Minimum and maximum void ratios in tyre-derived and recycled concrete aggregate mixtures

M. Kowalska, B. Bdzionek
Silesian University of Technology, Gliwice, Poland, magdalena.kowalska@polsl.pl, bartbdz816@student.polsl.pl

K. Gabryś, I. Zatorski
Warsaw University of Life Sciences, Warsaw, Poland, katarzyna_gabrys@sggw.edu.pl, s210358@sggw.edu.pl

ABSTRACT: Scrap tyre rubber is considered an appropriate material for use in geotechnical seismic isolation systems due to its beneficial dynamic properties, low unit weight, and sustainability. Rubber particles are commonly mixed with sand or gravel to reduce compressibility and increase shear strength. Such mixtures are typically prepared to a prescribed relative density, which necessitates the reliable determination of the minimum and maximum void ratios (e_{min} and e_{max}). This paper presents the e_{min} and e_{max} values obtained for mixtures of well-graded rubber granulate and well-graded recycled concrete aggregate, with the rubber content by weight X_M = 0, 11, 23, 55, or 100%. The investigation showed that vibration, recommended in standard procedures to determine e_{min}, was effective for RCA and RCA-TDA mixtures, but gave overestimated values for TDA. The method using a measuring cylinder proved to be the most effective for determining e_{max}. The relationships between X_M and e_{min} or e_{max} showed trends similar to those for typical mixtures of natural gravel and rubber of the same median sizes, i.e. no specific rubber content at which the limiting void ratios would reach a minimum. The obtained e_{max} values were higher than those observed in most sand-rubber and gravel-rubber mixtures, which results probably from both the higher e_{max} of the alternative aggregates used in this study and the different testing methodology.

1 INTRODUCTION

The minimum and maximum void ratios (e_{min} and e_{max}) are fundamental physical parameters of non-cohesive soils. In practical applications, they are needed to assess the current relative density, I_D (%), of a soil deposit and the possible maximum densification of granular fills at various stages of a building process and service life, e.g. during transport, after mechanical compaction, or as a result of operational loads. In laboratory tests, e_{min} and e_{max} are necessary to build comparative specimens by maintaining the same I_D value.

For a soil with known specific density ρ_s (g/cm^3), the minimum and maximum void ratios (dimensionless) can be determined based on the maximum and minimum dry densities, $\rho_{d,max}$ (g/cm^3) and $\rho_{d,min}$ (g/cm^3), according to equations:

$$e_{min} = \frac{\varrho_s}{\varrho_{d,max}} - 1 \quad (1)$$

$$e_{max} = \frac{\varrho_s}{\varrho_{d,min}} - 1 \quad (2)$$

The specific density can be determined following (ISO, 2015). However, there is no European standard that indicates common procedures for $\rho_{d,max}$ and $\rho_{d,min}$. This leaves room for some freedom of choice between using the existing national standards, e.g. American (ASTM, 2000a, 2000b), British (BS, 1990), German (DIN, 1996), or other established methods, like the ones described in the withdrawn Polish standard (PKNMJ, 1988). In most of these documents, to obtain the smallest possible dry density, it is recommended to either pour the dry material into a mould using a funnel that is slowly lifted over the bottom of the mould or to place the dry material in a measuring cylinder and allow the grains to fall freely after inverting the cylinder a few times. The methods to achieve the densest state consist of applying vibrations to the dry or wet material placed in a mould – using a vibrating table, hammer, or fork. Each set of instructions provides different recommendations regarding the applicability of these procedures, including the range of soil grading, method of soil preparation, dimensions and mass of the containers, as well as other accessories, or the number of repetitions.

The determination of e_{min} and e_{max} becomes challenging when non-standard geomaterials are considered. In this study, we focus on two sustainable waste-derived materials: scrap tyre rubber (also known as 'tyre derived aggregate', TDA), produced from end-of-life car tyres, and recycled concrete aggregate (RCA). Both materials exhibit mechanical and physical characteristics that differ from those of natural granular soils.

Due to its beneficial properties, such as durability, low weight, elasticity, energy absorption capacity, and high permeability, TDA is considered an appropriate material for use in geotechnical seismic isolation systems (GSIS). This has already been proved in several small and large-scale laboratory experiments (Abate et al., 2023; Gromysz and Kowalska, 2017; Hazarika et al., 2008; Kowalska and Vrettos, 2025; Rios et al., 2021; Tsang et al., 2021), numerical analyses (Chiaro et al., 2023; Tsang et al., 2012), and practical applications (Pitilakis et al., 2024; Raj and Suppiah, 2021; Takemiya, 2007). Most real or large-scale physical tests refer to the largest fractions of TDA (shreds and chips), which require less energy to manufacture and are less expensive. However, they contain significant amounts of steel and textile cord, which were believed to be responsible for the self-ignition cases reported in 1995 (Arroyo et al., 2011; Humphrey, 1996). This problem does not apply to finer fractions, such as granulate and powder, because they are made up almost entirely of pure rubber, with only minimal traces of other materials. Due to the size limits of various pieces of equipment, these fractions are preferred in small-scale laboratory tests.

To reduce the compressibility of the damping layers in GSIS, TDA is usually mixed with sand or gravel. As natural mineral soils are becoming expensive and less available, it is worth considering replacing them with an alternative material, e.g. RCA, which is sustainable, cost-effective, and behaves similarly to natural gravel.

In this research, our objective was to determine the minimum and maximum void ratios of well-graded TDA, RCA, and RCA-TDA mixtures, as well as the influence of testing methodology on the preparation of typical laboratory-size specimens of specified I_D.

The literature on RCA-TDA mixtures is scarce (Arulrajah et al., 2019; Gabryś et al., 2021; Shahjalal et al., 2023). To the authors' best knowledge, there have been no previous studies related to e_{min} and e_{max} in these materials. Earlier investigations of pure TDA or TDA-soil mixtures also rarely focused on the influence of rubber content, particle size distribution, or testing methodology on the e_{min} and e_{max} values. However, it has been shown, e.g. by Kim and Santamarina (2008) and Tasalloti et al. (2021), that rubber-only is usually characterised with higher void ratios than mineral soils or gravel-rubber mixtures. Additionally, it is already known that the ratio of the median particle sizes of rubber $d_{50,R}$ (mm) and soil $d_{50,S}$ (mm), $\eta = d_{50,R}/d_{50,S}$, plays an important role; at $\eta >> 1$ or $\eta << 1$, there exists a rubber content, giving minimum values of e_{min} and e_{max}, and indicating the segregation of the particles; at $\eta = 1$, both values tend to increase with the rubber content; c.f. Figure 3. It should be mentioned that in most published studies on TDA-soil mixtures, the components considered are uniformly graded.

2 METHODOLOGY

The choice of procedures to determine e_{min} and e_{max}, and the particle size distribution of the RCA and TDA, was dictated by the fact that the results were needed to prepare specimens of controlled I_D in the chambers of a triaxial apparatus and a resonant column. Both apparatuses accommodate specimens of specific dimensions (140 mm in height and 70 mm in diameter), imposing a limit on the maximum size of a particle (11.7 mm).

2.1 Material

The RCA came from the demolition of a shopping centre in Poland, specifically from the concrete ceilings and columns of its above-ground car park. It was washed to remove any impurities and dried. The TDA was produced by ambient-temperature shredding of car tyres. Their ρ_s were equal to 2.71 and 1.20 g/cm^3, respectively. Both materials were sieved through a set of sieves ranging in size from 0.063 to 6.3 mm. The required amount of each fraction was combined to produce well-graded air-dry RCA and TDA samples with the following identical parameters: d_{50} = 2.45 mm, coefficient of uniformity C_U = 15.7 (–), coefficient of curvature C_C = 1.2 (–). Three RCA–TDA mixtures were also produced with rubber content by weight X_M = 11, 23, and 55%, and $d_{50,R} = d_{50,S}$; see Figure 1. It shall be noted that the RCA particles were predominantly cubic, angular, and rough, whereas the TDA particles were cubic or elongate (in the finest fractions), subangular, and smooth.

Figure 1. RCA-TDA mixtures with X_M = 11, 23, and 55%.

2.2 Procedures

The minimum bulk density was determined on air-dry specimens using three methods:

(1) carefully placing the material using a spoon or scoop into a metal mould ~126 mm high and 71 mm in diameter (volume ~497 cm^3);

(2) filling the mould to approximately half or three-quarters of its height, slowly inverting it several times, placing it upright, and eventually filling the remaining volume using a spoon;

(3) slowly inverting a 1 litre plastic measuring cylinder graduated to 10 ml (62 mm in diameter) filled with soil a couple of times and carefully standing it on a flat surface.

In methods (1) and (2), the excess soil was removed with a straightedge. In method (3), before placing it on a table, the cylinder was slowly rotated to produce a possibly flat upper surface of the material and enable reading of the volume. Each measurement was repeated at least once. The $\rho_{d,min}$ was taken as the minimum of all measurements.

The maximum bulk density was determined for an air-dry specimen (water content of 0.5 to 3%) by compacting it in the same mould as in the methods (1) and (2). The material was placed in 4 to 5 layers, each of which was manually compacted with a tamper, with sufficient care to avoid crushing the RCA particles. Then a metal piston of a matching diameter, weighing ~532 g, was placed at the top of the specimen and strongly statically pressed by hand. The whole set was attached to a vibrating table and vibrated with a frequency of 50 Hz. The vibration amplitude was increased stepwise (0.1/0.4/0.6 mm) after no further sinking of the piston was observed in two consecutive measurements. At the end of the test, the piston was again pressed statically, by hand, before the final height measurement. Each reading was repeated twice. The $\rho_{d,max}$ was taken as the maximum of all recorded values.

3 RESULTS AND DISCUSSION

The $\rho_{d,max}$ and $\rho_{d,min}$ are presented in Figure 2 as a function of X_M. The relationships can be approximated with a simple polynomial function, which yields high values of the coefficient of determination (R^2 > 0.99). The red and green dotted trend lines in Figure 2 indicate the highest and lowest values of $\rho_{d,max}$ and $\rho_{d,min}$, respectively.

Both the maximum and minimum dry densities decreased gradually as the rubber content increased. It can be seen that the variability in the results obtained was higher for the loosest than the densest state; it was almost negligible for TDA.

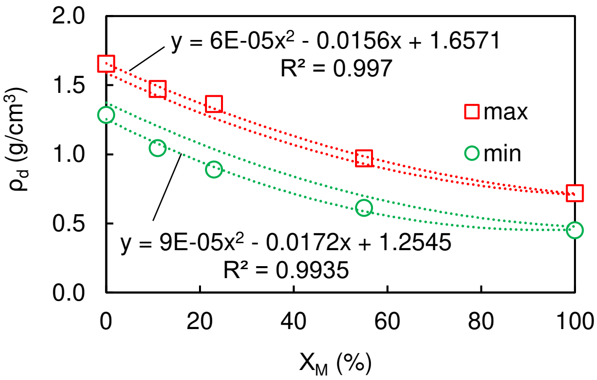

Figure 2. The maximum and minimum dry densities ($\rho_{d,max}$ and $\rho_{d,min}$) of the RCA-TDA mixtures.

In all cases, method (3) was the most effective in giving the smallest density of the material. It should be noted that the reading from the cylinder scale was subject to measurement uncertainty due to imperfect surface levelling. However, it is estimated that the resulting error did not exceed 0.02 g/cm^3.

For RCA and RCA-TDA mixtures, the maximum dry density was consistently achieved at the end of the test, i.e. after the height of the specimen stabilised at the amplitude of vibration of 0.6 mm. In contrast, the maximum dry density of the well-graded TDA was achieved when the piston was pressed statically by hand; subsequent vibration caused only a gradual loosening of the material. Additionally, TDA specimens exhibited partial rebound after removal of the static load and strong interlocking of particles, necessitating the use of tools to remove the material from the mould. Visual inspection, as well as sieve analysis before and after testing on a vibrating table, indicated negligible particle breakage in RCA specimens after vibration.

Figure 3 presents the $\rho_{d,max}$ and $\rho_{d,min}$, as well as the e_{min} and e_{max} values obtained in this study, compared with selected results from the literature. The data are coded using the type of material (RCA, TDA, S for sand, G for gravel) and its d_{50}, and the name of the researcher(s): Be for Benjelloun et al. (2022), F for

Fiamingo et al. (2025), M for Mashiri (2014), Pa for Pasha et al. (2019), and Pi for Pitilakis et al. (2024). It is worth noting that mixtures of well-graded RCA or natural sandy gravel and well-graded TDA have not been previously analysed. The presented literature results refer to the mixtures in which both soil and rubber are uniformly graded (C_U = 1–2.2, C_C = 0.9–2.2); the only exception is the research by Fiamingo et al. (2025), who investigated mixtures of well-graded sandy gravel (C_U = 27 and C_C = 2.2) and uniform TDA (C_U = 1.9 and C_C = 1.5).

The observed decrease in dry density with increasing rubber content (Figure 2) is consistent with published trends. Significant differences in the

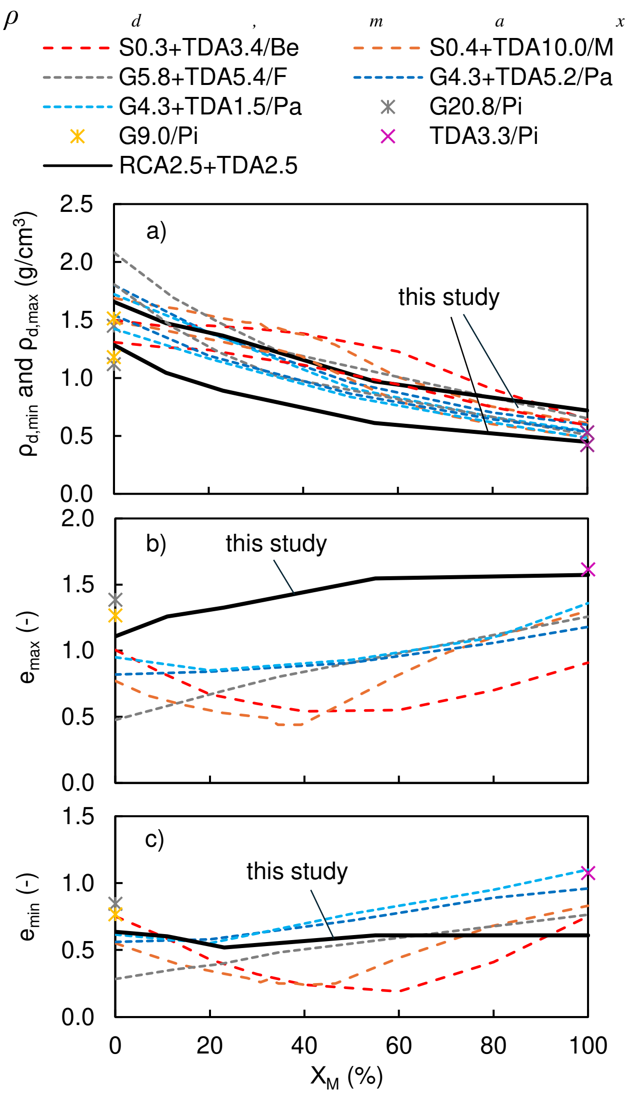

Figure 3. Influence of X_M on $\rho_{d,min}$ and $\rho_{d,max}$ (a), e_{max} (b) and e_{min} (c) of various soil-rubber mixtures.

and $\rho_{d,min}$ at $X_M \approx 0$ for the various materials presented are attributed mainly to variations in the particle size distribution (d_{50} = 0.3–20.8 mm), as well as the shapes and specific densities of the mineral particles. The results obtained for pure rubber exhibit a relatively small dispersion of values, likely due to the narrower range of rubber particle sizes considered in the literature (d_{50} = 1.5–10.0 mm).

Figure 3b and Figure 3c show that the relationships between the rubber content and the maximum and minimum void ratios of the mixtures of well-graded RCA and well-graded TDA are quite different from those for the other TDA-soil mixtures.

First of all, the addition of small rubber contents to RCA did not cause any significant decrease of e_{min} or e_{max}, typical for soil-rubber mixtures made of components of very different sizes, like in the studies of Pasha et al. (2019), Benjelloun et al. (2022), and Mashiri (2014), where η are equal to 0.35, 10.5, and 28.6, respectively. This also means that segregation of the RCA and TDA particles is very unlikely.

In the RCA-TDA mixtures, e_{max} exhibited a gradual increase with X_M, which is consistent with previous works presented in the literature for mixtures with $\eta \approx 1$, i.e. those investigated by Fiamingo et al. (2025) and Pasha et al. (2019), with η = 0.9 and 1.2, respectively. The values obtained in our research were higher than those reported for the sand-rubber and gravel-rubber mixtures in previous studies. This discrepancy could be attributed to the different testing procedures used or to the intrinsic properties of the tested materials. The latter can be supported by the fact that the values for the well-graded RCA and TDA are similar to those reported by Pitilakis et al. (2024) for uniformly graded medium gravel, with d_{50} = 9.0 and 20.8 mm, and rubber granulate with d_{50} = 3.3 mm, respectively. Unfortunately, the authors did not indicate the procedure used to obtain these results.

Interestingly, the e_{min} values for all RCA, TDA, and RCA-TDA mixtures were approximately 0.6, regardless of the rubber content. This similarity can be attributed to the well-graded nature of both components. Additionally, the relatively small e_{min} of the TDA alone can result from the static load applied to the specimens – a procedure not used in the standard approach, which turned out to be more effective than vibration. The application of a larger, controlled load is supposed to cause a further decrease in the observed trend for specimens with the prevailing rubber content.

4 CONCLUSIONS

For well-graded RCA and RCA-TDA mixtures containing 11–55% rubber by weight, vibration was effective in providing the maximum compaction of the material. In contrast, for well-graded TDA, higher density was reached under static loading, while vibration caused partial loosening of the structure. This is due to the high damping capacity of rubber and the deformability of the angular rubber particles, which strongly interlock when statically pressed but allow rearrangement and expansion under dynamic excitation. These findings suggest that the standard vibration-based procedures for determining e_{min} in natural soils are not suitable for rubber aggregates, as they can result in a substantial overestimation of the minimum void ratio.

The minimum void ratios obtained in the well-graded RCA and TDA specimens, as well as in their mixtures, were very similar (≈ 0.6) and did not show any clear dependency on the rubber content, typical for sand-rubber and gravel-rubber mixtures. Therefore, the grading characteristics of a material (C_U, C_C) appear to affect porosity more significantly than the stiffness of the individual particles. The results obtained for TDA suggest that, contrary to other materials, e_{min} should not be treated as a unique material constant; it may decrease with increasing applied stress and tend towards small values (even zero) under sufficiently high loads, reflecting progressive deformation and rearrangement of rubber particles. It is hypothesised that uniaxial loading in an oedometer or drained triaxial compression, up to the maximum stress expected in the analysed case, will provide more realistic e_{min} results if a true maximum density of TDA is required.

The procedure in which a measuring cylinder with soil is slowly inverted and gently placed on a table was the most effective in providing the loosest state in all well-graded RCA, TDA, and RCA-TDA mixtures. The e_{max} values obtained in this study showed a slight increase with rubber content – a relationship similar to that observed in other studies for sand-rubber and gravel-rubber mixtures characterised with $\eta \approx 1$. They were yet higher than those shown in most TDA-soil mixtures in the literature, which probably results from the higher coefficient of uniformity of the well-graded RCA-TDA mixtures. On the other hand, the e_{max} values for RCA and TDA (alone) were within the wide ranges obtained by other researchers for natural gravel and rubber granulate. The possible effect of C_U on e_{max} requires confirmation in further studies, as the observed discrepancies may also result from differences in research procedures applied, which are not always clearly specified in the literature.

Overall, this study provides practical guidance on determining the minimum and maximum void ratios of waste-derived aggregates and supports the development of more appropriate laboratory procedures for characterising and designing geotechnical systems that incorporate TDA.

ACKNOWLEDGEMENTS

The authors would like to thank the companies Orzeł S.A., ATB Truck S.A., and Holcim Polska for providing the research materials free of charge. This research was funded by the National Science Centre, Poland [Contract UMO-2024/53/B/ST8/02470].

DATA AVAILABILITY

The raw data presented in the study are openly available in RepOD at:
https://doi.org/10.18150/QFCBEZ

REFERENCES

Abate, G., Fiamingo, A., Massimino, M. R., Pitilakis, D., Anastasiadis, A., Vratsikidis and A., Kapouniaris, A. (2023). Gravel-rubber mixtures as geotechnical seismic isolation systems underneath structures: large-scale tests vs FEM modelling In *Geosynthetics: Leading the Way to a Resilient Planet* - Biondi et al. (eds). CRC Press, 1170–1176. https://doi.org/10.1201/9781003386889-147

Arroyo, M., San Martin, I., Olivella, S. and Saaltink, M. W. (2011). Evaluation of self-combustion risk in tire derived aggregate fills. *Waste Management* 31(9–10): 2133–2141. https://doi.org/10.1016/j.wasman.2011.04.014

Arulrajah, A., Mohammadinia, A., Maghool, F. and Horpibulsuk, S. (2019). Tire derived aggregates as a supplementary material with recycled demolition concrete for pavement applications. *Journal of Cleaner Production*, 230: 129–136. https://doi.org/10.1016/j.jclepro.2019.05.084

ASTM (2000a). *ASTM D4253-00 Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table*. ASTM International, West Conshohocken, PA.

ASTM (2000b). *ASTM D4254-00 Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density*. ASTM International, West Conshohocken, PA.

Banzibaganye, G. (2022). *Static and dynamic behaviour of sand-rubber chips mixtures* (PhD thesis). Technische Universität Kaiserslautern, Kaiserslautern. https://doi.org/10.26204/KLUEDO/6749

Benjelloun, M., Bouferra, R., Ibouh, H. and Jamin, F. (2022). Environmental challenge by geotechnic's method: example of the sandy soils reinforcement by rubber aggregates from used tires. *International Journal of Mechanical Engineering*, 7(2). https://hal.science/hal-03763430

BS (1990). *BS 1377-4: 1990 Methods of test for soils for civil engineering purposes — Part 4: Compaction-related tests*. British Standards Institution, London.

Chiaro, G., Palermo, A., Banasiak, L., Tasalloti, A., Granello, G. and Hernandez, E. (2023). Seismic response of low-rise buildings with eco-rubber geotechnical seismic isolation (ERGSI) foundation system: numerical investigation. *Bulletin of Earthquake Engineering*, 21(8): 3797–3821. https://doi.org/10.1007/s10518-022-01584-9

DIN (1996). *DIN 18126:1996-11 Baugrund, Untersuchung von Bodenproben - Bestimmung der Dichte nichtbindiger Böden bei lockerster und dichtester Lagerung*. Beuth Verlag GmbH. https://doi.org/10.31030/7209583

Fiamingo, A., Chiaro, G., Murali, A. and Massimino, M. R. (2025). Geotechnical characterization of soil-rubber mixtures with well-graded gravel. *Geosynthetics International*, 32(6): 818–834. https://doi.org/10.1680/jgein.24.00177

Gabryś, K., Radzevičius, A., Szymański A. and Šadzevičius, R. (2021). Shear Strength Characteristics of Recycled Concrete Aggregate and Recycled Tire Waste Mixtures from Monotonic Triaxial Tests. *Materials,* 14(23): 7400. https://doi.org/10.3390/ma14237400

Gromysz, K. and Kowalska, M. (2017). Reduction of vibrations applied on structures - results of chamber tests with the use of tire derived aggregate. *Procedia Engineering*, 193: 305–312. https://doi.org/10.1016/j.proeng.2017.06.218

Hazarika, H., Kohama, E. and Sugano, T. (2008). Underwater Shake Table Tests on Waterfront Structures Protected with Tire Chips Cushion. *Journal of Geotechnical & Geoenvironmental Engineering*, 134(12): 1706–1719. https://doi.org/10.1061/(ASCE)1090-0241(2008)134:12(1706)

Humphrey, D. N. (1996). *Investigation of Exothermic Reaction in Tire Shred Fill Located on SR100 in Ilwaco, Washington*. Federal Highway Administration, Washington, DC.

ISO (2015). *ISO 17892-3:2015 Geotechnical investigation and testing — Laboratory testing of soil — Part 3: Determination of particle density*. International Organization for Standardization.

Kim, H.-K. and Santamarina, J. C. (2008). Sand–rubber mixtures (large rubber chips). *Canadian Geotechnical Journal*, 45(10): 1457–1466. https://doi.org/10.1139/T08-070

Kowalska, M. Vrettos, C. (2025). Effect of layering and pre-loading on the dynamic properties of sand-rubber specimens in resonant column tests. *Acta Geotechnica*, 20(2): 607–624. https://doi.org/10.1007/s11440-024-02398-6

Mashiri, M. S. (2014). *Monotonic and cyclic behaviour of sand–tyre chip (STCh) mixtures* (PhD thesis). University of Wollongong.

Pasha, S. M. K., Hazarika, H. and Yoshimoto, N. (2019). Physical and mechanical properties of Gravel-Tire Chips Mixture (GTCM). *Geosynthetics International*, 26(1): 92–110. https://doi.org/10.1680/jgein.18.00041

Pitilakis, D., Anastasiadis, A., Vratsikidis, A. and Kapouniaris, A. (2024). Configuration of a gravel-rubber geotechnical seismic isolation system from laboratory and field tests. *Soil Dynamics and Earthquake Engineering*, 178: 108463. https://doi.org/10.1016/j.soildyn.2024.108463

PKNMJ (1988). *PN-88/B-04481 Building soils. Laboratory tests* (in Polish). Polish Committee for Standardization, Measures and Quality.

Raj, J. C. J. and Suppiah, S. (2021). Seismic isolation using scrap tire rubber pads. *Materials Today: Proceedings of the International Conference on Advanced Materials Behavior and Characterization* (ICAMBC 2020) 43: 1404–1407. https://doi.org/10.1016/j.matpr.2020.09.176

Rios, S., Kowalska, M. and Viana da Fonseca, A. (2021). Cyclic and dynamic behavior of sand–rubber and clay–rubber mixtures. *Geotechnical and Geological Engineering*. 39(5): 3449–3467. https://doi.org/10.1007/s10706-021-01704-3

Shahjalal, Md., Islam, K., Batool, F., Tiznobaik, M., Zahid Hossain, F. M., Sakil Ahmed, K., Alam, M. S. and Ahsan, R. (2023). Fiber-reinforced recycled aggregate concrete with crumb rubber: A state-of-the-art review. *Construction and Building Materials*, 404: 133233. https://doi.org/10.1016/j.conbuildmat.2023.133233

Takemiya, H. (2007). Pre-cast concrete WIB with tire shreds fill-in for a vibration mitigation measure. In Scrap *Tire Derived Geomaterials*, Hazarika, H., Yasuhara, K. (eds.), Taylor & Francis, London, pp. 383-393.

Tasalloti, A., Chiaro, G., Murali, A. and Banasiak, L. (2021). Physical and mechanical properties of granulated rubber mixed with granular soils - a literature review. *Sustainability,* 13(8): 4309. https://doi.org/10.3390/su13084309

Tsang, H.-H., Lo, S. H., Xu, X. and Neaz Sheikh, M. (2012). Seismic isolation for low-to-medium-rise buildings using granulated rubber–soil mixtures: numerical study. *Earthquake Engineering & Structural Dynamics*, 41(14): 2009–2024. https://doi.org/10.1002/eqe.2171

Tsang, H.-H., Tran, D.-P., Hung, W.-Y., Pitilakis, K. and Gad, E. F. (2021). Performance of geotechnical seismic isolation system using rubber-soil mixtures in centrifuge testing. *Earthquake Engineering & Structural Dynamics*, 50(5): 1271–1289. https://doi.org/10.1002/eqe.3398