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Influence of earthquake-induced excess pore water pressures on seismic bearing capacity of shallow foundations

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ABSTRACT: Earthquake-induced excess pore water pressures may produce significant strength and stiffness degradation in foundation soils. Accordingly, severe loss in the bearing capacity of shallow foundations can occur possibly leading to extensive damage in the superstructures. Then there is the need of simple, but rigorous, solutions to account for the effect of earthquake-induced pore pressures on seismic bearing capacity of shallow foundations. This effect is examined in the paper using the method of characteristics. First the effect of hydrostatic pore pressures on bearing capacity was investigated providing a corrective coefficient to account for it; then the effect of seismic-induced excess pore pressures was considered in the analyses. A seismic bearing capacity factor accounting for both the soil and the superstructure inertial effects as well as for the effect due to the excess pore water pressure was proposed and formulas of corrective coefficients were also derived to be used in the conventional bearing capacity trinomial formula.

1 INTRODUCTION

The evaluation of bearing capacity of a strip footing resting on a homogeneous dry soil subjected to a vertical and centred load is traditionally carried out using the formula introduced by Terzaghi (1943):

$$q_{\mathrm{ult}} = cN_{\mathrm{c}} + qN_{\mathrm{q}} + \frac{1}{2}\gamma BN_{\gamma} \tag{1}$$

In eq. 1 q_{ult} represents the ultimate load that the soil can sustain under the assumption of rigid plastic behaviour; N_{c}, N_{q} and N_{γ} are the bearing capacity factors, depending on the friction angle φ' of the soil; γ and c are the unit weight and the cohesion of the soil, respectively; B is the width of the foundation and q is the vertical pressure acting aside the footing at the level of the foundation base.

According to Terzaghi the three bearing capacity factors can be evaluated independently solving three different bearing capacity problems where the effect of the soil cohesion c, of the lateral surcharge q, and of the soil unit weight γ are in turn assessed, and these effects can finally be superimposed through eq. 1. The assumption underlying this superposition procedure is that the sum of the three terms is a conservative estimate of the true failure load.

For a weightless soil (γ = 0) the method of the characteristic lines allows evaluating exact closed form solutions of the bearing capacity factors N_{q} (Reissner, 1924) and N_{c} (Prandtl, 1920):

$$N_{\mathrm{q}} = \tan^2(\pi/4 + \varphi'/2)\, e^{\pi \tan \varphi'} \tag{2}$$

$$N_{\mathrm{c}} = \left(N_{\mathrm{q}} - 1\right) \cot\varphi' \tag{3}$$

that depend only on the angle φ' and are independent of the roughness of the foundation.

If the weight of the soil is considered ($\gamma \neq 0$) the values of the bearing capacity factor N_γ can be obtained through the method of characteristics only by numerical integration of the equilibrium equations (in fact in this case Hencky's theorem cannot be applied and the factor N_γ cannot be derived from analytical considerations). Also, unlike N_c and N_q, N_γ depends also on the roughness of the soil-foundation interface, which can be expressed by the soil-foundation friction angle δ.

A number of solutions are available for the assessment of N_γ obtained using the limit equilibrium method (e.g. Meyerhof 1951, Vesic 1973), the kinematic approach of limit analysis (e.g. Chen 1975, Michalowski 1997) and the method of characteristics (e.g. De Simone & Restaino 1985, Martin 2005), as well as by means of finite element or finite differences analyses (e.g. Griffiths 1982, Frydman & Burd 1997).

Recently Cascone & Casablanca (2016) using the method of characteristics, for both smooth and rough foundations, obtained values of N_γ in a perfect agreement with literature results obtained using the same method of analysis (De Simone & Restaino 1985, Martin 2005). These were shown to be exact since it was possible to achieve the coincidence of the stress and velocity fields as well as to extend the stress field beyond the plastic volume.

In the range $\varphi' = 15° \div 45°$ the numerical values of N_γ are satisfactorily fitted by the expression (Cascone & Casablanca 2016)

$$N_\gamma = \left(N_q - 1\right) \tan(k \cdot \varphi') \left(n + \frac{1 - n^3}{2}\right) \tag{4}$$

where $k = 1.3389$ can be approximated to 1.34 and $n = \tan\delta/\tan\varphi'$.

Bearing capacity expressed by eq. 1 can be corrected to account for the shape of the foundation, the depth and inclination of the footing base, the inclination of the ground, the inclination and the eccentricity of the applied load and also for the depth of the water table (e.g. Meyerhof 1963, Brinch-Hansen 1970, Vesic 1973, De Simone 1983). These solutions form the basis of the state of practice in the evaluation of bearing capacity of shallow foundations that, as an alternative, can be assessed by means of a bounding surface in the space of loading parameters (e.g. Butterfield & Gottardi 1994, Houlsby & Puzrin 1999).

Field and experimental evidence of seismic behaviour of shallow foundations pointed out the susceptibility of these geotechnical systems to suffer large deformations and failure (e.g. Richards et al 1993, Crespellani et al 1999, Chu et al 2006). In fact, during earthquake loading, in the soil under a footing the shear strength may be temporarily attained producing instantaneous failures resulting in the accumulation of permanent settlements. Seismic failure of shallow foundations resting on dry soils may be potentially attributed to: i) the increase in magnitude and change in direction of foundation loads due to the development of inertia forces on the superstructure; ii) the development of dynamic shear stresses in the soil, associated with the inertia of the soil mass.

In seismic design of shallow foundations, these situations can be coped with by introducing corrective coefficients into eq. 1 to embody the effects on bearing capacity of earthquake-induced inertia forces, arising in the superstructure and in the foundation soil (e.g. Paolucci & Pecker 1997, Kumar & Mohan Rao 2002, Castelli & Motta 2012, Cascone & Casablanca 2016, Casablanca et al. 2016, Pane et al 2016, Conti 2018).These effects are hereafter denoted as soil and superstructure inertial effects. As for the static case, also in seismic conditions a bearing capacity surface can be defined to verify footing stability conditions (Pecker 1997, Conti 2018, Mangraviti et al 2019).

In case of water table at or close to the foundation plane during the ground shaking excess pore water pressures may develop in the foundation soils and cyclic degradation may contemporarily occur. The consequent reduction of the soil shear strength and stiffness noticeably affects the seismic bearing capacity of the foundation and may lead to an overall bearing capacity failure of the soil-foundation system, even in cases where liquefaction does not occur.

Unlike soil and superstructure inertial effects, excess pore pressure effects do not vanish at the end of shaking but persist after the earthquake has ceased until complete excess pore pressure dissipation. Also, centrifuge tests on model foundations resting on liquefiable soils showed that during seismic shaking excess pore pressures beneath the footing are smaller than in the far field and, in addition, the load applied by the footing, producing an increase in effective stresses in the soil, prevents the complete loss of shear resistance. However, at the end of shaking, the hydraulic gradient generated during the seismic excitation produces a water flow that, due to coupling between pore pressures and deformations, causes excess pore pressure under the footing to increase again under static conditions (Mitrani & Madabhushi 2010) possibly inducing a post-seismic failure.

Despite evidence and despite many seismic codes (e.g. EC8-5) prescribe to account for excess pore pressures in seismic analyses, there is still the need of simple, but rigorous, solutions to account for the effect of earthquake-induced pore pressures on seismic bearing capacity of shallow foundations. However, to the authors' knowledge, only a few studies (e.g. Bouckovalas and Dakoulas 2007) consider the soil strength reduction possibly occurring during earthquakes due to seismic excess pore pressure build up in the evaluation of bearing capacity.

In this vein, a rigorous solution for the bearing capacity factor $N_{\gamma E}$ to be considered in seismic bearing capacity analyses of footings was derived using the method of characteristics. Herein the case of a strip footing resting on fully submerged soil is presented. The proposed bearing capacity factor $N_{\gamma E}$ accounts for the inertial (soil and superstructure) and excess pore pressure effects and was computed for both rough and smooth footings. By fitting numerical results, reliable formulas were obtained to evaluate an overall corrective coefficient e_γ that, applied to the static value of N_γ, allows accounting for bearing capacity reduction due to the combined inertial and excess pore pressure effects.

Since the static effective state of stress is relevant to examine the influence of seismic-induced excess pore water pressures, the effect of submergence on static bearing capacity was preliminarily investigated in the paper and an original solution was derived consisting in a corrective coefficient that takes into account hydrostatic pore pressures on the static bearing capacity factor N_γ.

2 METHOD OF ANALYSIS

The basic assumptions of the method of characteristics are that soil behaves as a rigid-plastic material, obeys the Mohr-Coulomb yield criterion and in a soil mass at incipient failure, equilibrium equations and plastic condition must be satisfied. This approach has been widely used in deriving bearing capacity factors of shallow foundations both under static and pseudo-static conditions.

The differential equations of equilibrium under plane strain conditions are:

$$\begin{aligned} \frac{\partial \sigma_{\mathrm{x}}}{\partial x} + \frac{\partial \tau_{\mathrm{xy}}}{\partial y} &= X \\ \frac{\partial \tau_{\mathrm{xy}}}{\partial x} + \frac{\partial \sigma_{\mathrm{y}}}{\partial y} &= Y \end{aligned} \tag{5}$$

where x and y are the horizontal and vertical directions, σ_{i} and τ_{ij} are normal and shear stresses acting on horizontal and vertical planes and X and Y are the horizontal and vertical body forces per unit volume. Under static conditions the body forces are $X = 0$ and $Y = \gamma$, γ being the soil unit weight, while under pseudo-static conditions they can be expressed as:

$$\begin{aligned} X &= \gamma \cdot k_h \\ Y &= \gamma \cdot (1 - k_v) \end{aligned} \tag{6}$$

where k_{h} and k_{v} are the horizontal and vertical seismic acceleration coefficients (the latter assumed positive when vertical seismic acceleration is directed upward) schematizing the inertial effect arising in the foundation soil.

Introducing the failure criterion into the differential equilibrium equations a set of two hyperbolic partial derivative equations is obtained. Using the method of characteristics, it is possible to transform the set of partial derivative equations into a set of ordinary differential equations in the variables s and ω, representing, respectively, the stress invariant $s = (\sigma_1+\sigma_3)/2$ and the angle formed by the maximum principal stress σ_1 with the horizontal axis x. Starting from the boundary conditions, the ordinary differential equations can be integrated to determine the stress state and the associated characteristic lines network.

The loading schemes adopted in the analyses are described in Figure 1. Specifically, Figures 1a, b and c refer to the evaluation of the seismic bearing capacity factors N^{s}_{cE}, N^{s}_{qE} and $N^{s}_{\gamma E}$, accounting for the effect of soil inertia (denoted by superscript s) while Figures 1d, e and f are relevant to the seismic bearing capacity factors N^{ss}_{cE}, N^{ss}_{qE} and $N^{ss}_{\gamma E}$ which account for the effect of the inertia of the superstructure (denoted by superscript ss).

In the reference schemes:

the seismic coefficients k_h and k_v represent the inertial effects arising in the soil mass involved in the plastic mechanism (soil inertial effect) and the ratio $\Omega = k_v/k_h$ is introduced to describe the influence of the vertical component of the ground motion;

the seismic coefficients $k_{h,i} = H_E/Q_o$ and $k_{v,i} = V_E/Q_o$ are defined as the ratio of the seismic horizontal (H_E) and vertical (V_E) inertia forces transmitted onto the foundation by the superstructure, to the static vertical force Q_o; $k_{h,i}$ and $k_{v,i}$ describe the effect of the inertia forces acting on the superstructure and, thus, depend on the seismic loading and on the dynamic response of the structure. Herein, this effect is introduced through the ratio $\eta = k_{h,i}/k_h = k_{v,i}/k_v$.

As it is known, using the characteristic lines method, three boundary-value problems must in turn be solved: the Cauchy problem, the Riemann problem and the Goursat problem, each of

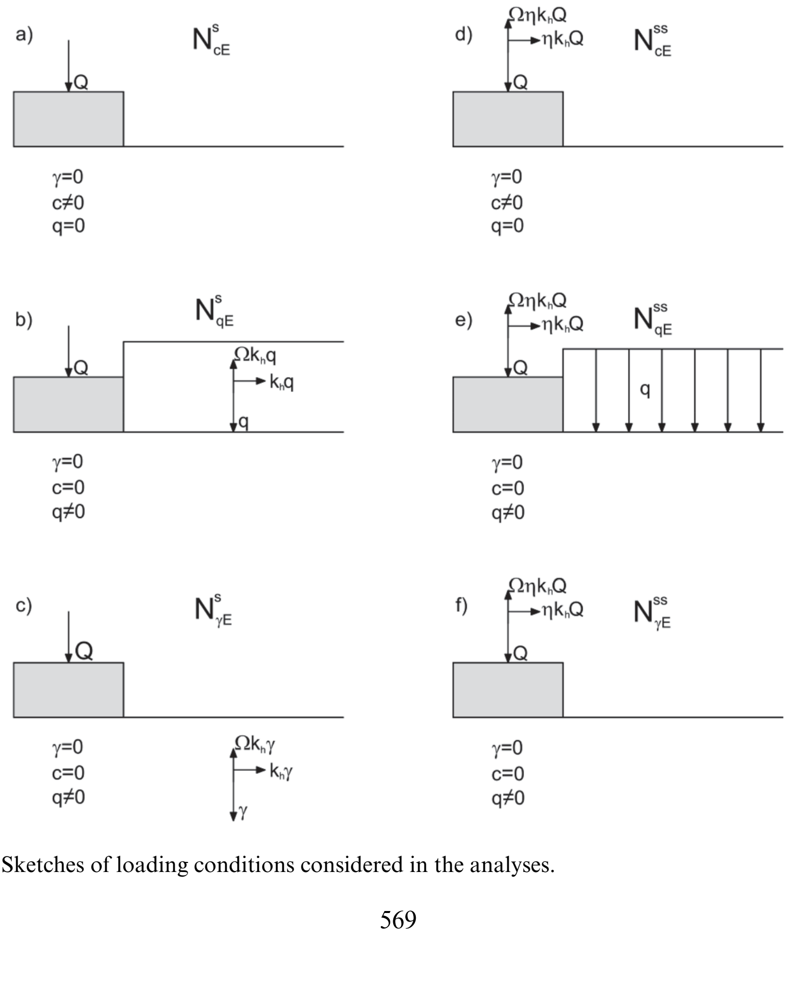

Figure 1. Sketches of loading conditions considered in the analyses.

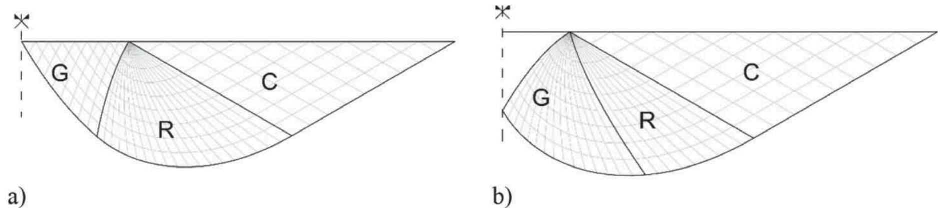

Figure 2. Characteristic lines network and Goursat, Riemann and Cauchy domains for a smooth (a) and a rough (b) footing.

them concerning one zone of the soil mass in the condition of plastic equilibrium, as shown in Figure 2a for the Hill mechanism representing the case of a perfectly smooth foundation and in Figure 2b for the Prandtl mechanism, relevant to the case of a perfectly rough foundation.

3 EFFECT OF HYDROSTATIC PORE WATER PRESSURES ON BEARING CAPACITY

Prior to examining the effect of seismic-induced excess pore water pressures it is necessary to consider the effect of submergence on static bearing capacity.

This problem has been studied by several researchers (Meyerhof 1955, Vesic 1973, Krishnamurti & Kameswara Rao 1975, Bowles 1982, Hansen et al 1987, De Simone & Zurlo 1987, Ausilio & Conte 2005, Sun et al 2013, Bouaicha et al 2018) who, using different methods of analysis, proposed formulas to evaluate an equivalent soil unit weight accounting for the effect of hydrostatic pore pressures u_o in the soil under the footing. Available formulas depend on the depth d_w of the water table under the footing base and on the depth of the plastic mechanism d_0, which, on turn, depends on the angle of shear strength. Though these formulas are formally different, it can be shown that some of them are perfectly coincident.

In this study the method of characteristics has been used to investigate the effect of hydrostatic pore water pressures on static bearing capacity of both smooth and rough shallow strip footings. Since the problems for the evaluation of the bearing capacity factors N_q and N_c do not depend on the effective state of stress acting along the failure surface bounding the plastic volume (in fact, in these problems a weightless soil is assumed), only the problem $c'=q=0$ (hereafter denoted as N_γ problem) was considered in the analyses.

Figure 3 shows a sketch of the problem for the case of a smooth footing, with the characteristic lines network deliberately plotted as a coarse mesh for the sake of clarity (in the analyses the mesh of the characteristic lines was much finer to achieve steady solutions); it is apparent that, consistently with the analysis for the problem of N_γ, the Riemann domain degenerates in a single line.

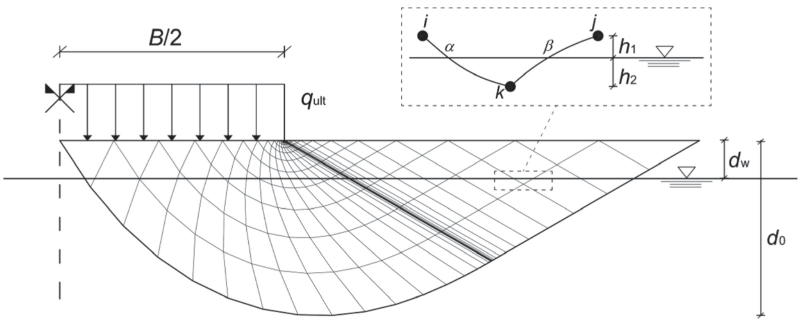

Figure 3. Reference scheme for the evaluation of the effect of hydrostatic pore pressures.

In the numerical finite difference integration of the equilibrium equations, for each point of the characteristic lines network of depth z under the foundation plane, the value of the hydrostatic pore pressures was assumed equal to $u_o = \gamma_w \cdot (z-d_w)$, γ_w being the specific weight of water. Also, a slight approximation has been introduced for those lines of the network that are crossed by the water table line. Specifically, following De Simone & Zurlo (1987) and with reference to Figure 3, the unknown state of stress in point k of the network is evaluated starting from the known stress variables at points i and j assuming an average unit weight γ_L defined as:

$$\gamma_L = \frac{\gamma h_1 + \gamma' h_2}{h_1 + h_2} \tag{7}$$

where γ' is the unit weight of the submerged soil and h_1 and h_2 are defined in Figure 3.

For various depth of the water table d_w and for φ' and γ varying in the ranges 15°–45° and 16–21 kN/m^3, respectively, the solution of the numerical problem allowed obtaining the characteristic lines network and the corresponding bearing capacity that can be expressed in the form:

$$q_{ult} = \frac{1}{2} B\gamma N_{\gamma w} \tag{8}$$

In eq. (8) $N_{\gamma w}$ is a generalized bearing capacity factor for the N_γ problem with water table at different depths from the foundation level.

From the characteristic lines network it was possible to detect the depth d_0 of the plastic volume that was found to slightly depend on the water table depth, at which a change in the gradient of effective stresses occurs. The depth d_0 increases for increasing d_w, reaching a maximum when the water table crosses the plastic volume at approximately mid depth ($d_w = d_0/2$) and then decreases again. d_0 takes the same (minimum) value in the two extreme conditions of foundation soil completely submerged or completely dry. However, this variation is negligible, in fact, maximum d_0 is about 10% larger than minimum d_0 for φ'=45°; this increase raises up to 20% for φ'=15°.

In Figure 4 the values of d_0 obtained for the case of fully submerged (or completely dry) foundation soil, normalized with respect to the width B of the footing, are shown for various values of φ' and for both cases of smooth and rough foundations. It can be observed that the plastic mechanism relative to a rough footing extends down to a depth that is approximately 2 times larger with respect to the case of a smooth footing. For smooth footings the ratio d_0/B is smaller than 1 for any value of φ' in the considered range 15°–45°; for rough footings d_0/B is smaller than 1 up to about φ' = 36° and is larger than 1 for greater values of φ'.

The results of the characteristics analyses for the depth d_0 are satisfactorily fitted by the following equation:

$$\frac{d_0}{B} = a\frac{0.5\cos\varphi'}{\cos(\pi/4 + \varphi'/2)} exp[b(\pi/4 + \varphi'/2)\tan\varphi'] \tag{9}$$

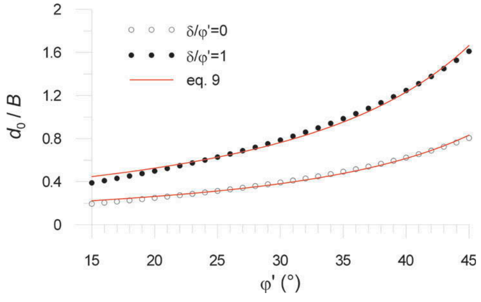

Figure 4. Normalized depth of the plastic mechanism for smooth and rough footings.

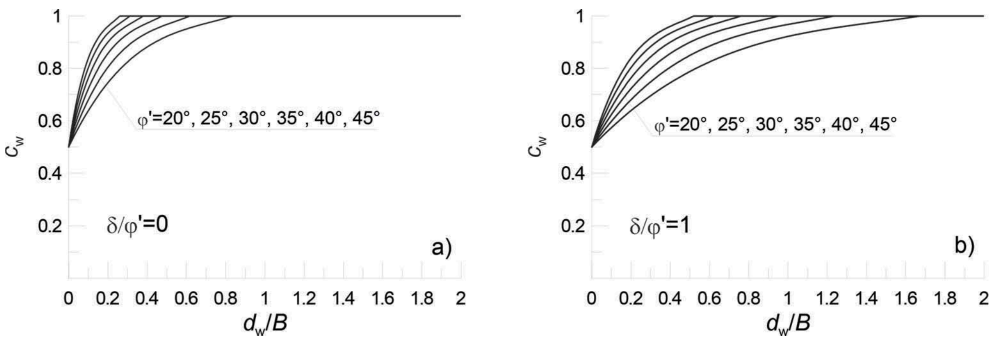

Figure 5. Corrective coefficient c_w accounting for hydrostatic pore pressure effect on bearing capacity (γ = 20 kN/m^3 and γ_w = 10 kN/m^3).

where a = 0.204 or a = 0.408 for smooth or rough footings, respectively and b=1.267 regardless the footing roughness.

In order to account for hydrostatic pore pressures on static bearing capacity, a coefficient c_w was introduced to correct the factor N_γ:

$$c_w = N_{\gamma w}/N_\gamma \tag{10}$$

Figure 5 shows the results for c_w for smooth (Figure 5a) and rough footings (Figure 5b), obtained for different values of φ' and for different depths of the water table assuming γ = 20 kN/m^3 and γ_w =10 kN/m^3. If the water table is at the level of the foundation (d_w = 0), as it is known, the bearing capacity factor is reduced by $c_w = \gamma'/\gamma$ (c_w = 0.5 for the case of Figure 5); conversely, when the depth of the water table increases, the effect of pore pressures on bearing capacity becomes smaller and as d_w approaches d_0, also the corrective coefficient c_w abruptly changes into one, meaning that the water table has no effect on bearing capacity. This condition is generally achieved for d_w/B lower than unity except for rough footings and large values of φ' which imply plastic volumes deeper than B.

The corrective coefficient c_w depends on the soil unit weight γ as shown in Figure 6a where the values of c_w computed for γ_w =10 kN/m^3 and γ in the range 16÷21 kN/m^3 are represented as a function of the ratio d_w/d_0, which in the term d_0 embodies the dependence on the angle of shear strength φ'. In the figure the symbols represent the results of the analyses carried out through the method of characteristics, while solid lines represent the best fit provided by the following equation, valid for $d_w/d_0 \leq 1$:

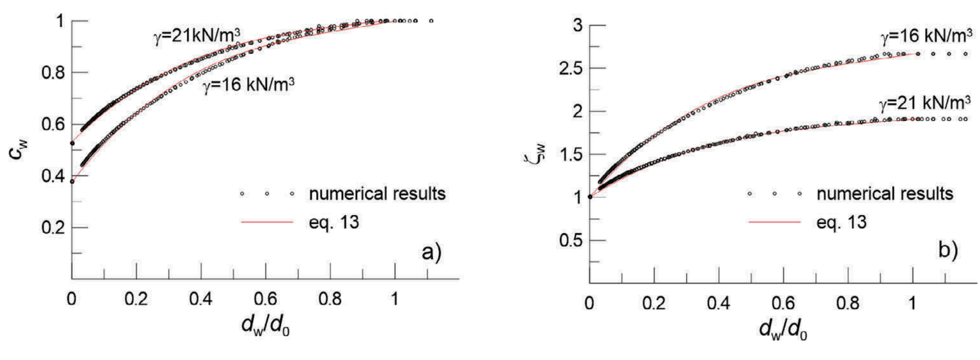

Figure 6. Corrective coefficients c_w (a) and ζ_w (b), computed for γ_w=10 kN/m^3 and γ=16 ÷ 21kN/m^3, and comparison with the predictions obtained through eqs. 11 and 13.

$$c_{\mathrm{w}} = \frac{\gamma'}{\gamma}\left\{1 + \frac{\gamma_{\mathrm{w}}}{\gamma'}\left[A\left(\frac{d_{\mathrm{w}}}{d_0} - \left(\frac{d_{\mathrm{w}}}{d_0}\right)^2\right) + \left(\frac{d_{\mathrm{w}}}{d_0}\right)^3\right]\right\} \tag{11}$$

where the coefficient A = 2.626.

It is worth noting that while the depth d_0 appearing in eq. 11 is that corresponding to a fully submerged (or completely dry) foundation soil (eq. 9), the values of c_{w} (eq. 11) are relative to the true d_{w}-dependent values of d_0.

Even if the dry soil condition is generally assumed as a reference for the evaluation of the water table depth effects (eq. 10), for the seismic analyses presented in the next section, it is convenient to refer to the condition of water table at the foundation level, putting the static bearing capacity in the form:

$$q_{\mathrm{ult}} = \frac{1}{2}B\gamma'\zeta_{\mathrm{w}}N_\gamma \tag{12}$$

and introducing the following corrective coefficient for the factor N_γ:

$$\zeta_{\mathrm{w}} = \frac{\gamma}{\gamma'}c_{\mathrm{w}} = 1 + \frac{\gamma_{\mathrm{w}}}{\gamma'}\left[A\left(\frac{d_{\mathrm{w}}}{d_0} - \left(\frac{d_{\mathrm{w}}}{d_0}\right)^2\right) + \left(\frac{d_{\mathrm{w}}}{d_0}\right)^3\right] \tag{13}$$

The coefficient ζ_{w} is plotted in Figure 6b as a function of the ratio d_{w}/d_0 for the same values of the soil unit weight γ considered in Figure 6a.

From the plots of Figure 6 it is apparent that the match between characteristics analysis results obtained for both c_{w} and ζ_{w} and the corresponding fitting eqs. 11 and 13 is satisfactory.

If the foundation soil is completely submerged ($d_{\mathrm{w}} = 0$, $c_{\mathrm{w}} = \gamma'/\gamma$), the coefficient is $\zeta_{\mathrm{w}} = 1$ and bearing capacity can be expressed as:

$$q_{\mathrm{ult}} = \frac{1}{2}B\gamma'N_\gamma \tag{14}$$

Conversely, if the water table is not interfering with the soil plastic volume and the soil under the foundation plane is completely dry (in this case in eqs. 11 and 13 $d_{\mathrm{w}}/d_0=1$ must be assumed), it is $\zeta_{\mathrm{w}}=\gamma/\gamma'$ ($c_{\mathrm{w}} = 1$) and the bearing capacity is given by:

$$q_{\mathrm{ult}} = \frac{1}{2}B\gamma N_\gamma \tag{15}$$

For any water table depth in the interval $0 < d_{\mathrm{w}} < d_0$, eqs. 11 and 13 allow accounting for the effect of hydrostatic pore water pressures and, for the more general case with $c' \neq 0$ and $q \neq 0$ the static bearing capacity can be computed as:

$$q_{\mathrm{ult}} = c'N_c + qN_q + \frac{1}{2}B\gamma'N_\gamma\zeta_{\mathrm{w}} \tag{16}$$

4 EFFECT OF SEISMIC EXCESS PORE WATER PRESSURES ON BEARING CAPACITY

4.1 *Problem statement*

During strong seismic motions soils may experience large strains, pore water pressure build-up and cyclic degradation of mechanical properties which may affect the stability conditions and the post-seismic serviceability of shallow foundations, possibly impairing the whole superstructure. Herein, the attention is focused on the effect of soil shear strength reduction due to the development of excess pore water pressures Δu and an attempt is made to estimate the effect of such reduction on the seismic bearing capacity.

Similarly to the static loading condition, since the seismic bearing capacity factors N^{s}_{qE} and N^{s}_{cE} do not depend on the effective stresses acting on the failure surface bounding the soil plastic volume, the N_{γ} problem ($c' = q = 0$) was again considered.

As usual, the effects of the earthquake induced pore water pressure is accounted for in the analyses through the excess pore pressure ratio $\Delta u^* = \Delta u/p'_o$ being p'_o the geostatic value of the effective mean stress acting in free-field condition.

To evaluate the influence of the excess pore pressure ratio on the seismic bearing capacity, the characteristics equations were integrated for several values of Δu^* varying in the range $0 \div 0.8$. The condition $\Delta u^* = 1$, which is generally considered for initiation of flow liquefaction in free-field condition with geostatic state of stress (null shear stresses), cannot be actually attained for the problem at hand. In fact, it can be demonstrated that, due to the static shear stresses induced by the foundation, in the N_{γ} problem ($c' = q = 0$), regardless the inertial effects of both soil and superstructure, the foundation reaches an incipient failure condition for a value of the excess pore water pressure $\Delta u_f^* = 1\text{-}1/F_0$, F_0 being the static safety factor of the foundation.

In the numerical integration of the equilibrium equations Δu^* was assumed to be independent of the static state of stress acting on the foundation soil and, thus, the same value has been considered for all the points of the characteristic lines network. Accordingly, for each point of the network the seismic values of the pore water pressure have been computed as $u_E = u_o + \Delta u$.

4.2 *Effect of soil inertia and excess pore pressures*

In order to examine the effects of soil inertia and of excess pore pressures, a parametric analysis was carried out assuming $\eta = 0$ (no superstructure inertia effect), the angle φ' in the range $15 \div 45°$, values of k_h in the range $0 – \tan \varphi'$ and different values of the ratio Ω.

Some results are shown in Figure 7 where, for the case $d_w = 0$, the seismic bearing capacity factor $N^{s}_{\gamma E}$ for smooth (Figure 7a-c) and rough (Figure 7d-f) footings is plotted versus the angle of shear resistance φ'. Specifically, the lines in Figure 7 refer to the values of $N^{s}_{\gamma E}$ evaluated for $\Delta u^* = 0 \div 0.8$ neglecting the effect of the vertical component of the ground motion into the foundation soils ($\Omega = 0$).

It is apparent that the effect of excess pore water pressures significantly affects the values of the seismic bearing capacity factor $N^{s}_{\gamma E}$. As an example, assuming the condition $\Delta u^* = 0$ as a reference, for the case $\varphi = 35°$, $k_h = 0.15$ and rough footing (Figure 7f), a reduction of $N^{s}_{\gamma E}$ of about 28% and 57% can be estimated for $\Delta u^* = 0.4$ and 0.8, respectively; in the case $k_h = 0.25$ these reductions increase up to 32% and 66% respectively, denoting a coupled effect of the soil inertial and excess pore pressure effects on the reduction of the seismic bearing capacity.

For the same case with $\varphi = 25°$ (Figure 7d) larger reduction of $N^{s}_{\gamma E}$ due to the excess pore water pressure can be predicted; specifically, for $\Delta u^* = 0.4$ and 0.8 and $k_h = 0.15$, the reduction in the seismic bearing capacity factor is equal to about 37% and 75%; in this case, if larger horizontal seismic coefficients are considered in the analysis, the contribution of soil weight to the overall seismic bearing capacity may vanish due to the combined soil inertial and excess pore pressure effects. This condition is attained, for example, for the case $k_h = 0.25$ and Δu^* larger than about 0.6 and for k_h=0.4 and Δu^* equal to about 0.2 (Figure 7d).

Finally, whatever are the values of the seismic coefficients, if no excess pore water pressure is considered in the analysis ($\Delta u^* = 0$), the values of $N^{s}_{\gamma E}$ obtained herein reproduce the seismic bearing capacity factor introduced by Cascone & Casablanca (2016) which accounts only for the soil inertial effects.

Starting from the results obtained through the numerical finite difference integration, a corrective coefficient e^{s}_{γ}, modified with respect to that proposed by Cascone & Casablanca (2016) to describe the combined effect of soil inertia and of earthquake-induced pore water pressure, was introduced normalizing $N^{s}_{\gamma E}$ with respect to its homologous static value N_{γ}.

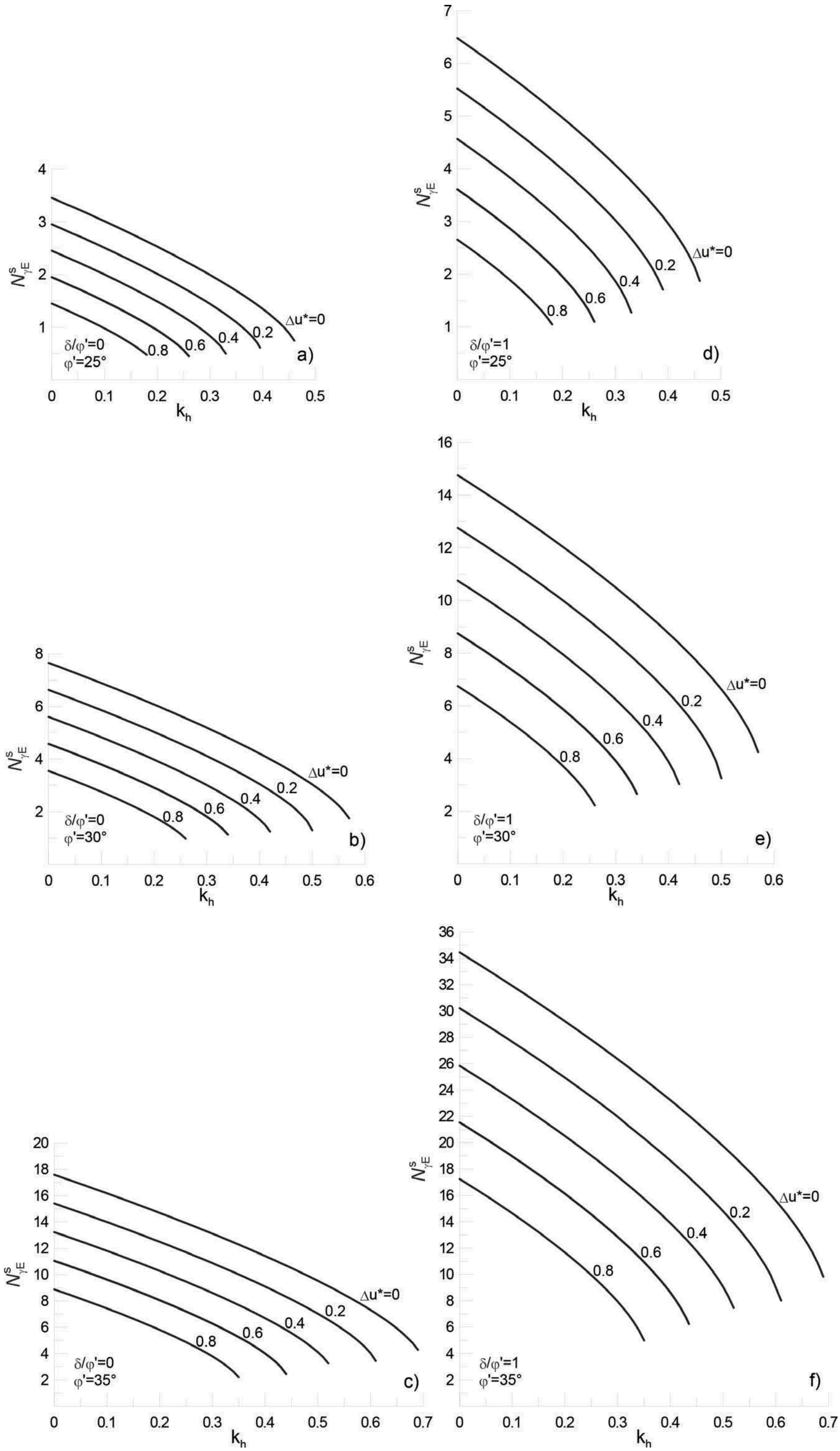

Figure 7. Seismic bearing capacity factor $N^s_{\gamma E}$ for smooth (a) and rough (b) footings with water table at the foundation level ($d_w = 0$) computed for different values of the excess pore water pressure ratio and φ' assuming $\Omega = 0$ and $\eta = 0$.

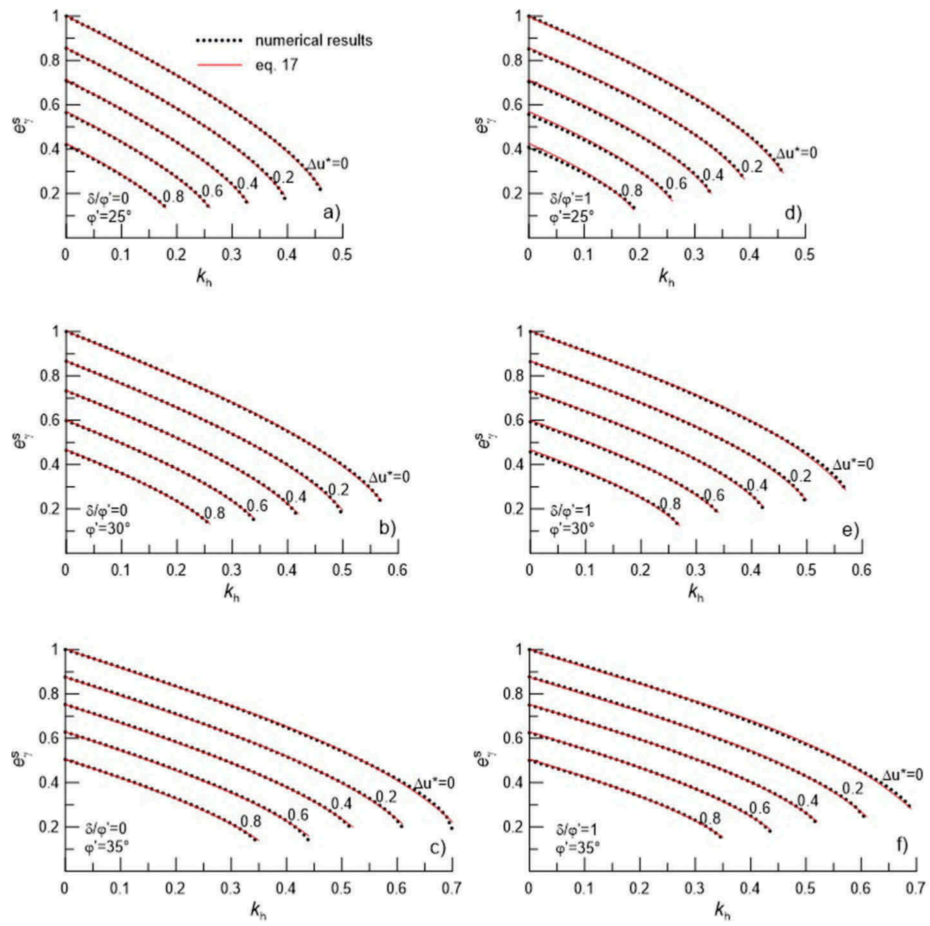

Figure 8. Corrective coefficient for the effect of soil inertia and excess pore water pressure computed for the conditions $d_w = 0$, $\Omega = 0$ and $\eta = 0$: a) smooth and b) rough footings.

For the case $d_w = 0$, some of the obtained numerical results are shown with dotted lines in Figure 8 for both smooth (Figure 8a-c) and rough (Figure 8d-f) footings, assuming $\Omega = 0$, $\Delta u^* = 0 \div 0.8$; as for the cases of Figure 7, the effect of the superstructure inertia was neglected ($\eta = 0$).

Curves in Figure 8 representing the coefficient e^s_γ exhibit approximately the same trend of those in Figure 7, relative to the bearing capacity factor $N^s_{\gamma E}$, however it can be observed that the values of e^s_γ for smooth and rough footings are barely distinguishable. Again, if no excess pore water pressure is considered in the analyses ($\Delta u^* = 0$) the values of e^s_γ coincide with those derived by Cascone & Casablanca (2016).

The numerical values of e^s_γ obtained for different values of Δu^*, k_h and k_v were fitted by the following empirical expression:

$$e^s_\gamma = \frac{N^s_{\gamma E}}{N_\gamma} = \left(1 - A\frac{k_h}{1-k_v}\cot\varphi^*\right)^B \sqrt{k_h^2 + (1-k_v)^2}\left[1 - \Delta u^*\left(1 - \frac{2}{3}\sin\varphi'\right)\right] \tag{17}$$

where φ^* is a reduced value of the angle of shear resistance of the foundation soil:

$$\varphi^* = \varphi'\left(1 - r_1 \cdot \Delta u^* \cdot e^{-r_2 \tan\varphi'}\right) \tag{18}$$

Table 1. Values of the regression coefficients of eqs. (19) and (22)

e^{s}_{γ}	($\delta/\varphi' = 0$)	$b_1 = 0.290$	$b_2 = -0.277$	$b_3 = 0.716$
	($\delta/\varphi' = 1$)	$b_1 = 0.198$	$b_2 = -0.014$	$b_3 = 0.528$
e^{ss}_{γ}	($\delta/\varphi' = 0$)	$d_1 = 3.056$	$d_2 = 2.683$	$d_3 = 0.562$
	($\delta/\varphi' = 1$)	$d_1 = 2.005$	$d_2 = 1.452$	$d_3 = 0.191$

and:

$$B = b_1 \tan^2\varphi' + b_2 \tan\varphi' + b_3 \tag{19}$$

For the case $d_w = 0$, the regression coefficients of eqs. 17-19 are $A = 0.92$, $r_1 = 1.193$ and $r_2 = 1.219$, regardless the roughness of the footing, while b_1,b_2 and b_3 are given in Table 1.

It is worth noting that for $\Delta u^* = 0$, φ^* equals φ' and eq. 17 reduces to the equation given by Cascone & Casablanca (2016). Curves of e^{s}_{γ} computed using eq. 17 are plotted in Figure 8 as solid lines, showing an almost perfect agreement with numerical results.

Finally, in Figures 7 and 8 it can be noted that, for certain combinations of seismic acceleration and excess pore pressure ratio, $N^{s}_{\gamma E}$ and e^{s}_{γ} may suddenly drop to zero, reaching the so-called fluidification condition (Richards et al 1990). For $\Delta u^* = 0$ this is observed when $k_h = \tan\varphi'$ (Cascone & Casablanca 2016), whereas for $\Delta u^* > 0$ the same condition is attained for $k_h = \tan \varphi^*$.

4.3 *Combined inertial (soil and superstructure) and excess pore pressure effects*

The influence of the superstructure inertia ($\eta \neq 0$) on $N^{ss}_{\gamma E}$ and its superposition with the influence of the soil inertia effect ($N^{s}_{\gamma E}$) have been already examined by Cascone & Casablanca (2016) with reference to the case $\Delta u^* = 0$. For each of these effects, these Authors introduced two corrective coefficients, e^{s}_{γ} and e^{ss}_{γ}respectively, and also showed that:

the reduction in bearing capacity factor N_{γ} due to superstructure inertia effect can be decoupled by the reduction due to the soil inertial effect;
the overall inertial (soil and superstructure) effect can be satisfactorily computed introducing a general corrective coefficient $e_{\gamma} = e^{s}_{\gamma} \cdot e^{ss}_{\gamma}$;
the superstructure inertial effect generally leads to a more remarkable reduction of the bearing capacity than the soil inertial effect.

Herein the coupling of the soil and superstructure inertial effects is reconsidered accounting also for the effect of excess pore water pressure. Specifically, a parametric analysis was carried out in order to verify if the superstructure and soil inertia effects are still decoupled, if the effect of the superstructure inertia is still dominant and, finally, to introduce an overall corrective coefficient accounting for both the inertial (soil and superstructure) and excess pore pressure effects.

Typical results are presented for the case $d_w = 0$ considering a smooth footing with $\varphi' = 30°$ and $\Omega = 0$ assuming Δu^* =0, 0.2, 0.4 and 0.6.The corrective coefficients are plotted in Figure 9 versus the horizontal seismic coefficient k_h. Specifically, curve A represents e^{s}_{γ} (soil inertia and, for $\Delta u^* \neq 0$, excess pore pressure effects, $\eta = 0$); curve B represents e^{ss}_{γ} (superstructure inertia, $1/\eta = 0$), curve C shows the numerical results obtained for the combined problem, including soil and superstructure inertia (assuming $\eta = 1$) and excess pore pressure and, finally, curve D represents the proposed superposition ($e_{\gamma} = e^{s}_{\gamma} \cdot e^{ss}_{\gamma}$).

In all the examined cases, the comparison between the curves C and D (which are almost coincident) allow affirming that, as for the case $\Delta u^* = 0$, also in presence of excess pore water pressure ($\Delta u^* \neq 0$) the reduction in bearing capacity factor N_{γ} due to superstructure inertial effect can be decoupled by the reduction due to the soil inertial effect which, in turn, is coupled with the excess pore pressure effect.

From the plots of Figure 9 b-d it can be also observed that since the superstructure inertial effect in not affected by excess pore pressure in the soil, for increasing Δu^* and k_h the combined soil inertial and excess pore pressure effect may result dominant and may lead to a noticeable reduction in bearing capacity.

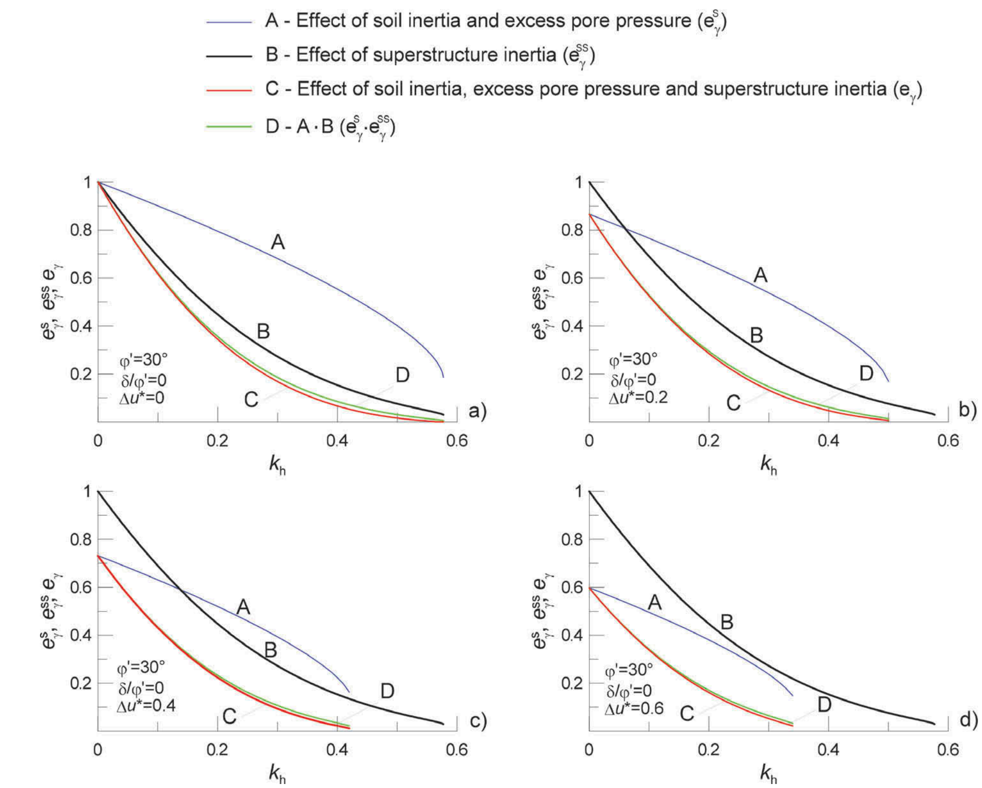

Figure 9. Influence of coupled soil inertial and excess pore pressure effects (curve A), of superstructure inertial effect (curve B), of the combined effects (curve C) and superposition of effects (curve D=A×B).

Based on the results described above, it can be stated that:

the effect of the superstructure inertia is generally decoupled from the soil inertia and weakening excess pore pressure effects, which, on turn, are coupled;
the corrective coefficient e^{ss}_γ already proposed by Cascone & Casablanca (2016) (eq. 22 of their paper) to account for the superstructure inertia holds also for the case $\Delta u^* \neq 0$;
the whole effect induced by soil and superstructure inertia and by excess pore water pressure can be satisfactorily computed through the product of the corresponding corrective coefficients.

Thus, the more general solution for the seismic bearing capacity factor related to the N_γ problem can be put in the form:

$$N_{\gamma E} = N_\gamma e^s_\gamma e^{ss}_\gamma \tag{20}$$

with e^s_γ given by eq. 17 and e^{ss}_γ provided by the following formula (Cascone & Casablanca, 2016):

$$e^{ss}_\gamma = \frac{N^{ss}_{\gamma E}}{N_\gamma} = \left(1 - C\ \frac{\eta \cdot k_h}{1 - \eta \cdot k_v} \cot \varphi'\right)^D \tag{21}$$

where $C = 0.65$ or 0.90 for smooth and rough footings, respectively, and

$$D = d_1 \tan^2\varphi' + d_2 \tan\varphi' + d_3 \tag{22}$$

with d_1, d_2 and d_3 listed in Table 1.

Finally, since the excess pore water pressure does not affect the N_q and N_c problems and a weightless medium is considered for the evaluation of N^s_{cE} and N^s_q (implying that for these two problems the only soil inertia effects may arise in the lateral surcharge q acting on the ground surface aside the foundation), then:

the seismic bearing capacity factor N^s_{cE}, coincides with its homologous static factor N_c (e^s_c=1);

N^{ss}_{cE}, N^s_{qE}, and N^{ss}_{qE} as well as the corresponding corrective coefficients e^{ss}_c, e^s_q, e^{ss}_q can be evaluated using the relationships by Cascone & Casablanca (2016) for the case Δu^*=0.

Accordingly, for the more general case with $c' \neq 0$, $q \neq 0$ and $\Delta u^* \neq 0$, with $d_w = 0$, the seismic bearing capacity can be evaluated as:

$$q_{ult} = c' N_c e^{ss}_c + q N_q e^s_q e^{ss}_q + \frac{1}{2} B \gamma' N_\gamma e^s_\gamma e^{ss}_\gamma \tag{23}$$

5 CONCLUDING REMARKS

During strong ground motions the bearing capacity of shallow foundations may dramatically reduce due to the concurrent effects of the inertia forces, arising in the foundation soil and in the superstructure, and of the possibly occurring reduction of foundation soil resistance due to the excess pore water pressure build-up.

Accordingly, there is the need of rigorous and practical solutions aimed to account for these effects on the evaluation of the seismic bearing capacity. To the authors' knowledge, only few studies focused on this topic and none of them examined the overall inertial and excess pore pressure effects on the reduction of seismic bearing capacity of shallow footings.

In this vein, the paper describes a procedure to introduce the influence of the excess pore water pressure on the evaluation of seismic bearing capacity of shallow strip footings by means of the method of characteristics. Specifically, the well-known N_γ problem is examined and the excess pore pressure ratio $\Delta u^* = \Delta u / p'_o$ is introduced in the analysis to assess the earthquake induced pore pressure Δu in the plastic volume involved in the failure mechanism as a function of the geostatic effective mean stress p'_o acting in free-field conditions.

Since the submergence of the soil under the footing affects the static state of stress, a preliminary static analysis is also presented in the paper and proper corrective coefficients, c_w and ζ_w, are proposed to account for the effect of hydrostatic pore pressures on static bearing capacity of both rough and smooth footings.

For seismic loading conditions, a bearing capacity factor $N_{\gamma E}$ was derived allowing to estimate the reduction of the bearing capacity term related to the N_γ problem owing to both the inertial (soil and superstructure) and excess pore pressure effects.

The results of the numerical analyses presented in the paper demonstrate that, as already observed by various authors for the case $\Delta u^* = 0$, also in presence of excess pore water pressures the soil and superstructure inertial effects are still decoupled. Conversely, the soil inertial and excess pore pressure effects are coupled since both affect the size of the plastic volume and the corresponding state of stress. Differently from the case $\Delta u^* = 0$ if large excess pore water pressures are expected, the combined soil inertial and excess pore pressure effect may result dominant in comparison with the superstructure inertia effect and may lead to a noticeable reduction in seismic bearing capacity.

Finally, starting from the numerical results a corrective coefficient e^s_γ to be applied to the static bearing capacity factor N_γ was derived to describe the combined effect of soil inertia and of earthquake-induced pore water pressure and a practical formula was derived for both smooth and rough footings. The soil inertial and excess pore pressure effect and the superstructure inertial effect can be evaluated separately through the coefficient e^{s}_{γ} derived herein and the coefficient e^{ss}_{γ} proposed by Cascone & Casablanca (2016); the overall seismic reduction of the bearing capacity factor N_{γ} can be expressed by superposition by using the corrective coefficient $e_{\gamma} = e^{s}_{\gamma} \cdot e^{ss}_{\gamma}$.

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