system architectures are considered and critically compared. The results show that the interaction domain presents a noticeable coupling in the M-V plane, which can lead to largely inaccurate estimations of on-site toppling resistance if not correctly accounted for.

1 INTRODUCTION

The stability of trees is becoming a critical issue in urban environments, owing (i) to an increased awareness versus the risks deriving from tree collapses (affecting structures, infrastructures, good and people) and (ii) to the increased frequency of extreme weather events (essentially driven by the ongoing climate changes). From a geotechnical point of view, the root-plate of a tree works as the direct foundation of a tall and slender "living structure", essentially subject to its own self-weight (W) and to possible lateral loads (F_w, as e.g. during windy loading conditions), resulting in a complex combination of toppling (M), vertical (V) and horizontal (H) actions, as sketched in Figure 1. In this perspective, Foundation and Geotechnical Engineering know-how can be consistently applied to investigate the toppling resistance of trees, looking for new engineering-based approaches for tree stability assessment, enriching the available criteria usually adopted by agronomists and plant scientists.

In the agronomic practice, in fact, one of the main non-destructive procedures to assess tree stability is represented by the static pulling test, consisting in the application of a monotonic sub-horizontal load to the tree stem, inducing a small rotation to the collar (usually not exceeding 0.25°).

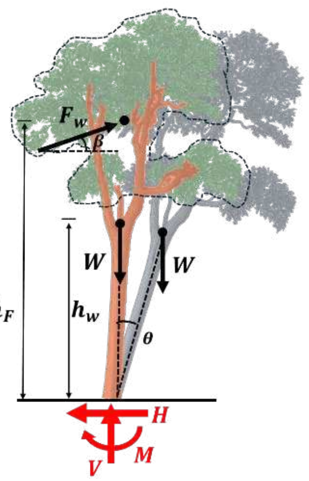


Figure 1. Combination of loads acting on a tree.

Test results are then interpreted by means of an empirically based approach (Wessolly and Erb, 1998), which however disregards the possible coupling effects among the M-V-H load components, as well as the well-known paths-dependency that characterizes the behaviour of geotechnical systems. Only one loading path is in fact investigated in standard pulling tests, whilst wind actions may in general be characterized by a high uncertainty on the inclination β (Figure 1), potentially inducing variable loading paths ranging from downward pushing (when $\beta < 0$) to upward lifting (when $\beta > 0$). In addition, it should be noted that during the loading process the progressive increase of trunk rotation θ (Figure 1) might induce significant additional 2nd order geometrical effects, increasing the total toppling action M exerted on the root plate and controlling the onset of the collapse.

A possible way to cope with such complex issues can be represented by the so-called "macroelement" approach, initially proposed for shallow foundations (Nova and Montrasio, 1991) and widely extended to a number of other applications in Civil and Geotechnical Engineering in the last thirty years (Galli, 2020). It is Authors' opinion that it could also represent a convenient strategy to model the complex mechanical response of a root plate, as also recently suggested by Mansur et al. (2024). To this goal, the definition of the ultimate condition for a root plate in the M-V-H loading space (the interaction domain) is a fundamental aspect that has not yet been investigated in scientific literature. More in general, the interest of Geotechnical researchers towards tree stability assessment procedures is progressively increasing in recent years (see also Marrazzo et al., 2026), and experimental methods, theoretical frameworks and numerical approaches which are currently in use in the Geotechnical domains can be profitably applied also to this very complex subject (Galli and Marrazzo, 2026).

This paper presents a preliminary set of 1g experimental toppling tests at different vertical load (and, for the sake of operational simplicity, zero horizontal load) on small scale simplified root prototypes, aiming at investigating possible coupling effects between M and V. The focus will be (for the first time, at Authors' knowledge) on the shape of the interaction domain for the root-plates, at least within the range of the tested V values.

2 EXPERIMENTAL SETUP

A simplified small-scale model was conceived, focusing in particular on a specific loading system capable of independently controlling M and V loading components, and allowing at the same time the prototype to freely develop large rotations. The goal of the tests was to objectively define a failure condition, also highlighting the contribution of second order effects. Similarly to what is done in many experimental works on small scale shallow foundations, where the structure is modelled by means of rigid elements (see e.g. Galli et al., 2015), here in the following the root architecture will be modelled by means of materials that can be considered infinitely rigid with respect to the surrounding granular material. An artificial granular material, with elastic soft grains, will also be employed to simulate the highly deformable vegetated soils at low confinement values, usually hosting the root systems. The failure condition will not then be associated to the breakage of one (of more) of the system's components, but it will be represented by the "global" failure of the model, with complete and uncontrolled uprooting.

The assumed experimental conditions evidently represent a huge simplification with respect to real systems (where partially saturated gravel-sand-fine mixtures are generally present, and root traits breakage can markedly reduce the overall uprooting resistance), but they will allow for very well repeatable testing condtions, and for an objective and meaningful comparison of the results. On the other side, the goal of the paper is not to derive possible scaling laws for real scale applications, but only to present a preliminary experimental research on M-V effects on such complex systems.

2.1 Simplified root architectures

Among the wide complexity of root geometrical shapes, shallow root plate geometries have been considered here in the following, since they represent a large amount of the cases of collapsed trees for uprooting mechanisms in urban areas. Shallow root plates are characterized by a dominant horizontal growth direction of the main root branches (1st order roots) with some possible sinker branches (Figure 2a).

Two simplified root geometries were conceived by assembling several simple structural elements on a wooden beam (32×32×150 mm), considered here as a rigid body and fixed in its turn to the loading system. Geometry 1 (Figure 2b) consists of a single wooden rod (10 mm in diameter and with a length l = 80 mm on each side) inserted in the middle of the base beam to simulate the presence of 1st order lateral roots only. Geometry 2 (Figure 2c) was instead created by adding four vertical wooden elements in the first order lateral root (3 mm in diameter and 40 mm in length) simulating the presence of sinker roots. Geometry 2 also accounted for the presence at shallow depths of a diffuse network of 2nd order lateral roots, by including a perfectly flexible and axially rigid rubber net. These root architectures were intended to model two extreme cases of a complex root geometry (Geometry 2, ideally representing a healthy and complete root system) and of a very simple one (Geometry 1, representing e.g. a diseased and partially damaged root plate).

This particular choice allows for an easier and more controllable installation, avoiding and excessive sample disturbance that could be induced when using more complex 3D root models (Marrazzo et al., 2024).

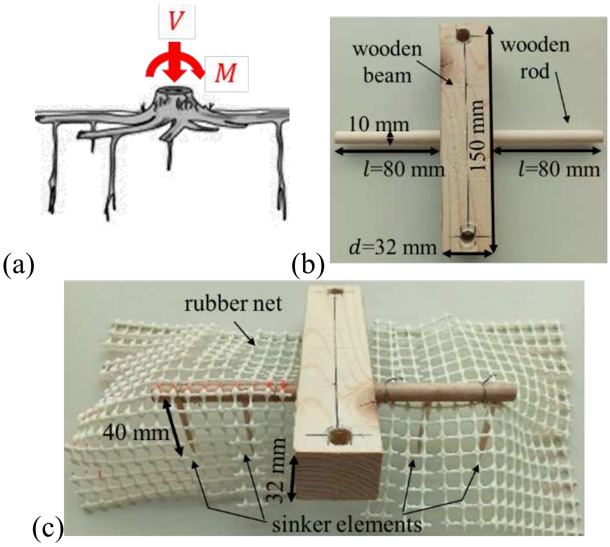


Figure 2. (a) shallow root-plate sketch; (b) Geometry 1; (c) Geometry 2.

2.2 Model preparation

The two root geometries were placed on a uniform layer of 120 mm of granular material deposited by a pluviation method into a rigid testing box (340×150×210 mm; Figure 3). The soil was ideally modelled using a granular material consisting of artificial deformable rubber grains, whose particle size distribution curve is comparable to that of fine gravel (Figure 4a).

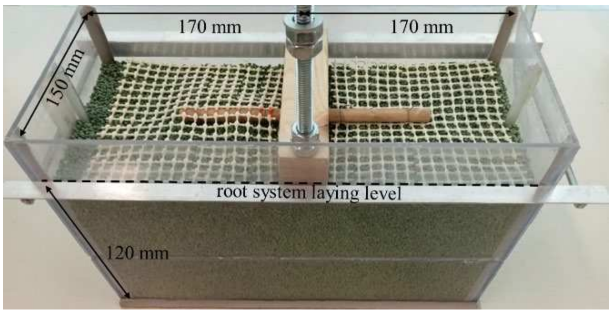


Figure 3. Wiew of the laying plan of the root model before the final cover

The rubber is characterized by a solid unit weight γ_s = 15.4 kN/m^3 and the grains are deposited at a porosity of about 40% (corresponding to a dry weight γ_{dry} = 9.2 kN/m^3. The peculiarity of this granular material is to show a relatively large deformability during booth the loading and the unlading paths, as witnessed by the results of a low pressure dry oedometric test shown in Figure 4b (for the sale of completeness, analytical power fittings are also reported). At the laboratory scale this allows for better capturing (with respect to usual laboratory clean sand or fines-sand mixtures) the condition of poorly compacted and highly deformable materials as vegetated soils at low burying depths are (see also Galli et al., 2023). A covering layer of 30 mm of granular material is then deposited on both sides of the model, to completely embed the root system.

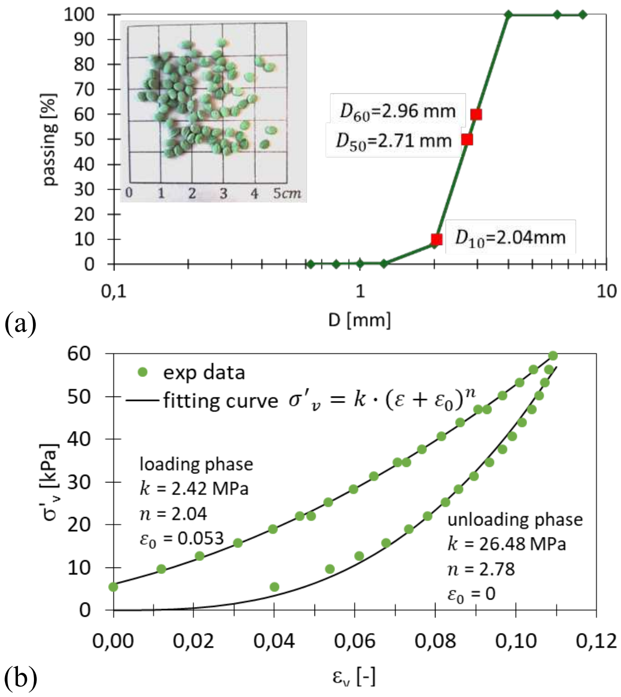


Figure 4. (a) grain size distribution; (b) load-unload dry oedometric test.

2.3 Loading system

The adopted loading system consists of a wooden plate fixed through two screw rods above the base beam. The plate hosts two tanks (each one with a storage capacity of 2 litres and initially filled with 1 litre of water) at a distance b of 190 mm from the centre of the model; the resulting elevation h of the model barycentre above the root laying level is 220 mm (Figure 5a). The tanks are connected to each other by a peristaltic pump (governed by an Arduino UNO R4 board) controlling the difference in water weights P_1 and P_2 and thus allowing to apply a toppling moment M at constant vertical load V. An imposed constant flow rate of 1.1 ml/s was set to the pump, which can be considered low enough to allow to for quasi-static testing conditions. The rotation of the model was measured at a frequency of 10 Hz, using a digital inclinometer with accuracy of 0.0055°.

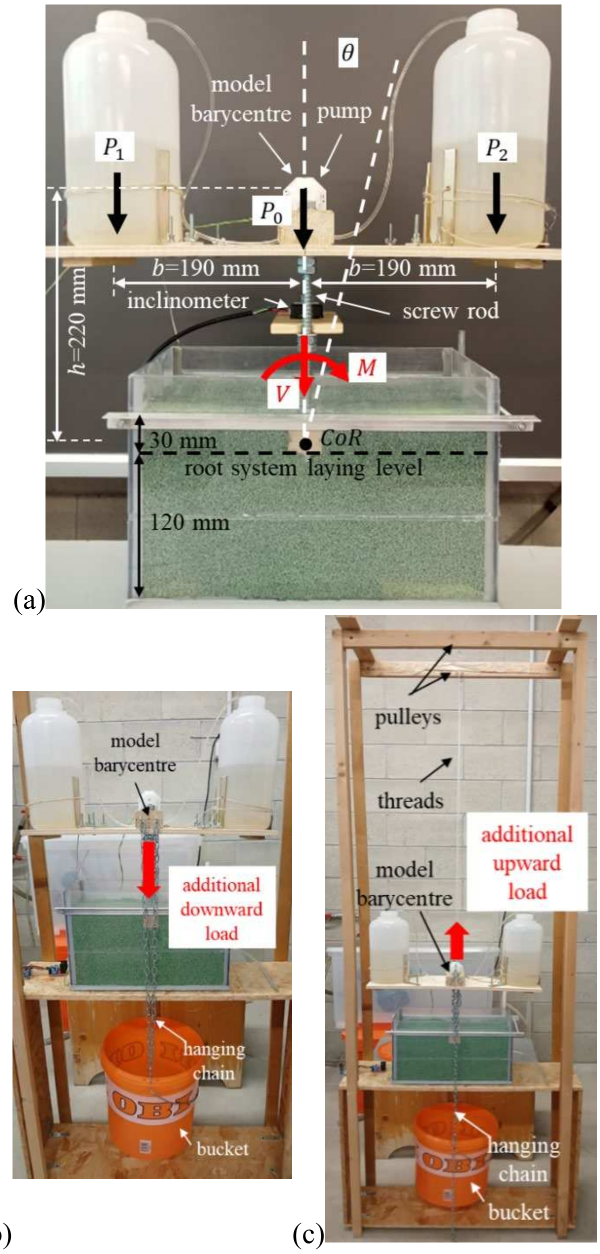


Figure 5. (a) overview of the prototype and of the loading apparatus at the beginning of test; (b-c) application of an additional downward or upward load.

This original loading system, despite evidently unable to apply a horizontal load H, was on purpose designed to allow for a precise definition of the applied moment even in the range of large rotations values, as those usually triggering the toppling collapse of the model. Traditional load-controlled systems, applying e.g. a pushing horizontal force on the stem (as in Zhang, 2023), at large rotation values would in fact suffer of spurious components (related to the progressive alignment of the stem with the horizontal direction) which make rather uncertain the evaluation of the applied moment, and can even prevent the onset of the collapse. Displacement-controlled loading systems have also been judged not to be adapt to this particular application, since they do not allow the model to develop uncontrolled rotations (i.e. the loss of controllability of the system) which represents the actual initiation of the collapse.

The applied toppling moment M is computed as the sum of two contributions, related to the load applied through the pump (M_{load}) and to the 2nd order effect (M^*) induced by the out-of-plumb of the weight W with respect to the centre of rotation, respectively:

$$M = M_{load} + M^*, \text{ where} \quad (1)$$

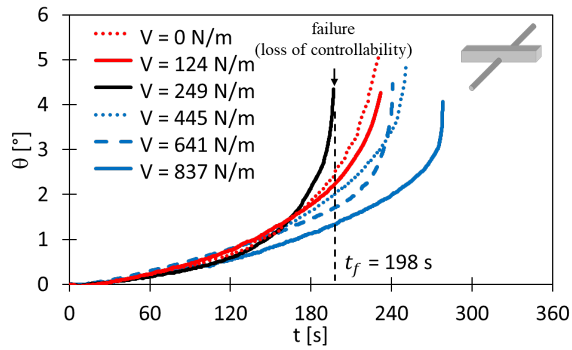
$$M_{load} = (P_2 - P_1) \cdot b, \text{ and} \quad (2)$$

$$M^* = W \cdot h \cdot \sin\theta. \quad (3)$$

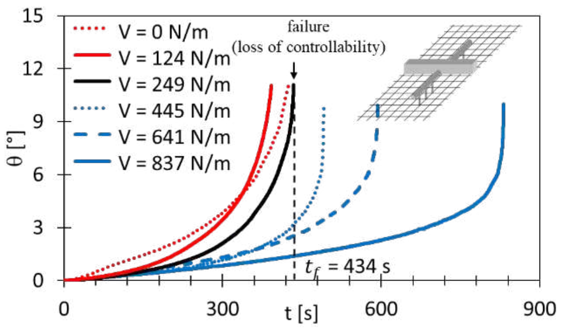
The value of the vertical load V is instead given by the total weight W (i.e. the net mass of the model P_0 = 1.80 kg and of the water in the two tanks P_1+P_2 = 2 kg). In order to test different V values, some additional dead loads can be placed at the beginning of the test into a bucket, hanged at the model barycentre through two hanging chains (Figure 5b), thus increasing the downward vertical load. Similarly (Figure 5c), the bucket can also be hanged through inextensible and perfectly flexible threads to a couple of pulleys, again fixed to the model barycentre, thus imposing upward actions with reduction of the vertical load. The resulting loading path in the M-V plane for the tests consists then in a preliminary vertical loading phase at zero applied moment M (exemplified by phase (a) of the loading path sketched in Figure 9a for one of the tests), and in a subsequent toppling phase with increasing moment values M at constant V load (phase (b) of the sketch of Figure 9a). Although not realistically representing the on-site loading paths for trees (where simultaneous variations of M and V (and H) can be induced by wind actions), these tests can meaningfully allow to investigate the failure condition in the M-V plane. In the following, for the sake of clarity, the values of both M and V will be normalized with respect to the length of the base beam (150mm), following the idea to assimilate the root systems to a unit element of an equivalent out-of-plane strip foundation.

3 RESULTS AND DISCUSSION

Six monotonic toppling tests were performed for each root geometry, at different vertical load values. Starting from V = 249N/m (corresponding with the total weight W of the model, with no applied dead loads), two tests were executed at lower vertical loads of V = 124 N/m and $V \approx 0$, by applying upward dead loads. Three additional tests were instead run at higher vertical loads V of 445, 641 and 837 N/m, respectively, by adding downward dead loads. A range of V values between 0 and about 330% of the weight W of the model was then investigated; no larger V values were tested, since an excessive disturbance would have been initially induced to the prototype, making the model markedly unstable.



(a)



(b)

Figure 6. (a-b) rotation records for the tests Geometry 1 and Geometry 2, respectively.

The recorded rotation histories are plotted in Figure 6a and b for Geometry 1 and 2, respectively, and allow to identify the precise onset of the failure as the time instant t_f corresponding with the loss of the controllability, when the curves assume a vertical tangent (i.e. an uncontrollable rotation increment $d\varphi \rightarrow \infty$). As an example, in the figures the failure instant for the case V = 249 N/m is indicated by a black arrow, The corresponding moment-rotation curves are plotted in Figure 7 and Figure 8 for Geometry 1 and 2, respectively. In particular, for the sake of comparison, both the values of M_{load} and M_{tot} are shown and – consistently with Figure 6 – the values at failure for the test with V = 249 N/m are indicated.

As expected, the results show that Geometry 2 has in general better mechanical properties than Geometry 1, owing to the major geometrical complexity, both in terms of ultimate resistance and initial stiffness of the curves. By comparing the $M_{load} - \theta$ curves for the two geometries (Figure 7a and 8a, respectively), it however appears that Geometry 1 seems to be less sensitive to the value of the vertical load V. The curves are in fact characterized by a very similar initial stiffness and by only a small scatter in the ultimate moment values, corresponding with the ultimate plateau of the curves. Geometry 2 on the contrary shows a more evident dependence on the V values, with progressively increase in both the stiffness and the ultimate moment, especially for downward additional loads. If the curves $M_{tot} - \theta$ are instead considered (Figure 7b and 8b), it now appears how for both the geometries the value of the vertical load V has much higher influence, either on the stiffness of the curves and on their moment values at failure. In particular, relevant differences in the curves can be already observed even for small rotation values (lower than 0.5°), i.e. close to the range usually investigated in real scale pulling tests.

By selecting the moment values at failure (both M_{load} and M_{tot}) of each test for geometries 1 and 2, the following interaction diagrams in the M-V plane can be plotted (Figure 9, where empty markers stand for M_{load} and full markers for M_{tot} values). Best fitting equations of the experimental data are also reported in the figures (dashed and dotted black lines, respectively; simple linear trends have been here considered). For the sake of the comparison, the theoretical computation of the limit moment M_L for an equivalent rigid foundation subjected to a centred vertical load V and resting on a rigid base (i.e. a solution that disregards the mechanical interaction between the soil and the root plate), has been included as continuous black lines. For the tests with V = 249 N/m the figures also highlight the computed value of M^*, thus exemplifying the quantitative contribution of the 2nd order effects.

In both cases the values of M_{tot} show a marked increasing trend with V, especially for Geometry 2 (the linear fitting coefficient between M_{tot} and V is 0.0489 m, i.e. more than twice the one of Geometry 1), meaning that higher geometrical complexity induces more pronounced mechanical interaction between the root and the soil. This trend, at least in the explored range of V values, is consistent with what is observed for shallow foundations for V values relatively far from the ultimate vertical bearing capacity.

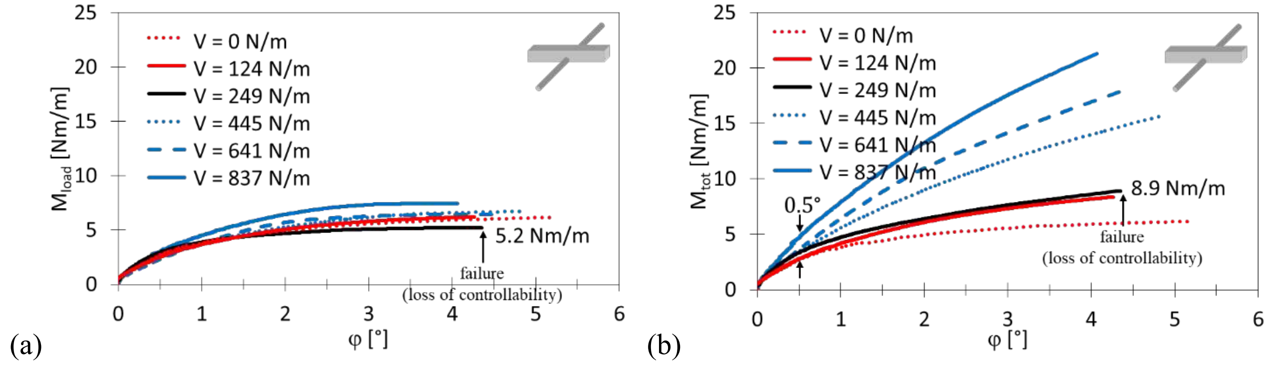


Figure 7. (a-b) $M_{load} - \varphi$ and $M_{tot} - \varphi$ curves for the tests on Geometry 1.

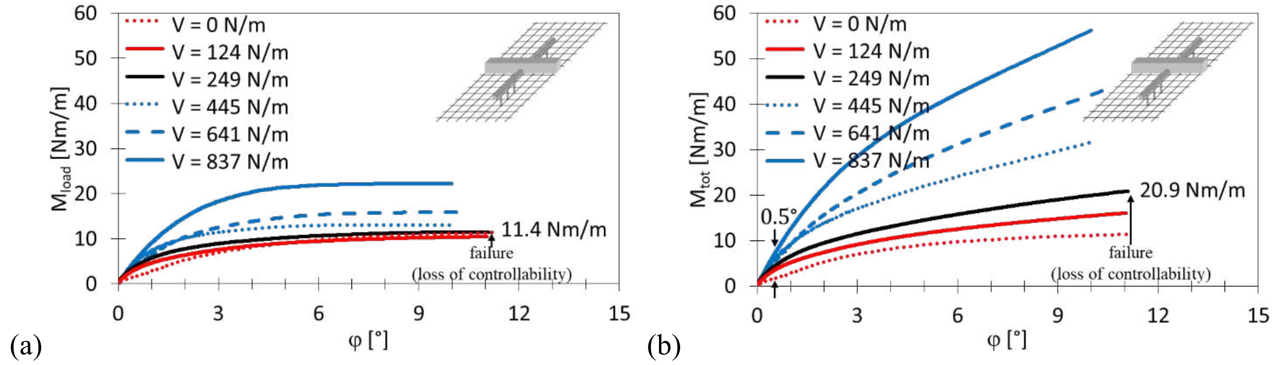


Figure 8. (a-b) $M_{load} - \varphi$ and $M_{tot} - \varphi$ curves for the tests on Geometry 2.

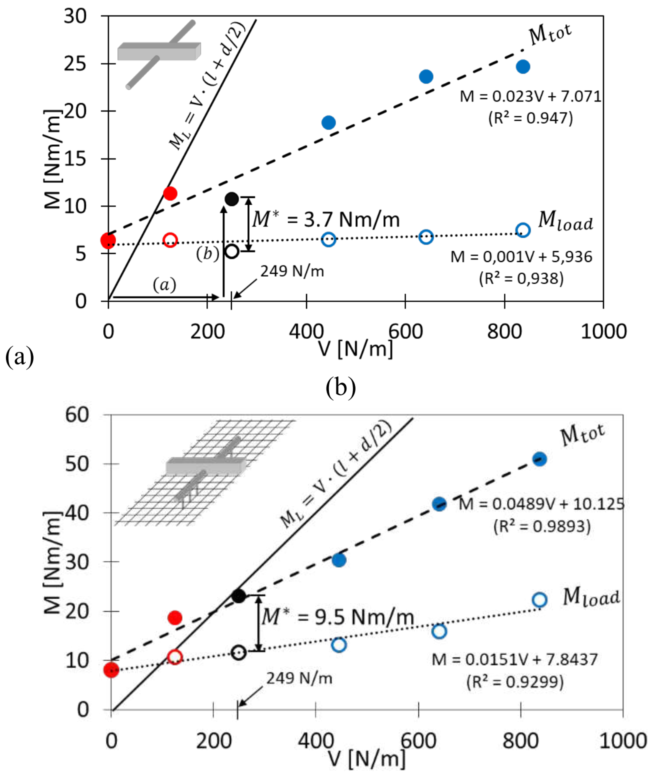


Figure 9. (a-b) ultimate loading conditions in the M-V plane for tests on Geometry 1 and Geometry 2, respectively.

With respect to the theoretical M_L solution, both geometries show higher toppling resistance at low vertical load, owing to the effect of the embedment of the root plate. At higher V values, however, the progressive sinking of the root within the soil reduces the ultimate resistance with respect to the theoretical solution, thus proving that soil-root plate interaction cannot be disregarded for meaningful predictions of the resistance of real root plates. The dependence on V is much less evident if M_{load} values are instead considered (values of the linear fitting coefficient of 0.001 m and 0.0151 m are obtained for Geometry 1 and 2, respectively), again suggesting that disregarding 2nd order effects would lead to largely inaccurate solutions.

4 CONCLUSIONS

The paper presented the results of an original small scale testing campaign on simplified root prototypes, focusing on the investigation of possible coupling effects among the toppling moment (M) and the vertical load (V). Two simplified root architectures were conceived with different geometrical complexity. Particular attention was paid to the design of an innovative loading system, precisely controlling the applied toppling component even at large rotation values, and to include 2nd order geometrical effects induced by out-of-plumb actions.

The tests, at different values of the vertical V load, clearly prove the influence of the vertical action on the toppling response of the systems, both in terms of stiffness of the moment-rotation curves and of their toppling resistance. This becomes quantitatively more evident for complex root architectures, owing to their interaction with a larger soil volume. The values of the ultimate toppling resistance, in particular, showed an increasing trend with the applied vertical load (at least for the tested range of V values), more pronounced for complex root architecture. The tests also show that assuming a large rotation scheme (i.e. including 2nd order effects) is essential: disregarding the out-of-plumb component of the applied vertical load would in fact lead to severely underestimate (or even to miss) the M-V coupling effects

Although limited to simplified small-scale tests (disregarding some fundamental aspects like e.g. the accurate reproduction of the root geometry, the choice of representative natural soils, the condition of partial saturation, …), these findings represent a consistent set of results, that can give a valid contribution toward the definition of the interaction domain for root plates and to the formulation of more complex macroelement laws for trees. At Authors' knowledge, these original results are still quite rare in scientific literature and they can also contribute to address future specific research to improve the mechanical interpretation of real scale pulling tests on trees or, more in general to assess their stability towards complex loading scenarios as those induced by the ongoing climate changes.

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