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On the distribution of site amplification factors

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ABSTRACT: A site-specific probabilistic seismic hazard analysis (PSHA) must consider the uncertainties in estimating site amplification functions (AFs). For this reason, AFs must be defined in terms of full probability distribution functions. The distribution of AFs is often computed using simulations, whereas the uncertainty in input parameters (i.e., input rock ground motion, shear-wave velocity profile, and parameters that control the nonlinear behavior of soil) is mapped into the uncertainty in the AFs, typically using Monte Carlo simulations. In the Monte Carlo simulations, sets of input parameters that capture the uncertainties in input variables are first generated. Then, site response analyses are conducted for each set of input parameters and the mean and standard deviation of the computed AFs are as assumed to represent the distribution of the AFs. Typically, these AFs are assumed to be lognormally distributed. In this paper, we show that this assumption is not necessarily correct and the AFs distribution in log domain can be skewed. Moreover, we develop an approach to consider this skewness and show the effects of relaxing the assumption of symmetric AFs on the results of PSHA.

1 INTRODUCTION

Bazzurro and Cornell (2004) demonstrated that the effect of local soil deposits on the seismic hazard at the surface can be considered rigorously using site amplification functions (AFs) and their distribution. Following this work, many researchers have studied the distribution of AFs (Rathje et al. 2010, Li and Assimaki 2010, Bahrampouri et al. 2018). However, all the studies to date have focused on the mean and standard deviation of AFs, assuming AFs are log-normally distributed. In this paper, we use equivalent linear site response analyses to show that the assumption of log normality is not necessarily correct, especially for high intensity motions. To relax this assumption, we use the skew-normal distribution (Azzalini and Valle 1996) for the logarithm of AFs. The skew-normal distribution is a generalized form of the normal distribution with an extra parameter that captures skewness. Moreover, we show that releasing this assumption can affect the resulting hazard curve.

The probability density function for the skew-normal distribution is:

$$f(x) = \frac{2}{\sigma}\, \phi\left(\frac{x-\mu}{\sigma}\right)\phi\left(\alpha\frac{x-\mu}{\sigma}\right) \tag{1}$$

where ϕ and Φ are the probability density and cumulative density functions of the normal distribution, μ is the location factor, σ is the scale factor, and α is the shape factor. Note that the parameter σ in the skew-normal formulation in the skew normal formulation is also a measure of spread, as the standard deviation in the normal distribution, but the two are not exactly equal. Moreover, if α is positive the distribution is skewed to the left and if α is negative the distribution is skewed to the right. If α is equal to zero, the distribution is symmetric and identical to the normal distribution. Therefore, the normal distribution is just a specific case of the skew-normal distribution. In Figure 1, three skew normal distributions are presented.
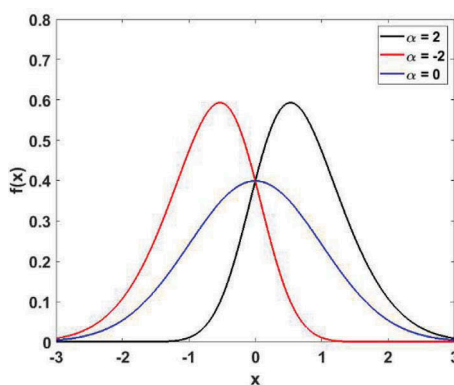
Figure 1.   probability density function of skew normal distribution for $\mu = 0$ and $\sigma = 1$

In the reminder of the paper, we study the distribution of AFs through an example of site response analyses. First, we introduce the shear-wave (V_S) profile and input ground motions used in this example. Then we study the resulting amplification function and the probability distribution function of the computed AFs. We also consider possible causes for the observed skewness. Finally, we demonstrate the effects of the shape of the AFs' distribution on the hazard curve on the surface of the hypothetical site.

2   SITE RESPONSE ANALYSES

In order to illustrate how the AFs can deviate from a normal distribution, we present a site response analysis for a hypothetical site. The V_S profile is obtained from a measured site, as described in Section 2.1. Site response analyses are computed using the Equivalent Linear method. The input motions are selected by assuming that this hypothetical site is located in Seattle. The results of the site response analyses are used to compute the hazard at the surface of the hypothetical site using the bedrock hazard curves for Seattle.

2.1   *Inputs*

In this study, we use a site characterized by Cox and Teague (2017) using MASW tests at Burnside Park, New Zealand. The 50 best-fit V_S profiles computed by Cox and Teague (2017) are presented in Figure 2. The profiles were selected based on how well they fit the empirical dispersion curve measured at the site (Cox and Teaue 2017). All of the 50 profiles have soft layers near the surface and have average V_S over the upper 30 meters, V_{S30}, smaller than 250 m/s. We assume all layers are sand and use the Darendeli (2001) model to estimate the modulus reduction and damping curves. Note that while the choice of modulus reduction and damping curves is important when conducting site response analyses, the choice of model does not affect the conclusions presented in this paper.

In selecting the input ground motions and estimating the hazard curve at the surface, we assume the profile is located in Seattle, Washington State. We select 300 ground motions that are compatible with contributing scenarios from the deaggregation map of Seattle (Frankel et al. 2000). The 300 ground motions are obtained from the Kiban Kyoshin network (KiK-net). We then use the 300 ground motions and the sigma spectra software (Kottke and Rathje 2008) to get 100 ground motions that are compatible with the uniform hazard spectra at 10%, 5%, and 2% probabilities of exceedance.
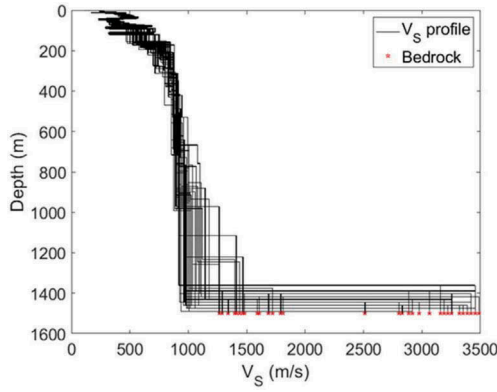
Figure 2. V_S profiles used in this study

2.2 *Results*

The AFs at the period (T) of 0.2 s are shown in Figure 3 as a function of the spectral acceleration of the input motion ($S_{a,Rock}$) at the same oscillator period (Stewart et al. 2014). The AFs are linear up to a threshold of $S_{a,Rock}$ around 0.2 g and then start decreasing with an increase in the intensity of the input motions. It is common to fit a functional form to the AFs (Stewart et al. 2014); however, since we are interested in studying the shape of the residual distributions, we used a non-parametric regression. Using a nonparametric regression makes sure that the shape of distribution is not due to the selected functional form. In Figure 3, the results of the nonparametric regression is presented as a blue line.

The vertical red line in Figure 3 is visually picked as the point of initiation of nonlinear behavior of the AFs. The distribution of residuals of the AFs for intensities higher than those indicated by the red line are plotted in Figure 4a. This plot shows that the AFs at large intensities is skewed and the normal distribution does not capture well the behavior of the residuals. We then fitted a skew-normal distribution to the data with maximizing likelihood. The value of the skewness parameters α is zero at small intensities and gradually increases to a value of α_H at larger intensities. The value of α_H is shown in Figure 4b. This plot shows that the AFs are right-skewed (i.e., negative α_H) at high intensities for the set of input motions and V_S in the example.
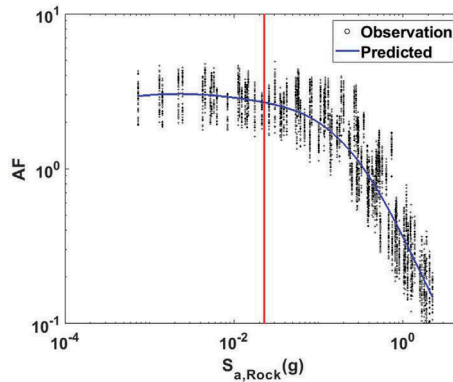

Figure 3.   AFs for 100 ground motions and 50 soil profiles at 0.2 s period. The blue line represents the prediction of nonparametric regression and the red line shows the point nonlinear behavior starts
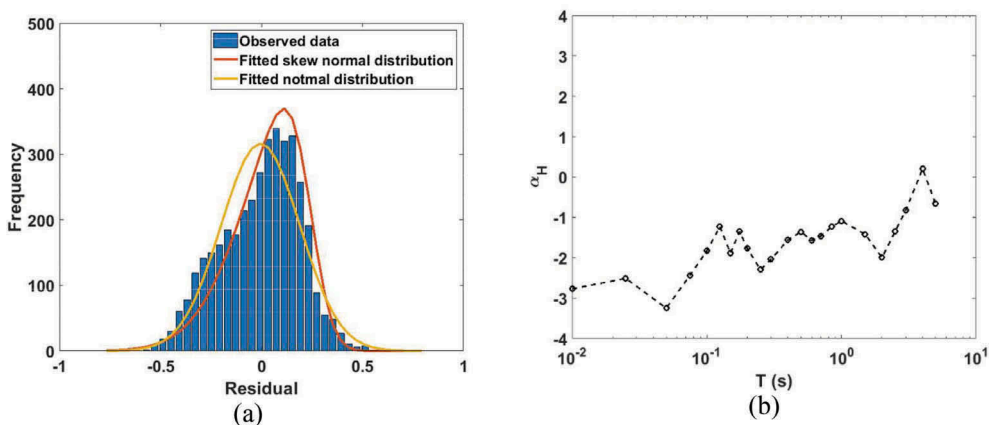
Figure 4.   a) The distribution of residuals of AFs (in log domain) at high intensities and comparison of normal and skew normal fit for them. b) The plot α at high intensities vs period

3   DISCUSSION

The authors did not anticipate the skewness of AFs distribution before observing it. Therefore, the real underlying reasons for this behavior are unknown to us. However, we could think of one mechanism that can make the distribution of AFs skewed at large strains. Each Vs profile in the Monte Carlo simulations has a different threshold where nonlinearity initiates. We can see that behavior schematically in Figure 5, where each gray line is the AF normalized by its linear value. As it is obvious in Figure 5, this source of uncertainty makes the distribution of AFs skewed to the right.

We used rock hazard curves for Seattle to visualize the effects of considering skewness in AFs on the hazard curve at the surface. The hazard at the surface is computed using the convolution approach (Bazzurro and Cornell 2004). Figure 6 shows the results of the convolution when we consider the skewness in the AFs distribution and when we assume symmetry. The difference between these two approaches is considerable. At moderate intensities the assumption of having skewed distribution gives higher exceedance rate because we are considering the fact the mode is larger than the mean. At high intensities the assumption of skewed distribution gives lower exceedance rate because the tail of skew normal with negative α is lighter on the positive side.
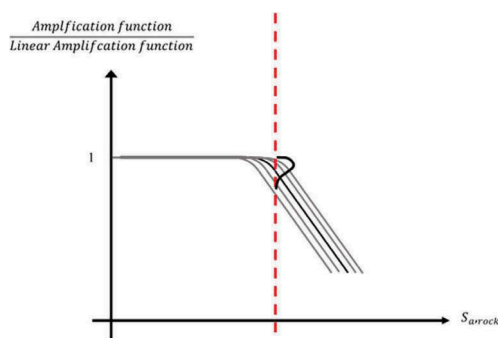

Figure 5.   Schematic plot of normalized AFs. The plot is presented to show the mechanisms for skewed AFs distribution at large intensities
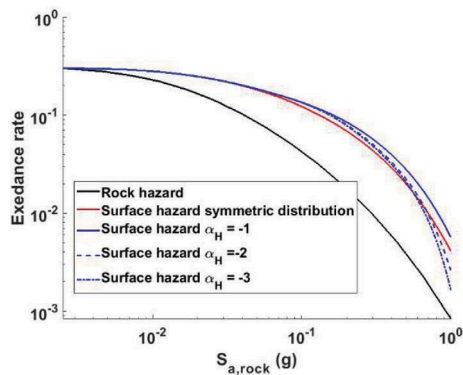
Figure 6.   The effect of skewness of AFs' distribution at large intensities on surface hazard curve

4   CONCLUSION

In this paper we postulated that the distribution of AFs at high intensities is not necessarily symmetric in the log domain. Moreover, we present a method for accounting for this skewness. It is important to note that with this limited analysis we neither can nor want to imply that the distribution of AFs are not symmetric all the time. Instead, we want to emphasize that there is the possibility of having non log-normal AFs distributions and relaxing the assumption of symmetry can improve the estimates of hazard curves at surface.

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