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On the stability of slender rock blocks subjected to horizontal and vertical seismic accelerations

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ABSTRACT: The rocking response of freestanding slender blocks to earthquake ground shaking is considered in the slope stability analysis of rock blocks, completely detached from the cliff and logged upon a horizontal plane. The horizontal and vertical seismic accelerations of a large database of recorded earthquake motions on rock soil (from US, Europe and Asia) have been used as input of a two-dimensional mechanical model implemented using a state space formulation. The model is able to solve the equations governing rocking problem accounting for the energy loss at every impact, to evaluate the block overturning (instability) or the rocking motion with smoothly rotations from one edge of the base of the block to the other. The records of the studied database cover a wide range of key characteristics in order to evaluate the effects on toppling of various seismic parameters such as PGA, PGV, Arias Intensity, energy flux, frequency content. The results in terms of stability of the dynamic numerical model are compared with the ones obtained by the pseudo-static approach, carried out considering static horizontal accelerations equal to the peak ground accelerations.

1 INTRODUCTION

The rocking response of free-standing rigid blocks during ground shaking was studied by many researchers with the aim of assessing the stability of different structures, such as masonry walls, ancient columns, electrical equipments and furniture. Starting from the pioneering work of Housner (1963) a number of contributions may be found in literature, using deterministic approaches, based on the integration of the equation of motion (Ishiyama, 1983; Shi et al., 1996; Makris & Konstantinidis, 2003, Zhang & Makris, 2001; Kounadis, 2010) or probabilistic approaches based on fragility curves (Dimitrakopoulos & Paraskeva, 2015, Giresini et al., 2018). Despite the numerous studies carried out on rigid body rocking (an extensive review work on this subject is reported in Makris, 2014), little literature on toppling of rock blocks under seismic actions is available (Keefer, 1984, Schurch & Becker, 2005, Anooshehpoor et al., 2004). According to Varnes (1978) classification, earthquake can trigger different types of landslide (falls, slides and topples) depending on topographic slope, bedrock geology and structure. While in slide mechanism block can move without necessarily inducing collapse, falls and toppling are brittle mechanisms. This means that using a pseudo-static approach for a slope stability analysis of rock slope, different seismic coefficient (peak acceleration divided by g) should be used for different mechanisms: higher for falls and lower for slides (Rampello et. al., 2010).

Also toppling is a brittle mechanism and it can be easily shown via static equilibrium that the horizontal ground acceleration needed to initiate the rotation of a free-standing prismatic column is g·(width/height). Nevertheless, upon rotation initiation, there is a safety margin between uplifting and overturning greater for greater blocks, as it has been recognized by Makris & Kampas (2016). Maiorano et al. (2015) proposed a reduction coefficient β for pseudo-static analysis, based on the results of 62 recorded horizontal earthquake motions applied to 100 slender rectangular blocks by means of a numerical integration of the equations governing the rocking problem, showing that the safety reserve is more significant for large blocks and rich frequency content time histories. In the present paper the model developed by

Maiorano et al. (2015) is extended to consider the combined effect of vertical and horizontal seismic motions related to earthquakes recorded in Central Italy in 2016 (Luzi et al., 2017).

2 PROBLEM DEFINITION

2.1 *Rocking analysis of free-standing rigid block subjected to horizontal and vertical accelerations*

The considered model (Figure 1) is a rectangular block with a uniformly distributed mass m and dimensions $2b$ times $2h$, rocking around O or O'. The block is characterized by a dimensionless slenderness parameter $a=b/h$, or the equivalent critical angle of rotation $\alpha=\tan^{-1}(b/h)$, and a size parameter $R = (h^2 + b^2)^{0.5}$ that is the radial distance from the center of rotation to the center of gravity. Depending on the characteristics of the horizontal and vertical acceleration of the stiff ground and the properties of the frictional interface, the block may rest, slide, rock, or slide-rock.

Assuming that the coefficient of friction is large enough to prevent sliding, the equation of motion for the rotation θ of a free-standing block with rotational inertia I_0 subjected to a horizontal $\ddot{u}_g(t)$and a vertical ground acceleration $\ddot{v}_g(t)$ when rocking respectively around O and O' is:

$$I_0\ddot{\theta}(t) + mgR\sin(-\alpha - \theta(t)) = -m\ddot{u}_g(t)R\cos(-\alpha - \theta(t)) - m\ddot{v}_g(t)R\sin(-\alpha - \theta(t)) \qquad \theta(t) < 0 \tag{1}$$

$$I_0\ddot{\theta}(t) + mgR\sin(\alpha - \theta(t)) = -m\ddot{u}_g(t)R\cos(\alpha - \theta(t)) - m\ddot{v}_g(t)R\sin(\alpha - \theta(t)) \qquad \theta(t) > 0 \tag{2}$$

The uplifting condition (rocking initiation) is given by:

$$\left|\frac{\ddot{u}_g(t)}{g}\right| \geq \left(1 + \frac{\ddot{v}_g(t)}{g}\right)a \tag{3}$$

Introducing the sign function equation (1) and (2) can be expressed in the compact form:

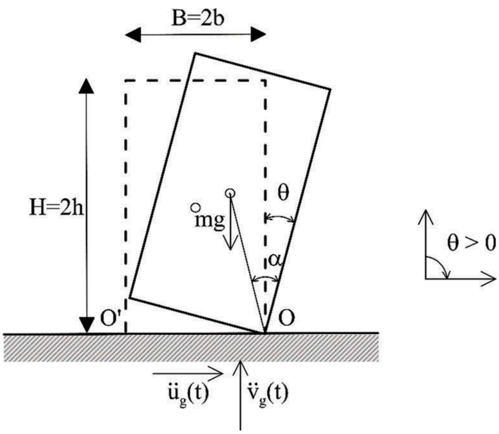

Figure 1. Rigid rectangular block under rocking motion.

$$\ddot{\theta}(t) = -p^2\left\{\left(1+\frac{\ddot{v}_g(t)}{g}\right)\sin[\alpha \mathrm{sgn}(\theta(t)) - \theta(t)] + \frac{\ddot{u}_g(t)}{g}\cos[\alpha \mathrm{sgn}(\theta(t)) - \theta(t)]\right\} \tag{4}$$

where $p = \sqrt{3g/4R}$is a frequency parameter (rad/s) of the block and is an expression of its size.

With reference to Figure 1, under a positive horizontal ground acceleration $\ddot{u}_g(t)$ able to induce uplift (equation 3), the block will initially rotate about the edge *O*' with a negative rocking rotation θ<0 and, if the acceleration changes sign, the block will reverse its motion, impact on the ground and assume positive rotation about the centre *O* and so on.

During rocking motion, the ratio of kinetic energy before and after the impact is:

$$\dot{\theta}^2\left(t_0^+\right) = r \cdot \dot{\theta}^2\left(t_0^-\right) \tag{5}$$

which means that the angular velocity after the impact is $\sqrt{r}$ times the velocity before the impact. Conservation of angular momentum before and after the impact gives the maximum coefficient of restitution (Housner, 1963):

$$r = \left(1 - \frac{3}{2}\sin^2\alpha\right)^2 \tag{6}$$

Values of the coefficient of restitution larger than the upper bound values by equation (6) result in loss of contect during impact (jump); on the contrary values smaller indicate inelastic behaviour during impact.

While blocks with small aspect ratio (low values of *a*) tend to conserve most of their angular velocity, less slender blocks (large values of *a*) dissipate more energy during impacts. Each collision dissipates some kinetic energy, so this dissipation mechanism provides an effective form of damping. In reality, an additional loss of energy, not considered in the present work, may occur depending on the nature of the material at the impact surface.

When only horizontal componet of earthquake is considered the uplifting condition is given by:

$$\left|\frac{\ddot{u}_g(t)}{g}\right| \geq a \tag{7}$$

and equation (4) becomes:

$$\ddot{\theta}(t) = -p^2\left\{\sin[\alpha \mathrm{sgn}(\theta(t)) - \theta(t)] + \frac{\ddot{u}_g(t)}{g}\cos[\alpha \mathrm{sgn}(\theta(t)) - \theta(t)]\right\} \tag{8}$$

2.2 *Numerical model*

The non-linear equation (4) is solved by a numerical integration with a SIMULINK extension in MATLAB (2017) by means of standard ordinary differential equations (ODE3) solvers with a fixed step of 0.001 s. Maiorano et al. (2015) considered only horizontal acceleration in the developed model (RHA model: rocking with horizontal accelerations); in the present paper the model (Figure 2) was extended to include also vertical acceleration (RHVA model: rocking with horizontal and vertical accelerations).

The integration of equation (4) in conjunction with the constraint imposed by equation (5) yields time histories of the rotation θ and the angular velocity. Overturning will occur when $\theta=\pi/2$, but different limit states may be defined in terms of normalized rotation θ_{max}/α as reported by Giresini et al. (2018). A value of the normalized rotation of 0.10, that is an order of magnitude smaller than the slenderness ratio, is considered without relevant risk of

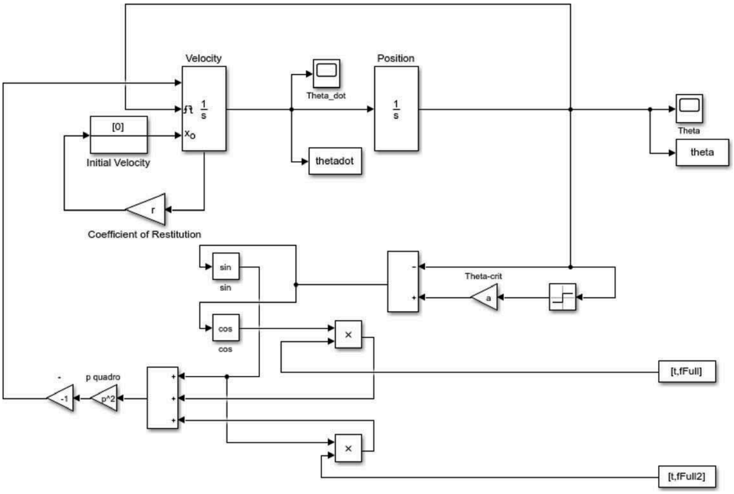

Figure 2. Mechanical model for the solution of motion equations in SIMULINK.

overturning. On the contrary a value of normalized rotation of 0.4 is called moderate rocking and a value of 1.5 may be considered as a near-collapse condition.

2.3 *Input data: block dimensions and seismic database*

The number of considered blocks has been extended to 170 (Table 1), considering 10 values of b ranging from 0.1 m to 1.0 m and 17 values of slenderness parameter $a=b/h$ ranging from 0.1 to 0.5 (angles α ranging from 5° to 26°).

The seismic database considered by Maiorano et al. (2015) has been extended including the records of Central Italy 2016 on subsoil type A or A* and PGA >0.1 g. In Table 2 the strong motion parameters such as peak ground acceleration PGA, Arias intensity I_a, peak ground velocity PGV, seismic wave energy flux density SED, mean period T_m, are reported for these records.

The rocking analyses of all the 170 blocks have been performed considering, for each station of the database of Central Italy event, the two horizontal time histories applied separately without considering the vertical component (RHA model) or combined with the vertical component of the acceleration (RHVA model). In previous studies (Makris & Kampas, 2016) it

Table 1. Blocks dimensions.

b/h	0,1	0,125	0,15	0,175	0,2	0,225	0,25	0,275	0,3	0,325	0,35	0,375	0,4	0,425	0,45	0,475	0,5
b (m)								h(m)									
0,1	1,00	0,80	0,67	0,57	0,50	0,44	0,40	0,36	0,33	0,31	0,29	0,27	0,25	0,24	0,22	0,21	0,20
0,2	2,00	1,60	1,33	1,14	1,00	0,89	0,80	0,73	0,67	0,62	0,57	0,53	0,50	0,47	0,44	0,42	0,40
0,3	3,00	2,40	2,00	1,71	1,50	1,33	1,20	1,09	1,00	0,92	0,86	0,80	0,75	0,71	0,67	0,63	0,60
0,4	4,00	3,20	2,67	2,29	2,00	1,78	1,60	1,45	1,33	1,23	1,14	1,07	1,00	0,94	0,89	0,84	0,80
0,5	5,00	4,00	3,33	2,86	2,50	2,22	2,00	1,82	1,67	1,54	1,43	1,33	1,25	1,18	1,11	1,05	1,00
0,6	6,00	4,80	4,00	3,43	3,00	2,67	2,40	2,18	2,00	1,85	1,71	1,60	1,50	1,41	1,33	1,26	1,20
0,7	7,00	5,60	4,67	4,00	3,50	3,11	2,80	2,55	2,33	2,15	2,00	1,87	1,75	1,65	1,56	1,47	1,40
0,8	8,00	6,40	5,33	4,57	4,00	3,56	3,20	2,91	2,67	2,46	2,29	2,13	2,00	1,88	1,78	1,68	1,60
0,9	9,00	7,20	6,00	5,14	4,50	4,00	3,60	3,27	3,00	2,77	2,57	2,40	2,25	2,12	2,00	1,89	1,80
1	10,00	8,00	6,67	5,71	5,00	4,44	4,00	3,64	3,33	3,08	2,86	2,67	2,50	2,35	2,22	2,11	2,00

was shown that the vertical ground acceleration generally has a marginal effect on the stability of a free-standing rocking column. As the authors highlighted, this is due to the fact that the vertical acceleration enters the equation of motion after being multiplied with sin $(\alpha - \theta) \ll 1$, whereas the horizontal acceleration enters the equation of motion after being multiplied by cos $(\alpha - \theta) \approx 1$.

In addition, it should be noted that a positive vertical acceleration (Figure 1) has a stabilizing effects, while a negative one, reducing the weight of the block, make it less stable. In the Central Italy seismic event the vertical accelerations were very high, reaching absolute values of PGA as high as 0.7-0.8 *g*. For this reason, the analyses with RHVA model were performed only considering this database.

2.4 *Results of the rocking analysis of rigid block subjected to horizontal and vertical accelerations*

A comparison between the rocking analysis by means of the two models RHA and RHVA is reported in Figure 3 in terms of *b* versus *h* plots for T1213HNE and T1213HNE+HNZ time histories. In this figure four regions of stability can be seen (Ishiyama, 1982): rigid bodies with dimensions within the zone I do not rock if subjected to horizontal and vertical accelerations; rigid bodies within zone I and II do not rock, if subjected only to horizontal acceleration; those with dimensions within zone III can rock but do not overturn, and bodies with dimensions within zone IV experience irregular behaviour and very likely will overturn. The limit between zones I and II is defined by Eq. (3) and represent a pseudo-static limit evaluated considering the peak acceleration PGA of the horizontal component and the minimum value of vertical acceleration (which does not necessarily coincide with the PGA and, in this case, is -0.728g). The limit between zones II and III is defined by Eq. (7) and depends only on the peak acceleration PGA of the horizontal component. The curves in Figure 3 representing the dynamic limits of the two rocking analyses RHA (only horizontal component) and RHVA (horizontal and vertical components) are very similar to each other, confirming that the vertical component is not significant in overturning analysis, at least for this acceleration time history.

For each station of Table 2, four rocking analyses have been carried out, two for the horizontal components separately and two for the horizontal components combined with the vertical one. Figure 4 shows the dynamics limits for all the acceleration time histories of Table 2. Only the events of the last four station are able to uplift and overturn some of the investigated blocks. Hence, there are no curves for the first nine stations of Table 2. Comparing these curves, it is evident that in the large part of time histories, vertical components do not affect the stability of the blocks. In some cases (compare for example CLO_HGN with CLO_HGN +HGZ) the presence of vertical acceleration can increase the stability of blocks. In other cases (T1213_HNE and T1213_HNE+HNZ) vertical component is slightly destabilising.

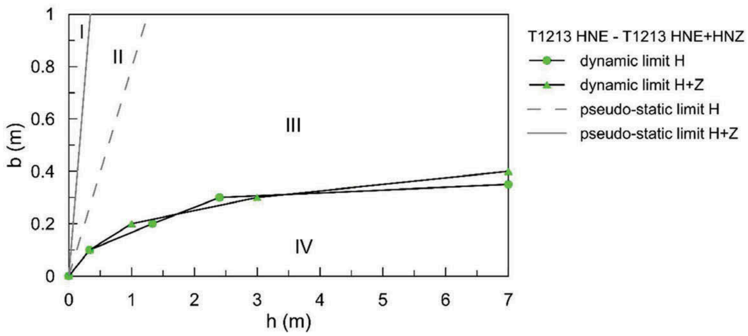

Figure 3. Stability plots in terms of block dimensions for T1213 time histories.

Table 2. Seismic database of Central Italy event 2016.

Event date	Lat	Lon	Depth	Mw	ML	Station code	R epi. [km]	Comp	PGA [g]	PGV [cm/s]	Tm [s]	SED [cm^2/s]
2016-10-26 19:18:06	42.91	13.13	7.5	5.9	5.9	MMUR	60.186	HNE	0.071	2.44	0.45	6
								HNN	0.104	2.68	0.26	4
								HNZ	0.043	2.01	0.30	3
2016-10-26 19:18:06	42.91	13.13	7.5	5.9	5.9	MNF	17.377	HGE	0.121	8.26	0.36	36
								HGN	0.085	9.67	0.24	63
								HNZ	0.087	7.97	0.26	61
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	MNF	25.979	HGE	0.133	6.42	0.38	51
								HGN	0.116	6.15	0.27	44
								HGZ	0.111	5.72	0.26	39
2016-10-26 17:10:36	42.88	13.13	8.7	5.4	5.4	CLO	8.545	HNE	0.102	5.23	0.31	21
								HNN	0.127	4.79	0.43	21
								HNZ	0.137	5.43	0.21	11
2016-10-26 19:18:06	42.91	13.13	7.5	5.9	5.9	MMO	16.204	HGE	0.170	7.30	0.28	49
								HGN	0.137	13.50	0.41	124
								HGZ	0.088	5.70	0.31	27
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	MMO	19.168	HGE	0.188	8.91	0.27	189
								HGN	0.188	11.41	0.28	254
								HGZ	0.139	11.44	0.29	151
2016-10-26 17:10:36	42.88	13.13	8.7	5.4	5.4	T1212	15.816	HNE	0.180	12.26	0.49	47
								HNN	0.195	10.50	0.43	28
								HNZ	0.059	2.88	0.38	9
2016-10-26 19:18:06	42.91	13.13	7.5	5.9	5.9	CLO	10.835	HGE	0.183	12.81	0.31	80
								HGN	0.193	12.31	0.43	160
								HGZ	0.219	8.54	0.21	42
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	T1220	30.932	HNE	0.238	15.59	0.46	409
								HNN	0.259	16.38	0.37	391
								HNZ	0.143	9.01	0.23	61
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	T1212	10.463	HNE	0.279	27.68	0.53	833
								HNN	0.278	24.65	0.46	601
								HNZ	0.166	8.90	0.46	134
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	ACC	18.559	HGE	0.434	44.07	0.39	1146
								HGN	0.392	39.43	0.45	1333
								HGZ	0.557	23.90	0.39	512
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	CLO	7.798	HGE	0.426	52.19	0.32	2849
								HGN	0.582	66.08	0.54	2935
								HGZ	0.797	68.61	0.32	4240
2016-10-30 06:40:18	42.83	13.11	9.2	6.5	6.1	T1213	11.981	HNE	0.794	60.73	0.26	1431
								HNN	0.764	30.45	0.20	856
								HNZ	0.886	32.53	0.17	516

Following the procedure reported by Maiorano et al. (2015) a rocking spectrum is proposed in Figure 5, in order to compare pseudo-static and dynamic limits. For each time history the minimum stable width of the block obtained by the dynamic analysis, b, divided by the limit width b_{ps}, obtained from a pseudo-static analysis considering only the horizontal component:

$$b_{ps} = \frac{PGA_h}{g} \cdot h \tag{9}$$

is represented in the figure as a function of a dimensionless parameter T_p/T_m, representative of frequency characteristics of the ground motion and block dimensions. T_p is the inverse of the frequency parameter p:

$$T_p = \frac{2\pi}{p} = 2\pi\sqrt{\frac{4R}{3g}} \tag{10}$$

while T_m is the mean period proposed by Rathje et al. (1998) of the horizontal component of the acceleration time history.

The ratio b/b_{ps} can be considered as a reductive coefficient β of the acceleration to be used in pseudo-static analysis. The upper bound curve is the same proposed by Maiorano et al. (2015) and, according to the results of present research, can be used also when horizontal and vertical components are considered acting simultaneously.

The rocking spectrum highlights the existence of a safety reserve more significant for large blocks (small values of T_p) and rich frequency content signals.

In the present paper another correlation for the reduction coefficient β is presented as function of the maximum absolute horizontal ground acceleration PGA_h (Figure 6):

$$\beta = \frac{b}{b_{ps}} = 1 - 0.65 \cdot \frac{PGA_h}{g} \tag{11}$$

in which b_{ps} is evaluated by means of eq. (9), regardless of the the considered earthquake compontent (only horizontal or horizontal combined with vertical compnent).

This reductive coefficient, even if very conservative, since obtained from the envelope of all the data, can be directly adopted in pseudo-static analyses.

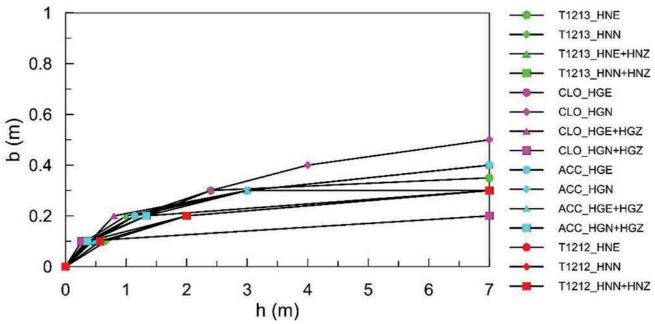

Figure 4. Stability plots in terms of block dimensions for the Central Italy database.

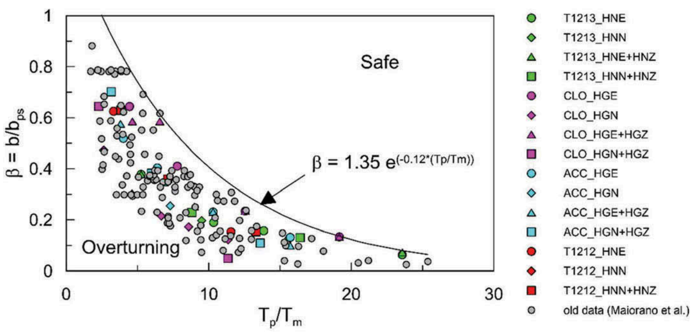

Figure 5. Rocking spectrum: reductive coefficient β (after Maiorano et al., 2015).

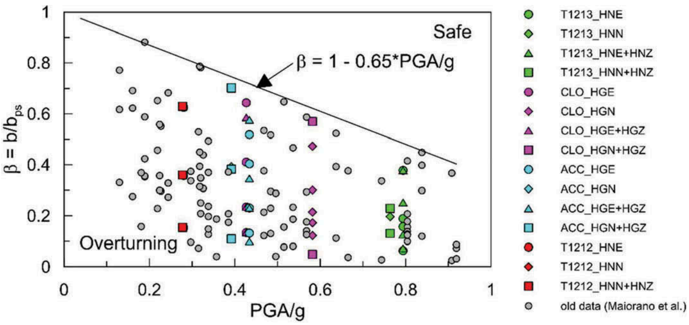

Figure 6. Reductive coefficient β as function of PGA of the horizontal acceleration time history.

3 CONCLUDING REMARKS

The paper investigates the rocking and overturning response of slender blocks on a stiff base subjected to horizontal and vertical acceleration time histories. The results of dynamic analyses performed on a number of horizontal and vertical acceleration time histories highlight:

- the negligible influence of the vertical seismic component related to the stability of slender rigid blocks under seismic excitations with respect to the overturn collapse phenomenon;
- the major part of the selected seismic events of Central Italy 2016, although featured by PGA higher than 0.10g, are not able to overturn any slender blocks.

As proposed in Maiorano et al. (2015), the signals able to uplift and overturn some block have values of SED lower than 300cm^2/s. A new reductive coefficient $\beta = b/b_{ps}$ as function of peak ground acceleration of the horizontal component PGA/g is proposed for pseudo-static analysis of the rocking behavior of rigid blocks.

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