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Dimensional analysis of backward erosion piping phenomena

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ABSTRACT: Backward erosion piping (BEP) is a mechanism of internal erosion that leads to the formation of shallow erosion channels in the sandy foundation of river embankments, or other water-retaining structures, at the interface with a more impervious layer. Under ultimate conditions this process may result in the failure of the earthen structure. Physical modelling enables the reproduction and study of this phenomenon in a controlled environment, thereby identifying the main variables involved in the process. To build a model capable of accurately reproducing the phenomenology at different scales, the achievement of dynamic similarity between the model and the real system, which in turn implies geometric and kinematic similarities, is crucial. Nonetheless, the requirement for dynamic similarity is generally difficult to achieve. In this context, dimensional analysis is of paramount importance, as it enables to define dimensionless groups of significant relevance for the physical problem under study. In this work, after a brief introduction to the BEP process, the theoretical approach required for dimensional analysis is applied to the results of physical model tests from the scientific literature, as well as to those obtained from the experimental apparatus recently implemented at the University of Bologna. Despite the dispersion of the results, which is certainly related to an insufficiently homogeneous experimental database, this preliminary study emphasizes the importance of the dimensional analysis tool and confirms the complex nature of BEP.

1 INTRODUCTION

1.1 Backward erosion piping

Backward erosion piping (BEP) is an internal erosion mechanism triggered by seepage through the shallow strata of sandy foundations of river embankments, or other water retaining structures. This process leads to the formation of shallow erosion channels within the sandy soil at the interface with an overlying, finer and then less permeable layer, the latter commonly referred to as the «blanket» (Bonelli, 2013).

During a flood event, the increase in river water level causes a significant increase in pore water pressure within the aquifer, and the resulting uplift pressure may create a localized exit point in the blanket. The formation of the downstream exit hole can also be attributed to pre-existing discontinuities or weak points in the blanket, either natural (e.g., local reduction in the blanket thickness) or artificial (e.g., man-made ditch for irrigation purposes). If the upward seepage pressures are high enough, sand fluidization will occur locally inside the hole (Figure 1(a)). This being a warning sign of initiation of the erosion process: sand looks like a boiling fluid, hence the term «sand boil».

Once triggered, the erosion may gradually develop towards the riverside, in a direction opposite to the groundwater flow, forming shallow pipes at the top of the sandy aquifer (Figure 1(b) and 1(c)). Sand particles are eroded and transported by the flow and, if the flow rate in the vertical part of the pipe is sufficiently high, they are ejected from the exit hole to form a ring similar to a volcano crater (Figure 2). When the piping channel reaches the upstream side, the erosion process accelerates due to a direct hydraulic connection between the upstream and downstream sides, causing a deepening and widening of the pipes. At this stage, under ultimate conditions, the process may result in a structural failure of the river embankment (Figure 1(d)), thereby potentially undermining the hydraulic vulnerability of the surrounding territory with dramatic consequences.

BEP is recognized as one of the major causes of severe damage, or even failure, to flood defenses worldwide (Foster et al., 2000), affecting several countries including the Netherlands, the United States, Hungary and China. In Italy, there are no specific data on the incidence of river embankments failures caused by BEP, as those available refer to all internal erosion processes and not only to piping. Nonetheless, a total of 130 sand boils of remarkable size have been catalogued in recent years along the major Italian watercourse, the Po River (Marchi et al., 2021), highlighting the evidence and importance of this phenomenology.

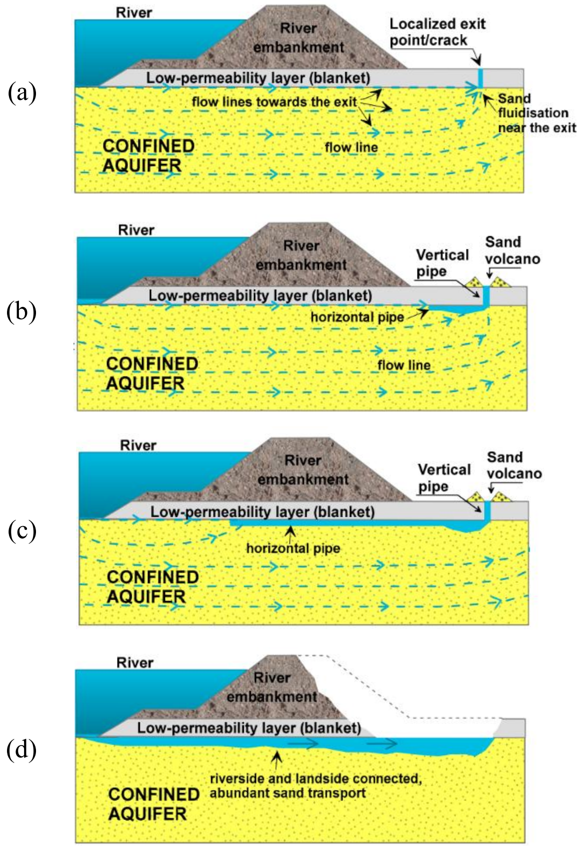

Figure 1. Model of BEP process: (a) seepage and erosion onset; (b) backward erosion initiation; (c) backward erosion progression; (d) river embankment failure (mod. from Tonni et al., 2024).

Figure 2. Sand boil along a stretch of the Po River (Italy) reactivated during the 2014 high-water event: the approximate diameter of the external sand volcano is 2.5 m (Marchi et al., 2021).

1.2 Critical hydraulic gradient

In the current study, the methodological approach of dimensional analysis has been applied to BEP phenomena by investigating the evolution of the critical hydraulic gradient as a function of selected dimensionless groups. Therefore, a common definition of this variable is of paramount importance. The critical hydraulic gradient is defined as follows:

$$i_{cr} = \frac{\Delta H_{cr}}{L_s} \qquad (1)$$

where ΔH_{cr} (m) is the critical hydraulic head difference between upstream and downstream sides and L_s (m) is the seepage length.

The critical one represents the maximum hydraulic head difference beyond which internal equilibrium conditions can no longer be achieved, leading to pipe progression towards riverside (Sellmeijer, 1988; Bonelli, 2013). Nonetheless, this head difference can be dominated by either the initiation phase (i.e., when the first micro-pipe begins to develop) or the progression phase of the erosion process (Pol, 2022). Due to this different erosion behavior, in section 4 the values of critical hydraulic gradients, obtained from laboratory experiments reported in the scientific literature, will be classified in initiation-dominated or progression-dominated tests.

2 EXPERIMENTS CONDUCTED ON BEP

To apply the methodological approach of dimensional analysis, results from laboratory tests at 1g – conducted by several researchers in the framework of physical modelling of BEP – have been extrapolated from the scientific literature. Among these, experimental data from the small- and medium-scale setups recently implemented at the University of Bologna have also been included.

In the context of physical modelling, regardless of the author, experimental apparatus used to simulate BEP have basically the same operating mechanism. A sand sample, representing the aquifer, is confined in a box, with a lid – simulating an impervious blanket – that is transparent to enable the visualization of piping channels. A hydraulic head is applied on the upstream side of the box, while an opening on the downstream side allows seepage through the sample and outflow of water. The main differences among the experimental setups are related to the geometric dimensions of the apparatus and their exit type. As regards the latter, the following configurations of experimental apparatus can be identified:

- area-type, representing river embankments on the top of an aquifer without blanket layer on the downstream side (Figure 3(a)). The experimental setups falling in this configuration, adopted for the current study, include those implemented by De Wit (1984) and van Beek et al. (2011);
- ditch-type, representing river embankments on the top of an aquifer with a blanket layer at the downstream side that is transversely intersected by a ditch (Figure 3(b)). The experimental setups falling in this configuration, adopted for the current study, include those implemented by De Wit (1984) and De Rijke (1991);
- hole-type, representing river embankments on the top of an aquifer with a blanket layer at the downstream side that is locally perforated by a circular hole (Figure 3(c)). The experimental setups falling in this configuration, adopted for the current study, include those implemented by De Wit (1984), Hanses (1985), van Beek et al. (2015), Negrinelli et al. (2016) and Pol et al. (2022), as well as the small- and medium-scale apparatus at the University of Bologna (Gragnano et al., 2023; Dodaro et al., 2024);
- slope-type, representing sand samples with a natural slope on the downstream side. This configuration has not been considered in the current work as it is not representative of any specific condition in the field, as noted by van Beek (2015).

The exit geometry strictly influences groundwater flow in the sandy aquifer and its erosion behavior (i.e., initiation- or progression-dominated) (Bonelli, 2013). In homogeneous sandy aquifers, experimental apparatus with hole-type exits generally exhibit progression-dominated erosion behaviors. On the contrary, ditch-, slope- and area-type setups tend to be initiation-dominated (Pol, 2022; Bonelli, 2013), thus implying that the critical head for progression is exceeded from the beginning of pipe formation.

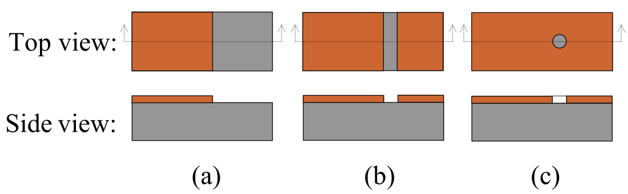

Figure 3. Top and side views of the considered apparatus configurations: (a) area-type; (b) ditch-type; (c) hole-type (mod. from Pol, 2022).

3 DIMENSIONAL ANALYSIS OF BEP

Physical modelling enables the reproduction and study of physical phenomena in a controlled environment. However, to build a model capable of accurately reproducing the phenomenology at different scales, the requirement of dynamic similarity becomes fundamental. This condition, which is satisfied when the forces acting in the model exhibit the same scale ratio with the corresponding forces in the prototype, in turn implies geometric and kinematic similarities. Nonetheless, the achievement of dynamic similarity between the model and prototype is generally difficult to achieve, and sometimes unnecessary. In this context, dimensional analysis is of paramount importance, as it enables to identify dimensionless groups of significant relevance for the physical problem under study.

To implement the theoretical framework of dimensional analysis, a sample of data sufficiently representative of the process under investigation has been collected and properly organized. For each laboratory test derived from literature, experimental findings – related to the critical hydraulic gradient (defined in section 1.2) and other variables for which numerical values have been provided by the authors – have been gathered to generate a database that was as homogeneous as possible.

After organizing the data based on the type of erosion behavior (i.e., initiation- or erosion-dominated), the methodological approach of dimensional analysis has been applied. To identify the so-called π-Groups (i.e., dimensionless variables, which are the output of dimensional analysis), the procedure outlined in Butterfield (1999) has been followed. More specifically, Buckingham's theorem (Buckingham, 1914) has been applied, taking into account the additional conditions formulated by Bridgman (1931) and van Driest (1946). A MatLab (MathWorks, 2020) script has also been implemented to select the repeated variables and automatically derive the resulting dimensionless groups.

More specifically, it is assumed that the critical hydraulic gradient can be described by a list of r variables collected in R, as follows:

$$R = \begin{Bmatrix} \Delta H, L_S, t_c, t_s, d_{50}, n, A_{exit}, \phi, \\ \rho_c, \rho_s, \rho_w, E_s, \mu_w, v_w, K_s, g \end{Bmatrix} \quad (2)$$

where ΔH (m) is the hydraulic head difference between upstream and downstream sides, L_s (m) is the seepage length, t_c (m) is the thickness of the clay blanket, t_s (m) is the thickness of the sand aquifer, d_{50} (m) is the median particle diameter of the sand grains, n (-) is the porosity of the aquifer, A_{exit} (m^2) is the cross-sectional area of the exit, ϕ (deg) is the angle of shearing resistance of the sand aquifer, ρ_c (kg/m^3) is the mass density of the clay blanket, ρ_s (kg/m^3) is the mass density of the sand aquifer, ρ_w (kg/m^3) is the water density, E_s (kPa) is the Young's Modulus of the sand aquifer, μ_w (Pa·s) is the water dynamic viscosity, v_w (m/s) is the water velocity in the exit point, K_s (m/s) is the hydraulic conductivity of the sand aquifer and g (m/s^2) is the gravitational acceleration. These 16 variables (r = 16) have been identified following the study conducted by Cote et al. (2023).

To describe the 16 components of R, 3 independent primary dimensions (m) are necessary (i.e., mass, length and time), thereby m = 3. In this way, the minimum number of independent dimensionless groups (p) necessary to specify the system is:

$$p = r - m = 13 \quad (3)$$

After applying the approach, for the admissible triads of repeated variables, more than 200 π-Groups have been derived and carefully analyzed. Most of them were excluded because they were either not deemed relevant to the objective of the study, or not quantifiable due to the lack of known numerical values of the variables involved. The following π-Groups have been identified as the most relevant:

$$\pi_1 = \frac{L_s}{d_{50}} \quad (4)$$

$$\pi_2 = \frac{d_{50} \cdot g}{K_s^2} \quad (5)$$

$$\pi_3 = \frac{A_{exit} \cdot \rho_w^2 \cdot K_s^2}{\mu_w^2} \quad (6)$$

where π_1 and π_2 derive from the selection of d_{50}, ρ_s and K_s as repeated variables, while π_3 derives from the selection of ρ_s, μ_w and K_s as repeated variables.

4 RESULTS AND DISCUSSION

The π-Groups, identified according to the procedure outlined in section 3, were plotted with respect to the critical hydraulic gradient. Following the legend shown in Figure 4, in the graphs presented hereinafter, data obtained from hole-type experimental apparatus are represented by circles, those from ditch-type apparatus by triangles, and those from area-type apparatus by squares.

For all laboratory experiments considered in the current study, the graph in Figure 4 illustrates the relationship between π_1 and the critical hydraulic gradient. From a general perspective, although the distribution of the data points exhibits a significant dispersion, initiation-dominated experiments tend to show critical hydraulic gradients higher than those derived from progression-dominated setups.

Additionally, the critical hydraulic gradient shows some dependency on π_1: as the latter increases (i.e., as the seepage length increases and/or the characteristic grain diameter of the sand aquifer decreases), the critical hydraulic gradient tends to decrease. However, the plot in Figure 4 cannot show a clear picture as the available data – coming from experimental setups exhibiting different erosion behaviors – have been considered all together.

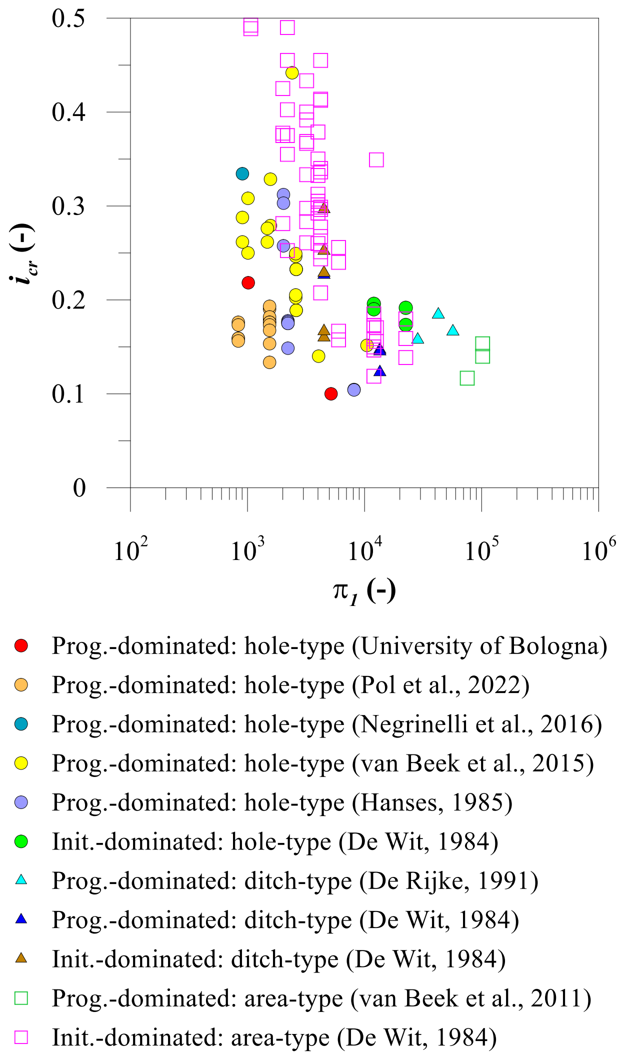

Figure 4. Critical hydraulic gradient as a function of π_1 for initiation- and progression-dominated experiments.

Therefore, only in relation to progression-dominated experimental setups, Figures 5 and 6 illustrate the distribution of the critical hydraulic gradient as a function of the dimensionless groups π_2 and π_3, respectively. Both graphs are still affected by an evident dispersion, which does not allow an effective assessment of the overall results. Such significant data scatter may be attributed – on the one hand – to the unavoidable non-homogeneity of the available experimental database and – on the other hand – to the intrinsic complexity of the studied physical phenomenon that is function of several π-Groups.

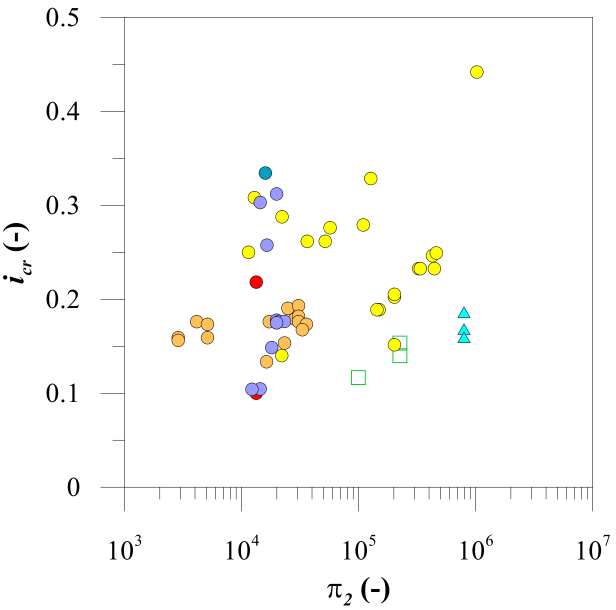

Figure 5. Critical hydraulic gradient as a function of π_2 for progression-dominated experiments (legend in Figure 4).

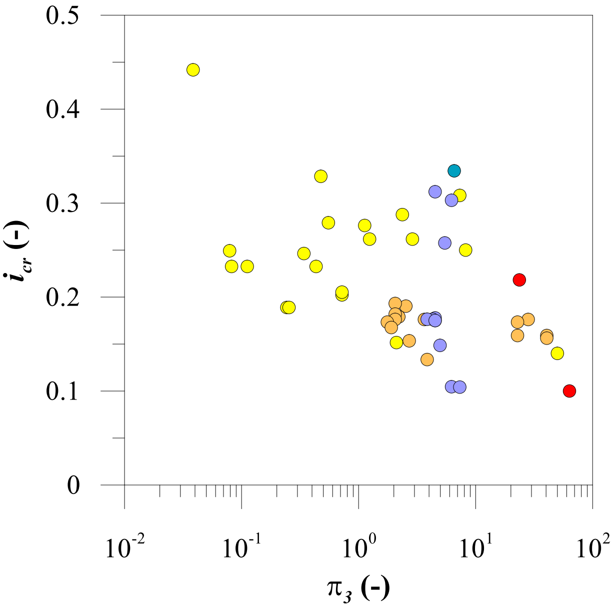

Figure 6. Critical hydraulic gradient as a function of π_3 for progression-dominated experiments (legend in Figure 4).

Furthermore, the variables d_{50} and K_s in π_2 are expected to be dependent on each other since, as the characteristic particle diameter increases, the hydraulic conductivity will also tend to increase.

As regards π_3, A_{exit} and K_s are the most important variables, as ρ_w and μ_w assume basically constant values within the considered experimental database. However, in this case, A_{exit} and K_s are not dependent on each other. In addition, it is important to notice that A_{exit} has a physical meaning only for hole-type apparatus, as it represents the cross-sectional area of the circular hole through which water outflows. In ditch- and area-type apparatus, A_{exit} would correspond to the whole cross-sectional area of the opening, but water is expected to outflow only through a limited portion of this area, thus making A_{exit} not representative of these exit types. That is also why ditch- and area-type configurations have not been considered in Figure 6.

5 CONCLUSIONS

The findings of the current work enable to make a few interesting observations.

Firstly, this study underscores the importance of dimensional analysis when taking into considerations data from different physical models, subjected to different boundary conditions. As expected, a marked variation – in terms of critical hydraulic gradient – among the types of exit geometries has clearly emerged. Regardless of the evident scatter of points, however, initiation-dominated experimental setups (i.e., ditch- and area-type apparatus) are affected by critical hydraulic gradients higher than progression-dominated setups (i.e., hole-type apparatus). When analyzing BEP experimental results, it is therefore very important to consider the specific boundary conditions, which basically depend on the outflow configuration of the apparatus. Moreover, the DA tool enabled to highlight the importance of medium- and large-scale apparatus, as they appear to provide a lower estimation of the critical hydraulic gradient when compared to small-scale setups.

Secondly, the analysis confirms the intrinsic complex nature of backward erosion piping, that depends on several dimensionless groups. Despite the rigorous approach from a dimensional point of view performed, the significant dispersion affecting experimental data does not thus enable to catch specific trends of the examined dimensionless groups as a function of the critical hydraulic gradient. This can be certainly ascribed, on the one hand, to the wide heterogeneity of the experimental database but, on the other hand, also to the complex phenomenon under investigation.

This preliminary study represents only a first step of future and more detailed investigations. Further experimental work is necessary to improve the data quality and homogeneity, so to be able to estimate all 13 dimensionless groups and to identify potential trends.

REFERENCES

Bonelli, S. (2013). *Erosion in geomechanics applied to dams and levees*, John Wiley & Sons. https://doi.org/10.1002/9781118577165

Bridgman, P. W. (1931). *Dimensional analysis*. New Haven: Yale University Press.

Buckingham, E. (1914). On physically similar systems: illustrating the use of dimensional analysis. *Physical Review*, 4(4), 345-376.

Butterfield, R. (1999). Dimensional analysis for geotechnical engineers, *Géotecnique* 49(3), 357-366.

Cote, L. V., Ovalle-Villamil, W., Cabrera, M., Sasanakul, I., Jommi, C. and Hick, M. A. (2023). Assessment of scaling principles and their effect on centrifuge modelling of backward erosion piping. *29th Meeting of European Working Group on Internal Erosion in Embankment Dams, Dikes and Levees and their Foundations*, 2-5 July 2023, Lyon, France.

De Rijke, W. G. (1991). *Verificatie piping model: proven in de Deltagoot*. Grondmechanica Report CO-317710/7, Delft, The Netherlands (in Dutch).

De Wit, J. M. (1984). *Onderzoek zandmeevoerende wellen Rapportage Modelproeven*. Grondmechanica Delft report CO-220887/10, Delft, The Netherlands (in Dutch).

Dodaro, E., Tumedei, E., Marchi, M., Gottardi, G. and Tonni, L. (2024). A novel medium-scale experimental setup to investigate backward erosion piping. *30th Meeting of the European Working Group on Internal Erosion in Embankment Dams, Dikes and Levees and their Foundations*, 9-11 September 2024, Bologna, Italy.

Foster, M., Fell, R. and Spannagle, M. (2000). The statistics of embankment dam failures and accidents. *Canadian Geotechnical Journal*, 37(5), 1000-1024. http://dx.doi.org/10.1139/t00-030

Gragnano, C. G., Camiletti, F., Forbicini, F., Gottardi, G., Marchi, M. and Tonni L. (2023). Physical modelling of backward erosion piping for the development of natural-based mitigation strategies. In: Ferrari, A., Rosone M., Ziccarelli, M., Gottardi, G. (eds) *Geotechnical Engineering in the Digital and Technological Innovation Era*, CNRIG 2023. Springer Series in Geomechanics and Geoengineering. Springer, Cham, 87-94. https://dx.doi.org/10.1007/978-3-031-34761-0_11

Hanses, U. (1985). *Zur Mechanik der Entwicklung von Erosionskanälen in geschichtetem Untergrund unter Stauanlagen*. PhD thesis, Grundbauinstitut der Technischen Universität Berlin, Berlin, Germany (in German).

Marchi, M., García Martínez, M. F., Gottardi, G. and Tonni, L. (2021). Field measurements on a large natural sand boil along the river Po, Italy. *Quarterly Journal of Engineering Geology and Hydrogeology*, 54(4), 1-10. https://doi.org/10.1144/qjegh2020-097

MathWorks. (2020). *MatLab (R2020b)*, The MathWorks, Inc. https://www.mathworks.com

Negrinelli, G., van Beek, V. M. and Ranzi, R. (2016). Experimental and numerical investigation of backward erosion piping in heterogeneous sands. *ICSE 2016 (8th International Conference on Scour and Erosion)*, 12-15 September 2016, Oxford, UK.

Pol, J. C. (2022). *Time-dependent development of backward erosion piping*. PhD thesis, Delft University of Technology, Delft.

Pol, J. C., Kanning, W., van Beek, V. M., Robbins, B. A., and Jonkman, S. N. (2022). Temporal evolution of backward erosion piping in small-scale experiments. *Acta Geotechnica*, 17(10), 4555-4576. https://doi.org/10.1007/s11440-022-01545-1

Sellmeijer, J. B. (1988). *On the mechanism of piping under impervious structures*. PhD thesis, Delft University of Tecnology, Delft.

Tonni, L., Marchi, M., Bassi, A. and Rosso, A. (2024). A Sand Boil Database for Piping Risk Management in the Po River, Italy. *Water*, 16(10), 1-25. https://doi.org/10.3390/w16101384

Van Beek, V. M., Knoeff, H. and Sellmeijer, H. (2011). Observations on the process of backward erosion piping in small-, medium- and full-scale experiments. *European Journal of Environmental and Civil Engineering*, 15(8), 1115-1137. https://doi.org/10.1080/19648189.2011.9714844

Van Beek, V. M. (2015). *Backward erosion piping: initiation and progression*. PhD thesis, Delft University of Technology, Delft.

Van Beek, V. M., van Hessen, H. M., Vandenboer, K. and Bezuijen, A. (2015). Developments in modelling of backward erosion piping. *Géotechnique*, 65(9), 740-754. http://dx.doi.org/10.1680/geot.14.P.119

Van Driest, E. R. (1946). On dimensional analysis and the presentation of data in fluid flow problems. *Journal of Applied Mechanics, Transactions of the ASME*, 13(1), A34-A40. https://doi.org/10.1115/1.4009512