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Developing and optimizing site-specific G_0 correlations using $SCPT_u$ data

N. Barounis, J. Philpot & T. Smith
Cook Costello, Christchurch, New Zealand

ABSTRACT: The small-strain shear modulus G_0 and small-strain shear wave velocity V_s are fundamental soil properties that are important when defining the static and dynamic characteristics of a site. When a Seismic Cone Penetration Test ($SCPT_u$) is undertaken during the in-situ geotechnical investigations, the V_s profile can be measured alongside the conventional CPT_u data (q_c, f_s, u_2). This paper proposes a methodology for developing and optimizing site-specific G_0 correlations based on the V_s profile, which is in a form that makes use of only the conventional CPT_u data. This correlation can be utilised on other similar sites where seismic geophysical testing has not been undertaken. The proposed method relies upon the empirical relationship between the small-strain shear modulus and in-situ void ratio. The method is applied to a site in Christchurch, New Zealand where high quality $SCPT_u$ data has been obtained and the results are compared against other empirical methods.

1 INTRODUCTION

The small-strain shear modulus G_0 and the small-strain shear wave velocity V_s are fundamental soil properties that play a key role in geotechnical earthquake engineering. The small-strain shear modulus is widely incorporated into static and dynamic soil-structure interaction considerations, and the small-strain shear wave velocity is widely utilised to undertake seismic site classification.

Direct measurement of the V_s profile, or empirical methods of estimating the V_s profile based on in-situ tests, are the preferred approaches for undertaking seismic site classification in New Zealand. The V_s profile is measured directly if seismic geophysical testing is carried out during the in-situ geotechnical investigations; however, these tests are typically not undertaken for low importance level projects in New Zealand due to budget constraints. There are numerous empirical methods already available for estimating G_0; however, these methods are typically limited to a certain location or soil type.

This paper proposes a methodology for developing and optimizing site-specific G_0 correlations based on $SCPT_u$ data. The site-specific G_0 correlations can then be applied using only conventional CPT_u data collected onsite and can also be utilised on other sites with similar soil conditions where seismic geophysical testing has not been undertaken. The method is intended to produce G_0 correlations that can capture important features observed in the V_s profile so these are carried through to the subsequent analysis.

2 THEORETICAL BACKGROUND OF THE PROPOSED METHODOLOGY

2.1 *Definition of the small-strain shear modulus*

The small-strain shear modulus and small-strain shear wave velocity have the physical relationship presented in Equation 1:

$$G_0 = \rho V_s^2 \tag{1}$$

where G_o = small-strain shear modulus; ρ = soil density; and V_s = small-strain shear wave velocity. Equation 1 typically requires seismic geophysical testing to be undertaken in order to obtain the V_s profile of the soil.

2.2 *Empirical relationship between G_0 and the void ratio*

An empirical expression between G_0 and the void ratio (Yang & Liu 2016) takes the general form of Equation 2:

$$G_0 = AF(e)\left(\frac{\sigma\prime_m}{p_a}\right)^n \tag{2}$$

where A = a material coefficient; $F(e)$ = a void ratio function; σ'_m = mean effective overburden pressure; p_a = atmospheric pressure (in same units as σ'_m); and n = an empirical model fitting parameter. It should be noted Equation 2 in a form such that G_0 may be estimated using only CPT_u data, provided the material coefficient, empirical model fitting parameter and the general form of the void ratio function can be defined for a given site.

Many previous studies undertaken for estimating G_0 based on the void ratio indicate the general form of the void ratio function $F(e) = e^{-x}$.

2.3 *Estimation of in-situ void ratio using CPT_u*

The in-situ void ratio can be determined for completely saturated soils using Equation 3:

$$e = \frac{G_s\gamma_w - \gamma}{\gamma - \gamma_w} \tag{3}$$

where G_s = specific gravity of soil; γ_w = unit weight of water; γ = bulk unit weight of soil (in same units as γ_w). G_s can be assumed or determined in a laboratory. The method from Barounis et al. (2017) relies on the correlation for estimating soil unit weight using CPT_u (Robertson 2010). Equation 3 is only applicable for fully saturated soil (S_r = 1.0), i.e. soil that is permanently located below the groundwater table, and is also only applicable for inorganic soils.

2.4 *Modified normalized small-strain rigidity index*

The modified normalized small-strain rigidity index (Robertson 2016), which is applicable for both sands and clays, is defined in Equation 4:

$$K_G^* = \left(\frac{G_0}{q_n}\right)Q_{tn}^{0.75} \tag{4}$$

where K^*_G = modified normalized small-strain rigidity index; q_n = net cone resistance; Q_{tn} = normalized cone resistance. Robertson concludes that the soil has significant microstructure when K^*_G exceeds 330. Typical soil characterization approaches and the classification of soil behavior type becomes less reliable when the soil has significant microstructure and some judgement is required (Robertson 2016).

2.5 *Data averaging*

The proposed methodology makes use of both the V_s profile obtained from geophysical testing and conventional CPT_u data. It should be noted that shear wave velocity measurements from $SCPT_u$ are typically recorded at 500mm depth intervals, whereas CPT_u data is typically recorded at 10-20mm depth intervals. The shear wave velocity measurements can be considered to be average values for each 500mm layer. Hence, whenever the two datasets are being evaluated against each other, it is important to evaluate the V_s measurements against an average of the CPT_u data in the same 500mm layer.

3 DEVELOPING AND OPTIMIZING THE SITE-SPECIFIC G_0 CORRELATION

3.1 *Mean effective overburden pressure*

The mean effective overburden pressure is defined in Equation 5:

$$\sigma\prime_m = \frac{(\sigma\prime_1 + \sigma\prime_2 + \sigma\prime_3)}{3} \tag{5}$$

where σ'_1 = major effective overburden pressure; σ'_2 = intermediate effective overburden pressure; and σ'_3 = minor effective overburden pressure. Assuming the vertical effective overburden pressure is the major component of the effective overburden pressure, and that the soil is in a laterally isotropic state, Equations 6 and 7 can be established:

$$\sigma\prime_1 = \sigma\prime_v \tag{6}$$

$$\sigma\prime_2 = \sigma\prime_3 = \sigma\prime_h = K_0\sigma\prime_v \tag{7}$$

where σ'_v = vertical effective overburden pressure; and K_0 = at-rest effective stress coefficient. Substituting Equations 6 and 7 back into Equation 5 yields:

$$\sigma\prime_m = \frac{\sigma\prime_v(1 + 2K_0)}{3} \tag{8}$$

If K_0 and σ'_v have been estimated versus depth, then the mean effective overburden pressure can be estimated using Equation 8. Alternatively, an assumption of $K_0 = 0.5$ can be assumed for a simplified analysis involving normally consolidated soils. Applying this simplifying assumption produces Equation 9:

$$\sigma\prime_m = \frac{2\sigma\prime_v}{3} \tag{9}$$

3.2 *Simplification of Equation 2*

Substituting Equation 9 back into Equation 2 and noting that the atmospheric pressure at sea level is approximately 101kPa, the relationship for estimating G_0 can be rewritten for normally consolidated soils as per Equation 10:

$$G_0 = AF(e)\left(\frac{2\sigma\prime_v}{303}\right)^n \tag{10}$$

3.3 *Determining physical parameters*

In order to solve Equation 10 and develop the site-specific G_0 correlation, both the seismic geophysical testing and conventional CPT_u data are required. Using the V_s measurements and the average soil density, G_0 can be estimated for each 500mm soil layer using Equation 1. The average vertical effective overburden pressure and the average in-situ void ratio can be estimated from CPT_u data for each 500mm soil layer.

At this stage, all of the physical parameters are known quantities; however, the material coefficient, empirical model fitting parameter and the general form of the void ratio function t still need to be established for the given site. It is clear that a rational method needs to be established for determining these empirical factors; therefore, a correlation optimization process is undertaken.

3.4 *Defining and Optimizing the G_0 correlation.*

In order to define the site-specific G_0 correlation, the measured G_0 values for each 500mm layer is plotted against the stress-normalized void ratio parameter. Based on how the data tends to plot on this graph, a power curve relationship can be utilised to obtain the best-fit correlation between these two variables (Figure 3). The final form of the site-specific G_0 correlation is presented in Equation 11:

$$G_0 = A\left(\frac{e}{\left(\frac{2\sigma\prime_v}{303}\right)^m}\right)^{-x} \tag{11}$$

where m = an assumed empirical model fitting parameter (input); and x = the best-fit empirical correlation parameter obtained from fitting a power curve to the data points (output). The m-exponent in Equation 11 has been introduced as a means of identifying the optimal correlation, and also to produce a G_0 correlation that is mathematically consistent with the relationship that was presented in Equation 2. It is important to note the assumed m-exponent affects the denominator of the stress-normalized void ratio parameter.

As the stress-normalized void ratio in the above equation is raised to a negative power (x-exponent), Equation 11 can be simplified through mathematical manipulation and rewritten to be in a similar form to that of Equation 10:

$$G_0 = \frac{A}{e^x}\left(\frac{2\sigma\prime_v}{303}\right)^{mx} \tag{12}$$

By making direct comparisons between Equations 10 and 12 (to demonstrate mathematical equivalence), the following can be noted:

$$F(e) = \frac{1}{e^x} \tag{13}$$

$$n = mx \tag{14}$$

The n-exponent was defined below Equation 2. It is important to note this parameter is a product of the assumed m-value and the resulting x-value. In order to produce the optimal correlation, the m-value is varied (typically between 0.3-0.8) to produce a range of different empirical correlations. The optimal m-value will produce the strongest possible correlation for the dataset (refer to Figure 4). The optimization process is discussed further during the application of the proposed method.

3.5 *G_0 dependency on empirical parameters*

Figure 1 indicates the dependency of G_0 with respect to each of the parameters A, e^x and n if all other variables are held constant for each of the respective graphs. For example, the plot on the left of Figure 1 holds the values of e^x and n as constants, while varying the A parameter. As expected, G_0 is directly proportional to the magnitude of A and is inversely proportional to e^x. The n-exponent controls the overall shape of the G_0 curve. As the n-exponent increases, the G_0 curve becomes increasingly more linear; whereas, for lower n-exponent values the G_0 values increase rapidly at low pressures, approximately tending towards a constant value at larger pressures.

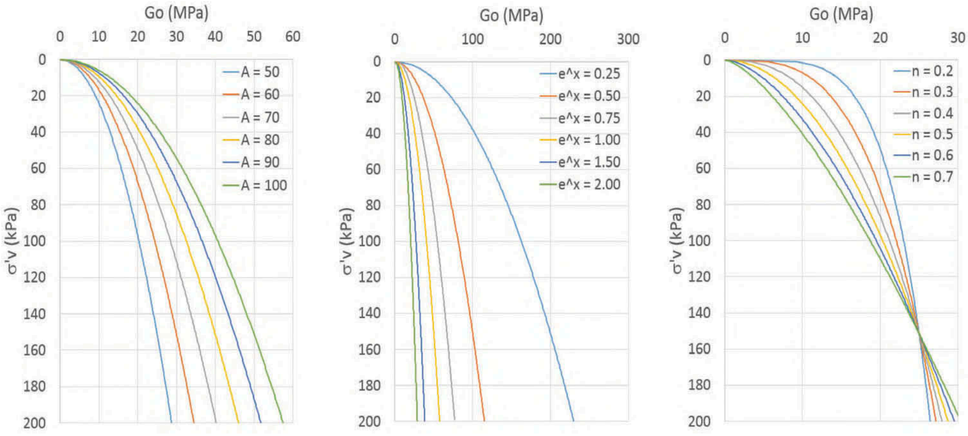

Figure 1. G_0 dependency on: A (left); e^x (center); and n (right).

4 APPLICATION OF THE PROPOSED METHOD

The proposed methodology has been applied to a site in Christchurch, New Zealand where high quality data is available and there is a shallow groundwater table. The site is located in a geological setting of dominantly alluvial sand and silt overbank deposits. A borehole log at the site indicates the soil comprises very soft to firm silt with traces of organics and peat. There are also intermittent layers of sandy silt and sand present to the termination depth of the borehole (15.7m below ground level).

Figure 2 below shows the normalized soil behavior type plot (SBT_n), the measured V_s profile for the site and the stress-normalized void ratio plot. It can be seen that the soil profile is complex with interbedded layers of clays, silts and sands overlying a dense sandy gravel layer, which is consistent with the borehole results. The shear wave velocity profile encounters a marked stiffness contrast between the upper 17.5m of soil and the underlying 2.5m of dense sandy gravel. The stress-normalized void ratio parameter tends to decrease with increasing depth and there is also a step change observed in the dense sandy gravel layer. It should be

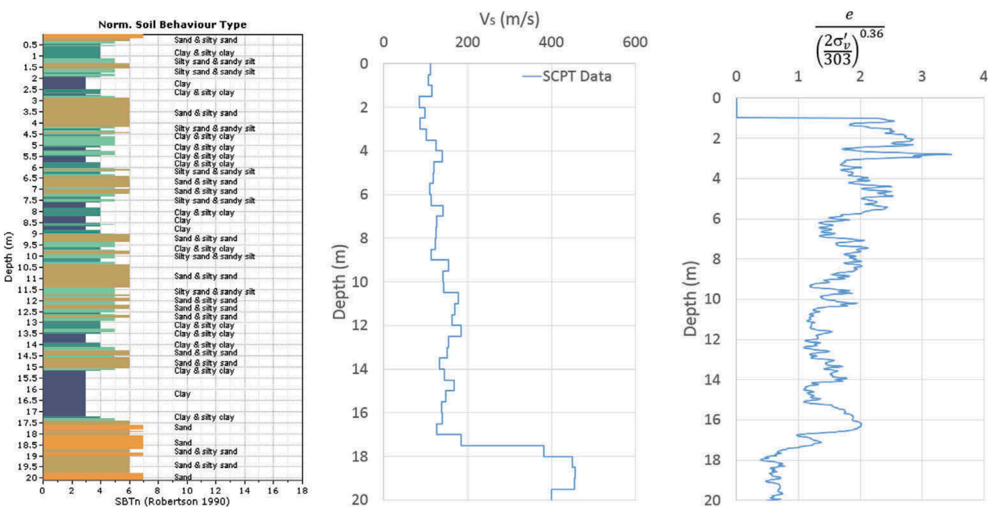

Figure 2. SBT_n plot (left), V_s profile (center), and stress-normalized void ratio (right) for the subject site.

noted that a specific gravity of 2.65 was assumed for the entire soil profile to allow the void ratio to be estimated.

Figure 3 presents the optimized site-specific G_0 correlation for the site based on one $SCPT_u$ investigation. It should be noted that there are typically a few soil layers where no distinct correlation between the trend of the V_s measurements and the CPT_u data is observed; hence, those data points need to be filtered out prior to obtaining the best-fit power curve relationship to prevent the site-specific G_0 correlation from being contaminated by those outliers in the data.

The optimal site-specific small-strain shear modulus correlation is reproduced in Equation 15:

$$G_0 = \frac{100.51}{e^{2.27}} \left(\frac{2\sigma\prime_v}{303} \right)^{0.82} \tag{15}$$

Figure 4 shows the correlation optimization plot for this site as a function of the empirical model fitting parameter (n-exponent). It can be seen that all of the assumed empirical models have a strong correlation with an R^2-value of between 0.92-0.93; however, the optimal correlation was obtained at an n-exponent value of approximately 0.82. Figure 4 is

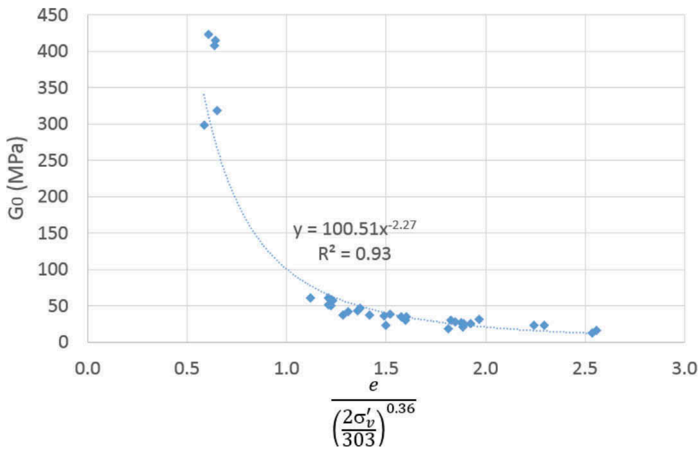

Figure 3. Site-specific small-strain shear modulus correlation.

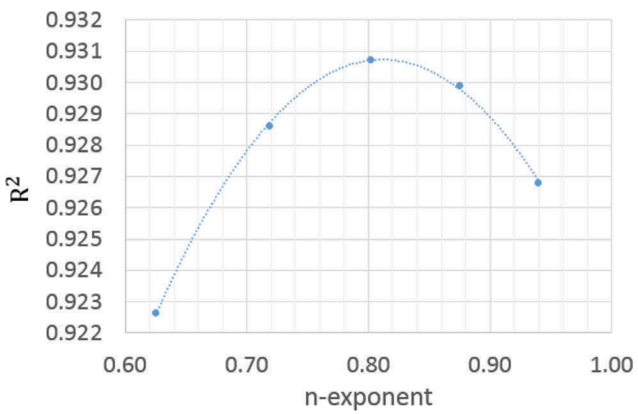

Figure 4. Correlation strength for each of the assumed empirical models

Table 1. Summary of parameters for each of the assumed empirical models

m	x	n	A	R^2
0.25	2.50	0.63	92.21	0.923
0.30	2.40	0.72	96.25	0.929
0.35	2.29	0.80	99.85	0.931
0.40	2.19	0.88	103.01	0.930
0.45	2.09	0.94	105.78	0.927

produced by assuming a range of m-exponent values, plotting the data (Figure 3) for each of these m-exponents in order to obtain the associated G_0 correlation and R^2-value. The x-value is obtained from the best-fit power curve relationship and the n-value is produced by multiplying the assumed m-exponent and resulting x-exponent using Equation 14. Table 1 summarizes the results of assuming various m-exponents (inputs) to obtain the optimal site-specific G_0 correlation.

Several empirical correlations for estimating the small-strain shear modulus are currently available. Knappett et al. (2010) presents an empirical equation for estimating G_0 for sands, which is based on the void ratio and mean effective overburden pressure. McGann et al. (2015) presents an empirical equation for estimating the V_s profile based on CPT_u data and is applicable for use in the Christchurch region.

Figure 5 presents the small-strain shear modulus, shear wave velocity and modified normalized small-strain rigidity index plots for the Christchurch site. A comparison between the $SCPT_u$ data and the empirical methods for estimating G_0 and V_s using the McGann et al. (2015), Knappett et al. (2010) and the proposed methodology are also shown in Figure 5. It can be seen that all methods are producing reasonable estimates for G_0 in the clays, silts and sands present in the upper 17.5m of the soil profile. The proposed methodology has the additional benefit of being able to capture unique characteristics of the V_s profile, such as the stiffness contrast observed in the dense sandy gravel layer located below 17.5m depth. As the soil in the upper 1m is above the groundwater table, the soil is unlikely to be completely saturated; therefore, the Knappett et al. (2010) and proposed methods cannot be utilised to estimate G_0 until the soil is completely saturated.

It can be seen that the K^*_G values based on the $SCPT_u$ data and the proposed methodology exceed 330 in the dense gravel layer. Based on this, the dense sandy gravel layer has some significant microstructure (Robertson 2016).

It is expected that the site-specific G_0 correlation developed for this site will be suitable for use on other similar sites; however, it is unlikely to be appropriate for use on other sites where there is no dense gravel layer with significant microstructure. The proposed methodology is expected to be appropriate for estimating the empirical parameters (A, m and x) presented in Equation 11 for any site with seismic geophysical testing and a shallow water table. This is investigated further in an accompanying technical paper, which attempts to develop and optimize a regional G_0 correlation based on $SCPT_u$ data.

The percentage residual error plot shown in Figure 5 confirms that all methods of estimating V_s are performing well for the loose/soft soil in the upper 17.5m of the soil profile. On average, residual errors of less than 10% are produced by all methods for the fully saturated soil located in the upper 17.5m of the soil profile. Excluding the dense gravel layer and soil located above the groundwater table, there are very few instances where the methods are producing residual errors in excess of 20%, and the largest errors are generally produced near significant transition layers in the soil profile. The correlation produced using the proposed method is also producing V_s estimates with residual errors of less than 20% in the dense sandy gravel layer; whereas, the residual errors increase up to 40-50% at this depth for the other two correlations.

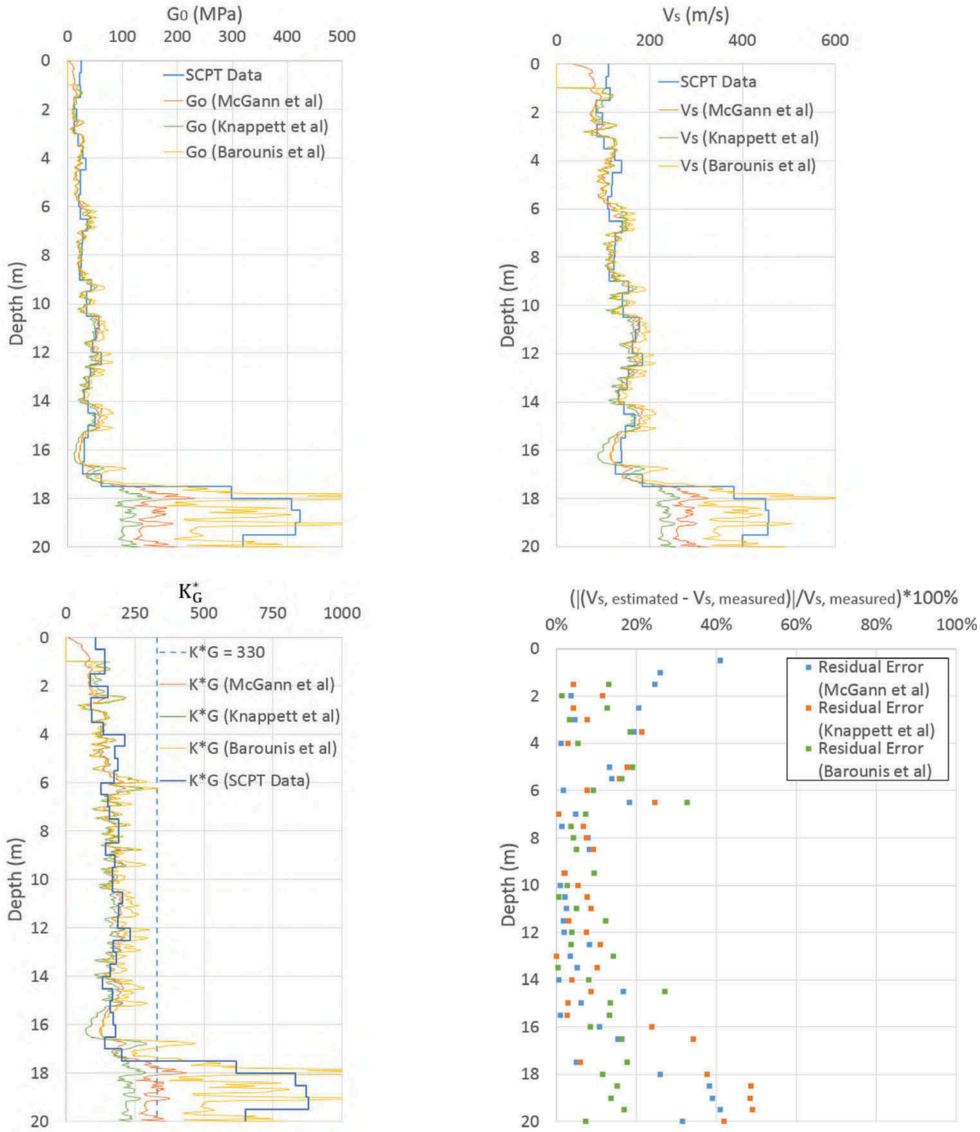

Figure 5. Small-strain shear modulus (top left), small-strain shear wave velocity (top right), modified normalized small-strain rigidity index (bottom left), and percentage residual error (bottom right) plots for the site.

5 CONCLUSION

This paper has presented a methodology for developing and optimizing site-specific small-strain shear modulus correlations. The method has been applied to a site in Christchurch, New Zealand, with results showing that the method is accurately estimating the G_0 profile as compared to measurements obtained from the $SCPT_u$ data. The site has a unique characteristic, which is a dense sandy gravel layer that has significant microstructure located below 17.5m depth. The proposed method is accurately estimating the G_0 profile and is able to capture the stiffness contrast in the dense sandy gravel layer that has significant microstructure. Based on this, it is expected that the site-specific G_0 correlation developed for this site will be suitable for use on other similar sites; however, it is unlikely to be appropriate for use on other sites where there are no dense sandy gravel layers with significant microstructure.

The method could be modified and extended to also make use of DMT data, or instead use SDMT as the basis for developing the G_0 correlation; however, further research is required to validate this.

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