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Plane-strain apparatus to study the effect of Vibrojet® pile installation on soil state

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ABSTRACT: The demand for quieter pile installation methods to minimise environmental impact is increasing significantly with the growing number of planned offshore wind farms. One of these methods is the Vibrojet® technique, combining an axial vibrational driving force with jetting-reduced resistance. To jet controllably while preserving bearing capacity, it is important to understand the jetting impact in and around the pile. Jetting is commonly used and well-studied within the dredging industry. Vibrational driving is currently being studied all over the world, however, the behaviour of a jet in this pile installation environment is not yet well understood. We present a novel experimental apparatus that allowed us to research this. The apparatus mimics the conditions inside the pile and around the pile tip that occur when installing a pile using the Vibrojet® technique. It enables digital image correlation, conductivity concentration measurements, and pore pressure and total stress measuring. Complementing the description of the apparatus, we provide two examples of how this apparatus has been used in testing. Lessons learned from this study will shed light to other studies where soil-fluid interaction is investigated.

1 INTRODUCTION

Offshore wind energy is emerging as a key energy source, with countries around the world setting ambitious targets for developing offshore wind farms. With growing ecological awareness, the need for quieter installation methods for their offshore foundations arises to ensure less environmental impact during installation. One of these quieter installation methods for monopiles is the Vibrojet® technique, combining a vibrational driving force with jetting-reduced resistance. The research presented herein limits itself to reducing only inner wall friction. For this novel technique, it is important to understand the effect of the jet in and around the pile.

This is a hybrid problem combining the dynamic pile-soil interaction and resulting soil displacements and density changes (highly complex soil behaviour influenced by acceleration of soil grains, large cyclic strain amplitudes, initial void ratio in relation to overburden stress, and by change in pore water pressure (Viking, 2002)) with the hydraulic jets eroding soil around them and changing hydraulic head both inside the pile and in the approximate soil body. The former closely relates to vibrohammer installations, while the latter relates most to situations

encountered in the dredging industry.

Many studies have used element testing for studying individual phenomena. General formulations of hydraulic fluidisation of soil were derived describing minimum flow needed for expanding a bed and deriving porosity based on flow (Richardson and Zaki, 1954; Wen and Yu, 1966; Summerfelt and Cleasby, 1996; Kramer et al., 2019). For dynamic excitation of soil, main influences such as vibration frequencies and strain amplitude and their effect on compaction, liquefaction and cyclic mobility were investigated (Barkan, 1962; Bernhard, 1967; Youd 1972; Castro and Poulos, 1977; Denies et al., 2010; Denies and Holeyman, 2017). For example, Barkan (1962) found that permeability increases proportionally to the acceleration of the vibrations.

During pile driving, however, the source of excitation and, if in place, the source of water are changing their position. Therefore, a lot of research was focussed on moving sources. Lab set-ups used to research (vibro-)pile driving at 1 g are mainly small full piles (Bernhard, 1967; Rodger and LittleJohn, 1980; O’Neil and Vipulanandan, 1989; Martinelli et al., 2025) and half-pile set-ups (Vogelsang et al., 2015; Remspecher et al., 2019; Fridman et al., 2025), and less frequently 2D-pile set-ups (White and

Bolton, 2004; Thanh Chi, 2008; and Vogelsang et al., 2015). Vogelsang et al. (2016a) described the differences between the three set-up types as follows. A full-pile set-up captures 3D pile-soil interaction. It is a robust, simple set-up with mostly on-pile measurements; reliable in-soil measurements are difficult but achievable. A half-pile set-up enables visual assessments of soil deformation at the symmetry plane, often evaluated with Digital Image Correlation (DIC), but results can be affected by soil-window friction and deflection. A 2D-pile set-up, also enabling DIC, has the advantage of a relatively large base area which is favourable for the reproducibility. With smooth and rigid side walls, realistic simulation can be obtained with small model dimensions, but any radial effects are neglected.

For investigating jetting for the dredging industry, mostly flumes are used: long (rotating) tanks and horizontally moving set-ups (for example Winterwerp et al., 1992). For looking into the effect of a single jet jetting vertically in a sand bed, research has been performed for “free” jets placed in the middle of a body (Aderibigbe and Rajaratnam, 1996; Niven and Khalili, 1998) and for wall jets (Niven and Khalili, 1998; van Rhee and Bezuijen, 1998; de Brum Passini et al., 2018). Similar pros and cons as for the full-pile and half-pile set-ups apply to the “free” and wall jet set-ups, respectively.

A novel experimental apparatus is presented herein to mimic the situation inside the pile and around the pile tip that occurs when installing a pile with the Vibrojet® technique. This 2D-pile apparatus enables a novel DIC method using UV response

seeding particles, that allows for increasing the area of interest. Furthermore, a relatively uncommon technique of conductivity concentration meters is applied to measure the porosity of the soil (Cengiz et al., 2025). The apparatus is additionally equipped with total stress and pore pressure sensors. Next to the details on the apparatus, two examples of how this apparatus has been used are provided.

2 TEST APPARATUS

2.1 Overview and validity

To research the mechanisms that play a role in Vibrojet® pile installations - for instance jet erosion in a closed environment, soil state change by vibrations and by jetting, change in hydraulic head gradients - in and around the pile, a novel apparatus was designed and manufactured. The apparatus comprises a soil container and its supporting structure, a jet system (structural and hydraulic components), a pile-hammer configuration, and the instrumentation and control systems. A schematic top view of the apparatus can be found in Figure 2-1.

The soil container is very slim in comparison to the length and height, representing a plane-strain slice of soil. Two configurations that were used in the testing campaigns are shown. In configuration A, the jet was placed next to the soil container wall, mimicking only the inside of the pile; the jet is next to the pile wall and the opposite container wall

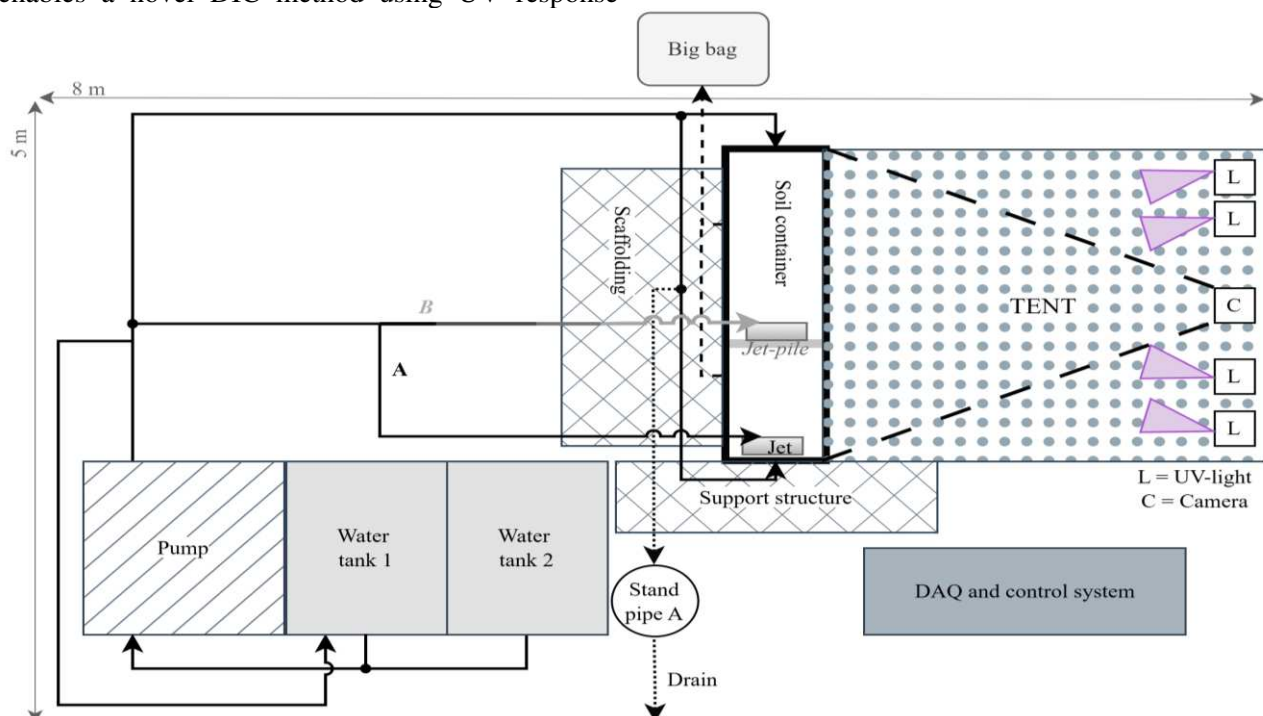


Figure 2-1. Schematic top view of the apparatus showing both configurations A and B, jet = nozzle manifold

represents the pile axis. A linear guide was present to guide the jet downwards while penetrating the soil. In configuration B, the jet was mounted to a plate, mimicking a pile wall section plus jet ring; the side of the plate where the jet ring is mounted represents the inside of the pile. Around the pile tip and at the outside of the pile wall, there is sufficient soil for proper analysis. A guiding slot was built in the tank to guide the pile section while it was penetrating the soil. An additional reinforcement frame was needed to handle the vibrational loads. Water is pumped from the water tanks via the jet into the tank, fluidising the soil inside of the pile. Photos of both configurations are provided in Figure 2-2.

The jetting and vibrational processes for a Vibrojet® pile installation are assumed axial-symmetrical. If, therefore, only a section of the pile and environment would be constructed in the lab, this would be a wedge shape, while the soil container is given a rectangular shape. The larger the pile diameter, the better this plane-strain assumption applies. Besides that, the difference between the circular sector and a strip is largest at the axis of the pile. Towards the pile wall this difference becomes smaller and smaller. Since the Vibrojet® technique focusses on monopiles and on fluidising close to the pile wall, the strip shape of the soil container will

yield enough accuracy to investigate the magnitude and behaviour of the mechanisms that play a role.

This apparatus is used for 1 g tests. For the erosion surface processes and hydraulic influences studied, the results can be taken one-to-one, since for the surface processes effective stress is the same for any water depth, and permeability and hydraulic gradient are independent of scale if natural sands are used. For investigation of particle displacement and soil density change around the pile over an installation, scaling of the pile-soil needs to be considered. An important mechanism is the shaft-soil interaction. Because of this not only L/D and D/t must be considered during scaling, but also the effects of effective stress on the shear modulus (Wood, 2004). For any deeper failure mechanisms, like sliding, although not part of the performed tests series, the extrapolation of the stresses and strength of the soil would also require scaling, and results should be considered carefully.

2.2 Components

2.2.1 Soil container

The soil container is a rigid steel construction comprising a soil compartment underlain by a water compartment separated by a filter plate, Figure 2-3.

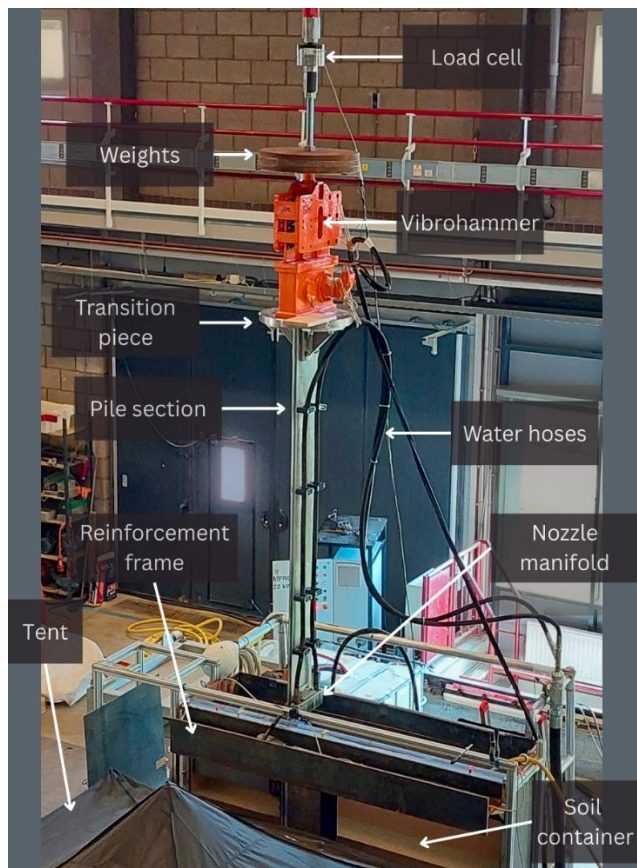
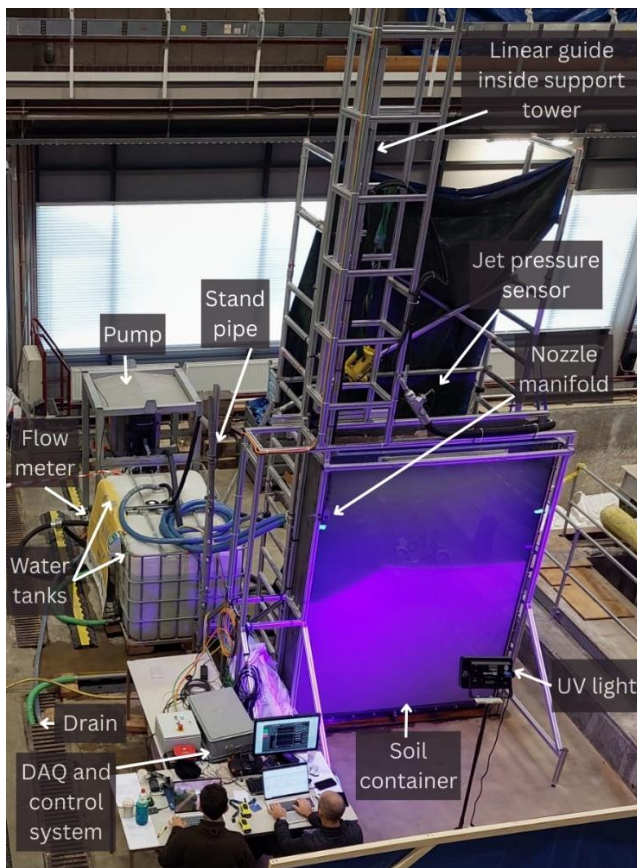


Figure 2-2. Photos of apparatus for configuration A (left) and B (right)

Controlling the pressure in the water compartment via a standpipe allowed to control the head gradient over the soil sample. The filter plate, analogous to the false floor with distribution orifices of Summerfelt and Cleasby (1996), was constructed as a sandwich structure of two perforated steel plates with a geotextile in between. Because of structural strength the perforations were until 27.5 mm of the edge of the filter plate. It was observed that this impacted visuals at the front plate and measurements at the back plate up to 100 mm above the filter plate.

The back plate and side plates contained a raster of holes for the sensors. At the top of the back plate a cutout was present over the width, to allow the water to overflow into a gutter. Originally, a filter (similar to the filter plate) was designed at the cutout to prevent soil being flushed out of the tank. During trials the filter constantly got clogged, therefore, the gutter was connected to a big bag to catch the escaped particles, which were returned into the soil container after a test. The front plate was made of 50 mm thick Perspex to enable visual inspection and recording of the test.

All plates were bolted together using O-ring cord and sealant to ensure water tightness. To protect the Perspex plate from local high bolt forces a steel strip was put between the bolts and the Perspex. A zinc-based coating was applied to prevent corrosion. A support structure prevented the container from toppling.

2.2.2 Linear Guide + Support Structure

In configuration A, a linear guide was mounted vertically in a 6.2 m high tower-shaped support structure, see Figure 2-2. A nozzle manifold with hydraulic feedlines was mounted on the linear guide carriage. The carriage was driven by a servo motor with stepless speeds ranging from 1 to 160 mm/s.

This apparatus enabled a 2.8 m vertical stroke, from any intermediate start- and stop position. In top position, the manifold was above the soil container, allowing for nozzle exchange. In bottom position, the manifold reached the soil container bottom. Both positions were set and secured using limit switches.

2.2.3 Vibrohammer + guided pile section

In configuration B (Figure 2-2), a 25 mm-thick pile section, representing a monopile wall segment, was driven by a vibrohammer plus water jetting. No linear guide nor support tower were used; the nozzle manifold was mounted on the 2.7 m-high pile section, hanging from a gantry crane, see Figure 2-3. A bespoke guidance frame was built into the soil con-

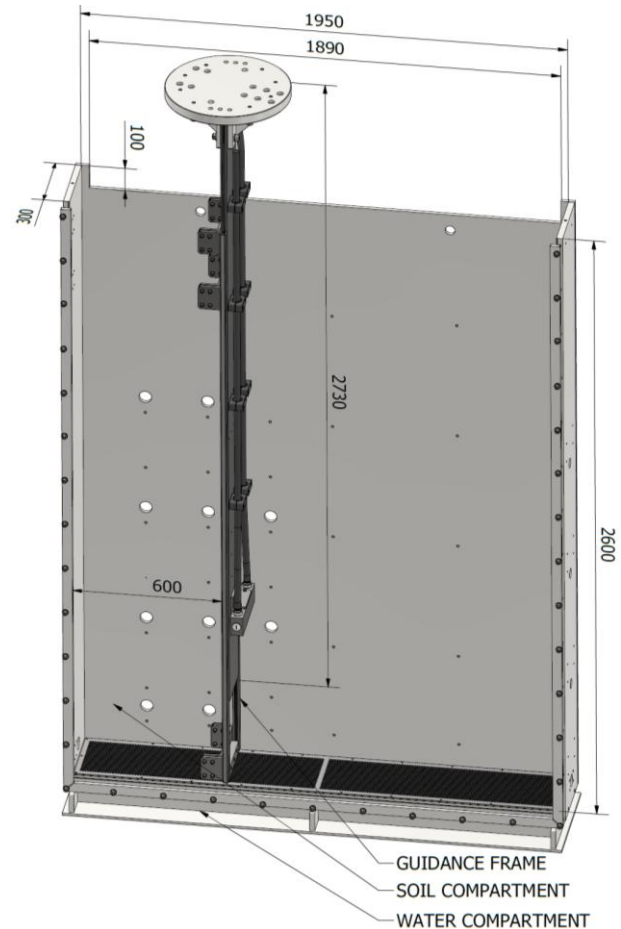


Figure 2-3. Soil container design and inner dimensions; plus configuration B pile section, nozzle manifold and guidance frame

tainer. On top of the pile section, a transition piece was bolted for the vibrohammer connection interface.

An APE-3 vibrohammer, having an eccentric moment of 0.35 kgm and drive force up to 18 kN, was utilised. Additionally, three 50 kg weights were added above the vibrohammer for dampening. The vibrohammer was powered by a Variable Frequency Drive (VFD) controlled 11 kW hydraulic power unit (HPU), at frequencies between 20 and 30 Hz.

2.2.4 Hydraulic water supply system

In both configurations, a hydraulic water supply- and circulation system was in place. The piping and instrumentation diagram is shown in Figure 2-4.

An EVMS20-12 pump with a VFD driven 15 kW-motor was used, delivering maximum 20 bar pressure or 510 L/min flowrate. System pressure was monitored using pressure sensors downstream from the pump. 3-Way valves facilitated three main functionalities: 1) Bypass loop: the hydraulic system is started in bypass loop to create a stable flow. Next, the valves are switched to one of the following two:

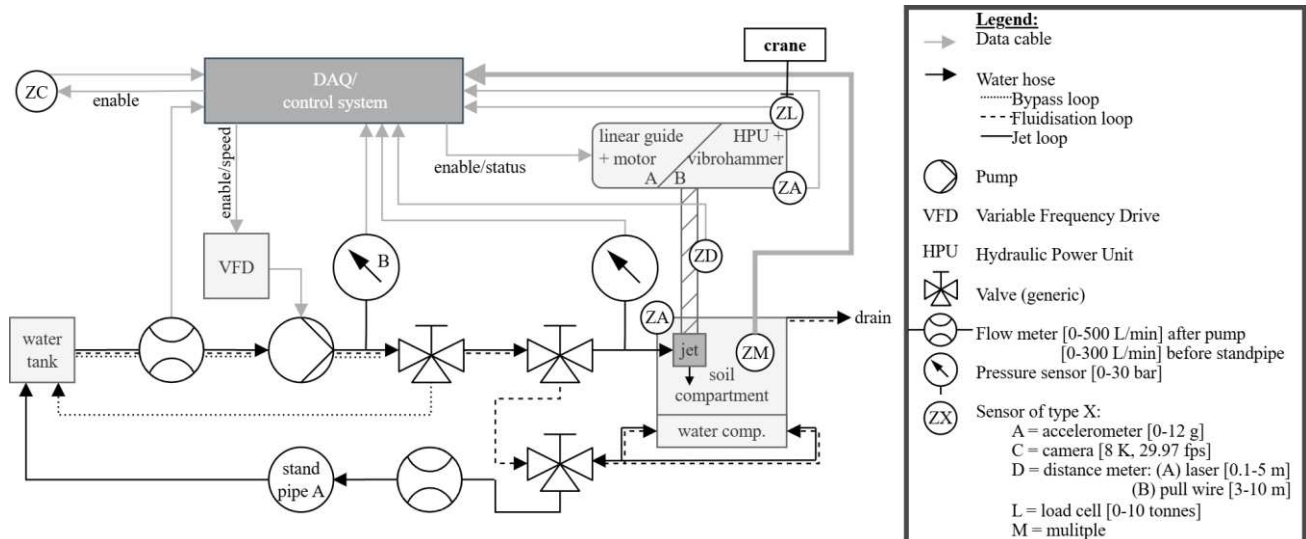


Figure 2-4. Piping- and instrumentation diagram; jet = the nozzle manifold

2) Fluidisation loop: water supply is directed towards the water compartment, through the filter plate into the soil compartment. The sand sample is mixed well by the uniform upward flow as part of sample preparation; 3) Jet loop: water flow is supplied towards the manifold via pressure rated hoses, resulting in jetting out of the nozzles during the tests. Flow is directed via a single 2 inch hose split into two $\frac{3}{4}$ inch hoses to enable the required bending radius. The hoses move vertically with the nozzle manifold (Figure 2-2).

Configuration A includes a standpipe consisting of standard modular PVC-pipe segments, with an overflow recirculating to the water tanks. For configuration B, the standpipe was not implemented. This was not required due to the open connection underneath the pile segment between inside and outside the pile.

2.3 Instrumentation

A patterned grid of through-holes was made in the back plate, enabling sensor position optimisation for the two configurations. Sensors were placed in with bespoke caps, so they were flush with the inner surface. Unused holes were plugged with dummies. The soil container was equipped with pore pressure sensors and total stress sensors to enable determination of effective stress. An absolute pressure sensor in the water compartment served as reference sensor. The conductivity concentration measurements strings (CCMs) were glued into the tank and accompanied by a temperature sensor, since they are highly sensitive to temperature. Positioning of these sensors can be found in Figure 2-5. For controlling the tests several additional sensors were included, see Figure 2-4.

The soil container front plate was illuminated by multiple LED UV panels (Figure 2-2) in the blacklight wavelength range (320-400nm), which is safe to work

in. A high-quality digital single-lens reflex camera suitable for filming was used. The camera was connected to the control and monitoring system from where it could be triggered.

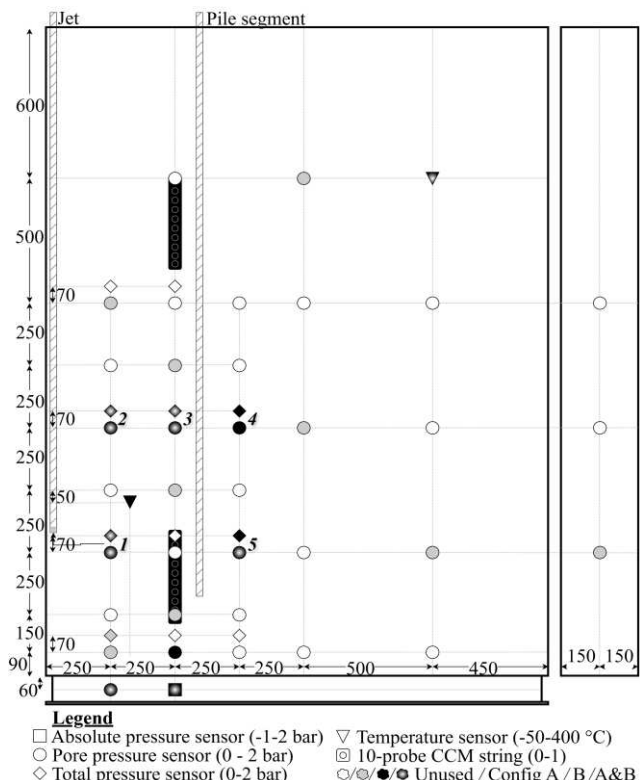


Figure 2-5. Sensor types and positioning for both configurations (inner dimensions); jet, for config. A, and pile segment, for config. B, shown at their target depths; bold skewed are the pair numbers as adopted in the results

2.4 Control system

Test control and monitoring were carried out using a Dewesoft data acquisition system, which served as the central interface for operating and observing key

components during both test configurations. DewesoftX software provided synchronised control over the test sequence and ensured consistent setup throughout the campaign. Emergency stop functions were available on all powered systems.

The data acquisition (DAQ) system controlled the start, stop, and speed of the pump via its VFD. Pump frequency adjustment could be made in fine increments, allowing continuous control over the flow conditions. Camera triggering and the initiation and stopping of data logging were managed within the same interface, ensuring full time-synchronisation between equipment operation and recorded data. In configuration A, the DAQ system also operated the linear guide, controlling vertical position and speed. A draw-wire sensor tracked the linear guide's vertical position. In configuration B, the pile section was lowered using the remote controlled manually operated gantry crane, with a laser sensor used to measure its position and speed. In both cases, all data was logged in DewesoftX.

Flow direction between test loops was adjusted manually using dedicated valves. The vibrohammer used in configuration B was operated independently through a separate HPU. Its vibration frequency and amplitude were adjusted manually and remotely, with stepless frequency control allowing fine-tuning.

2.5 Materials and sample preparation

A fine ($D_{50} = 150 \mu\text{m}$), medium ($D_{50} = 310 \mu\text{m}$) and coarse ($D_{50} = 600 \mu\text{m}$) sand were tested within the apparatus. These sands were selected to ensure that the overall range of particle size distributions and permeability spanned a representative spectrum relevant to offshore foundations. The fines content, characteristic of offshore sands, is not represented due to practical experimental causes as fines wash out during placement and handling of the material.

The sand sample was prepared by funnelling the needed amount of air-dry sand into the soil container. 0.3-0.6% of the total dry mass were UV response seeding particles, which were pre-mixed into the sands. To ensure a homogeneous sample, the sand was fluidised with an upward flow, after which the sand was left to settle to a loose sample. An accompanying benefit of the fluidisation method is that all air pockets can easily escape. After this, the soil was compacted to the intended relative density by using a needle vibrator (M2500). The vibrating needle was lowered to the bottom of the tank and then slowly drawn back up. This was done on multiple places along the sample and was continued until the soil surface reached the target level. Special care was taken that the needle would not hit any of the back-plate sensors.

3 EXAMPLE TEST RESULTS

3.1 Example with configuration A

Jet pit development over time under an impinging vertical jet was researched with this configuration. At initial state the jet flow is built-up in the bypass loop, then the flow is switched to the jet loop, initiating jetting inside the soil compartment. After a few seconds the flow stabilises, and the jet starts moving downward with constant rate until target depth $\pm 0.85 \text{ m}$ into the sand sample (phase 1). In Figure 3-1 the jet nearing target depth is shown for the tracer experiment, which was outside of the official quantitative test series but was performed to enhance visibility for explanatory purposes. At this point in time the stand-off distance of the jet pit in front of the jet has stabilised to a fixed value specific to the applied settings ~equilibrium at penetration speed. After this moment, the jet was kept in a fixed position while continuing jetting (phase 2). This allowed for the jet pit to develop to its maximum size ~equilibrium at zero speed. Then the jet was retracted, jet was stopped, and the soil was left to settle.

An example of the stand-off distance development



Figure 3-1. Picture of the jet pit in the tracer experiment, including jet position and schematics of stand-off-distance.

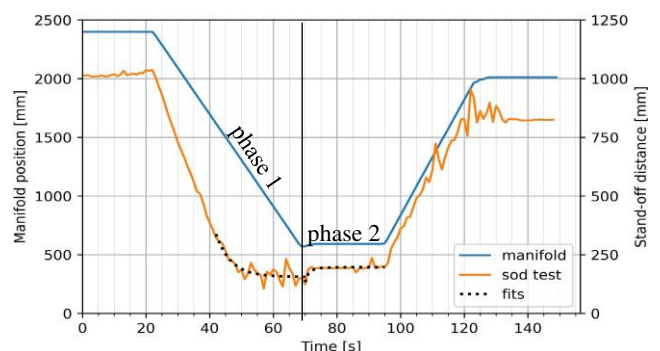


Figure 3-2. Example result over time for one of the tests; the manifold position, the stand-off distance (sod) to the manifold derived from the DIC analysis and the exponential fits to derive stationary plateaus.

over a full test is shown in Figure 3-2 for the medium sand at 77% RD and a penetration speed of 40 mm/s at low flow conditions. Indicated are the fitted profiles to gain the equilibrium stand-off distances for the given penetration speed and zero speed.

3.2 Example with configuration B

The soil state on the outside of the pile was researched for both vibro- and Vibrojet® installations. During these tests the pile section was driven into the soil sample at a fixed crane-controlled speed. Only when soil resistance was building up, a lower speed could be observed. In Figure 3-3 the effective stresses derived for the five sets of total and pore pressure sensors around the pile tip are shown over the duration of both a vibro- and a Vibrojet® test. Firstly, the hammer and jet (if tested) were turned on, after which the drain valve was opened. The latter led in both cases to a relatively big and sudden increase in effective stress. After that, penetration was initiated. At some point, penetration goes slowly and crane manipulation are started to reach target depth, which can be observed by the vibrations in the signal. When the jet stops, a small jump in effective stress is observed.

A distinct difference is the behaviour of pair 4 between the two tests. In the vibrohammer test the effective stress at this sensor pair builds up over the installation, which is attributed to the compaction due to the vibrations. In the Vibrojet® test, however, this sensor shows a “flat line” without any effective stress. This is because this sensor pair lies within the inside of the pile segment, where the jet is active. From approximately 120 s, the jet is approaching closely until the sensor pair is within the jet pit and measures thus no strength of the missing soil skeleton. For pair 5, similar behaviour can be observed, but delayed with respect to pair 4, you do see a nice recovery from jet pit to fixed skeleton after the jet is stopped.

For all the sensors pairs that were located on the outside of the pile segment (pairs 1 to 3), the above-described behaviour is not observed. This is clear evidence that the jet zone stayed inside the pile segment during installation.

An example of the CCM-sensor results is presented in an accompanying paper of Cengiz et al. (2026).

4 CONCLUSIONS

A novel apparatus is presented herein that can be used to conduct research to the situation inside the pile and around the pile tip that occurs when installing a pile with the Vibrojet® technique. The examples show that the apparatus facilitates strong opportunities in analysing with DIC method for increased area of

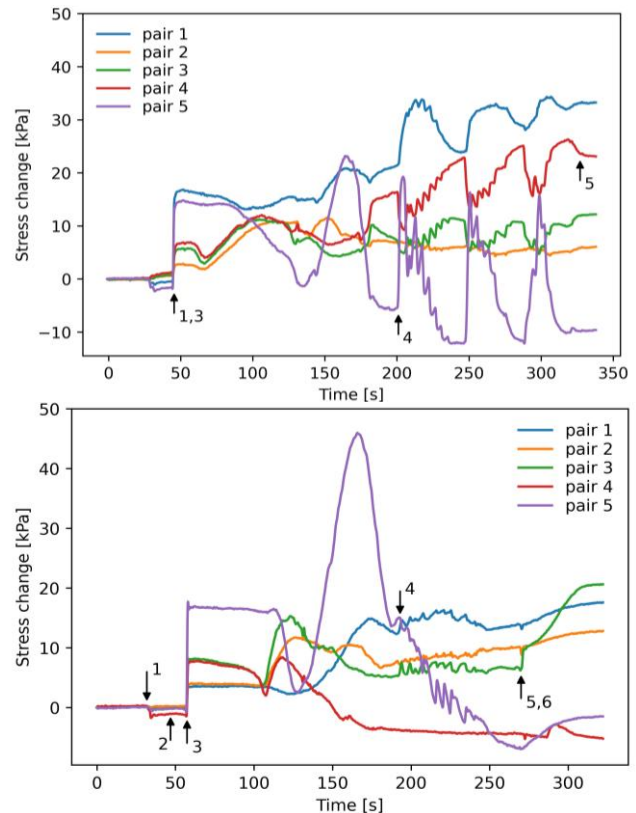


Figure 4-1. Effective stress change over time, obtained from pairs of total stress and pore water pressure sensors for a vibrohammer (top) and Vibrojet® (bottom) installation. Six events are indicated; 1 vibrohammer starts, 2 jet starts, 3 drain valve opens, 4 crane manipulations start, 5 vibrohammer stops, and 6 jet stops.

interest and the possibility to connect the DIC to other sensor types, because of the ‘flat’ tank design. Further, it shows that for these kinds of mechanisms, the plane-strain assumption is very powerful even when researching an installation technique for pipe piles. Also, this assumption can be particularly useful when the target is to compare multiple techniques or multiple settings.

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