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Comparison of post-liquefaction settlements at a liquefaction test site considering numerical and empirical methods

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ABSTRACT: Assessment of earthquake-induced liquefaction is an important topic in geotechnical engineering due to the significant potential for damage to infrastructure. Assessments of liquefaction triggering and estimation of post-liquefaction deformations is commonly done using empirical models, which often assume laterally homogeneous soil layers. Numerical models offer the potential to examine the effects of spatially variable properties on liquefaction-induced deformations, but this approach is not common in practice. This study compares the predictions of post-liquefaction reconsolidation settlement of a spatially variable site using empirical and numerical approaches. The numerical model is first calibrated usingan empirical model and uniform soil profiles. The recalibrated numerical model is then used to model the response of a soil site with spatial variabilityin Hollywood, South Carolina. Numerical analyses are performed using 2D profiles to examine the effects of excess pore pressure diffusion on the predicted settlements. Implications of the findings for practice are discussed.

1 INTRODUCTION

Liquefaction of granular soils may lead to large reconsolidation settlements as the excess pore-pressures generated during a seismic event dissipate within partially- or fully-liquefied layers. This settlement can cause severe damage to superstructures and buried infrastructure, especially if significant differential settlement occurs across a site. Previous researchers have developed empirical models based on the results of laboratory experiments and field observations to estimate liquefaction-induced settlements in the free field (e.g., Tokimatsu & Seed 1987, Ishihara & Yoshimine 1992, Sento et al. 2004, Yoshimine et al. 2006, Idriss & Boulanger 2008). These models often use profiles of penetration resistances from standard penetration tests (SPTs) or cone penetration tests (CPTs) and estimates of earthquake loading to predict the shear-induced volumetric strains that will occur in a soil profile. These one-dimensional models assume the soil to be laterally homogeneous, which represents a significant simplification for most sites. Numerical models offer a means to consider the effects of spatial variability (e.g., Popescu et al. 1997, Montgomery & Boulanger 2017) on the magnitude of reconsolidation settlements, but these approaches are not commonly used in practice and few comparisons of numerical and empirical estimates for reconsolidation settlement have been performed.

In this paper, numerical models are used to simulate the response of two-dimensional (2D) cross-sections of a site in Hollywood, South Carolina, USA using the finite difference software FLAC (v7.0; Itasca 2014). A number of CPT tests, downhole shear wave velocity tests, and mud-rotary borings have previously been performed at the site (Stuedlein et al. 2016, Gianella & Stuedlein 2017). This extensive characterization program provided good estimates of spatial variability at the site, which was examined by Bong & Stuedlein (2018) to estimate the magnitude of liquefaction-induced differential settlement that would be expected at the site using empirical

models. This paper will extend the work performed in this previous study by using numerical simulations with the constitutive model PM4Sand. Early analyses showed that the default calibration of PM4Sand produced settlements that were approximately 8-10 times smaller than the empirical settlement estimates, so the calibration was modified to bring these results into better agreement. The recalibrated model was then used to simulate the response of the Hollywood site to a hypothetical earthquake. Comparisons were made between the empirical and numerical settlement results. The results showed that numerical models tended to predict smaller settlements due to non-uniform strains within the columns. The 2D numerical results also predicted smaller differential settlements than the empirical models, likely due to pore pressure diffusion (or redistribution) during shaking. The implications of these findings for practice are discussed.

2 HOLLYWOOD, SC TEST SITE

This study focuses on an experimental geotechnical test site in Hollywood, SC. The Hollywood test site was used to evaluate liquefaction mitigation using driven displacement piles and controlled blasting techniques (Stuedlein et al. 2016; Gianella & Stuedlein 2017), pile spacing and installation effects on driving and penetration resistance (Stuedlein & Gianella 2016), time-dependent regain of small-strain stiffness (Mahvelati et al. 2016; 2018), spatial variability of silty fines (Bong & Stuedlein 2017), and liquefaction-induced settlements (Bong & Stuedlein 2018). The site has been extensively characterized using CPT tests, downhole-and surface wave-based shear wave velocity tests, and mud-rotary borings with split-spoon samples. Figure 1 presents a plan view of the test site showing the locations of the SPT, CPT, and SCPT soundings.

2.1 *Site stratigraphy and geotechnical properties*

The generalized stratigraphy at the Hollywood site is shown in Figure 2. The upper 2.5 m is a loose to medium dense silty or clayey sand fill, which overlies a potentially-liquefiable layer comprised of loose to medium dense poorly graded sand with lenses of silty sand. This layer is the focus of the current study. The liquefiable layer is underlain by a 1.5 m thick soft to medium stiff clay layer which in turn is underlain by a dense sand layer. The thickness of the various layers is generally uniform across the site. The depth to groundwater exhibits seasonal variation, and can be as shallow as 2 m below the ground surface.

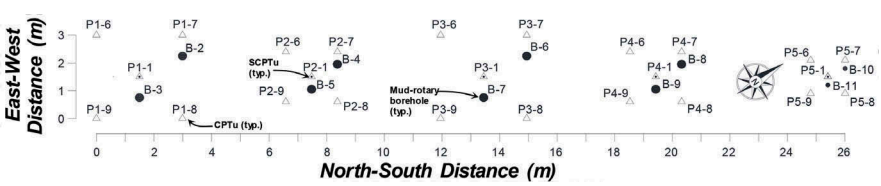

Figure 1. Plan view of the Hollywood test site showing the locations of the various explorations performed (modified from Bong & Stuedlein 2018).

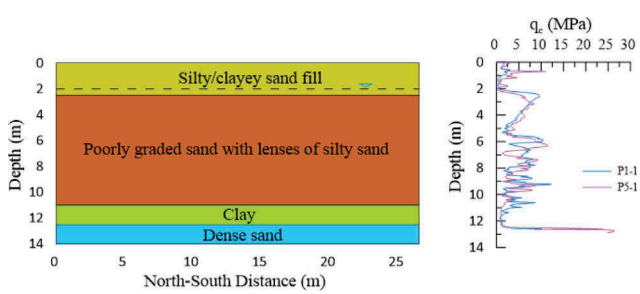

Figure 2. Cross-section of the Hollywood site showing the stratigraphy and selected CPT profiles.

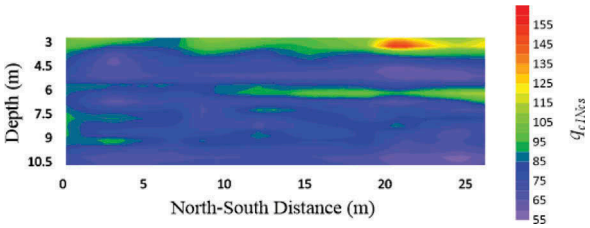

Figure 3. Contours of q_{c1Ncs} for a 2D soil section at 0.25 m distance in E-W direction (2nd section).

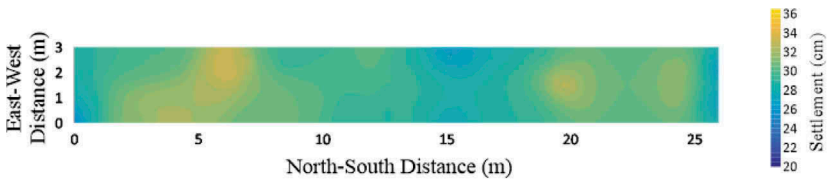

Figure 4. Contours of ground surface settlements for the Hollywood site estimated using the empirical model proposed by Ishihara & Yoshimine (1992) and Yoshimine et al. (2006) for *PGA* of 0.15g.

2.2 *Empirical estimates of liquefaction-induced settlement*

Settlements were estimated for the Hollywood site using the empirical approach recommended by Ishihara & Yoshimine (1992) and updated by Yoshimine et al. (2006). This method uses the factor of safety against liquefaction, maximum shear strain, and penetration resistance (or D_r) as the basis for computation of the volumetric strain of discretized soil layers. Integration of the volumetric strains over the discretized layer thicknesses yield the settlement expected at the ground surface. For this study, the CPT-based liquefaction triggering framework outlined by Idriss & Boulanger (2008) was used to estimate the factor of safety against liquefaction.

The analysis of settlement is commonly performed using the soundings available for a given site. In an alternate approach, Bong & Stuedlein (2017) used random field modeling to produce a three-dimensional geostatistical model of the corrected cone tip resistance (q_{c1Ncs}) and fines content within the liquefiable layer at the test site. The geostatistical model was discretized at intervals of 5 cm vertically with depth (from 2.5 m to 11 m) and 25 cm in horizontal directions (from 0 m to 3 m in E-W direction and 0 m to 26 m in N-S direction). A cross-section through the 3D model at 0.25 m (East-West direction) is shown in Figure 3 as an example of the spatial variability in penetration resistance at the site. The current study uses a 2D numerical approach, and the original fields were averaged to produce grids with a 50 cm spacing in the vertical direction and a 50 cm spacing in the N-S direction. This averaging was done in order to reduce the number of elements in the simulations and decrease the time required for each run. The 13 cross-sections in the E-W direction were analyzed numerically and using the empirical approach discussed above.

Empirical settlements were estimated for the test site for an earthquake with a moment magnitude (M_w) of 7.0 and peak ground acceleration (*PGA*) of 0.15g (Figure 4). The results indicate that the site is expected to experience a maximum settlement of 34 cm and a minimum settlement of 24 cm, with a corresponding average of 30 cm and maximum differential settlement of 10 cm across the site. These results are slightly different from the estimates by Bong & Stuedlein (2018) due to the averaging and the slightly higher *PGA* used in this study.

3 NUMERICAL MODELING OF LIQUEFACTION AT HOLLYWOOD, SC

Numerical simulations were performed to investigate the effects of ground motion variability and diffusion of excess pore pressures within the spatially variable deposit on the estimated settlement. The simulations were performed using the commercial finite difference software FLAC (v7.0; Itasca 2014). The simulations used 2D sections running along the N-S direction (Figure 2) with a

model width of 26 m and a total depth of 14 m. The simulations were performed in three stages. In the first stage, the model was brought to static equilibrium with the base fixed against movement and only vertical movements along the sides of the model. The water table was fixed at a depth of 2 meters and no flow was allowed through the sides or base of the model. In the second stage of the analysis, earthquake shaking is applied to the base of the model as a horizontal acceleration time history. During this stage, the base of the model is fixed against vertical movement and periodic boundary conditions are used on the sides. In the final stage, the model is allowed to reconsolidate with the same boundary conditions as static equilibrium.

3.1 *Soil properties*

The fill layer, liquefiable layer, and the dense sand layer at the model base were modeled using PM4Sand (Boulanger & Ziotopoulou 2015). PM4Sand is a bounding surface plasticity model which is able to model the cyclic behavior of sandy soils, including both the cyclic mobility and reconsolidation phases of liquefaction. This constitutive model has previously been used to examine the effects of spatial variability on the liquefaction-induced deformations of gently sloping ground (Montgomery & Boulanger 2017) and earth dams (Boulanger & Montgomery 2016). PM4Sand requires three main input parameters. The first parameter is the apparent relative density (D_r) which can be estimated based on q_{c1Ncs} using equation 1 (Idriss & Boulanger 2008):

$$D_r = 0.465\left(\frac{q_{c1Ncs}}{0.9}\right)^{0.264} - 1.063 \tag{1}$$

The second parameter is the shear modulus coefficient (G_o) which is estimated based on the stress normalized shear wave velocity (V_{s1}) of the soil. The final parameter is the contraction rate parameter (h_{po}) which must be calibrated using single element direct simple shear simulations to obtain the desired cyclic strength. In this study, the desired cyclic resistance ratio was calculated using the CPT-based triggering relationship proposed by Idriss & Boulanger (2008). An upper limit of 0.8 was put on the triggering curve for very dense soils.

The properties of the liquefiable sand layer were represented using the random fields of q_{c1Ncs} values described in Section 2.2 (e.g., Figure 3). The D_r was calculated from the q_{c1Ncs} value using Equation 1 and V_{s1} was correlated to the q_{c1Ncs} using Equation 2. This equation was developed by adjusting the default PM4Sand correlation between G_o and D_r to better match the average V_{s1} measured at the Hollywood site (Mahvelati et al. 2016).

$$V_{s1} = 45.5\,(q_{c1Ncs})^{0.3244} \tag{2}$$

The final parameter (h_{po}) was obtained by performing single element direct simple shear simulations in FLAC for different q_{c1Ncs} values and adjusting h_{po} to achieve the desired cyclic strength. This process was repeated for 34 values of q_{c1Ncs} ranging from 21 to 254, which covers the range of q_{c1Ncs} values in the random fields. Values of h_{po} for intermediate q_{c1Ncs} values were obtained through linear interpolation. The permeability for each zone of the liquefiable layer was calculated using the geostatistical model of the fines content developed by Bong & Stuedlein (2017) and a modified version of the correlation proposed by Rawls & Brakensiek (1985). The original relationship was scaled down by two orders of magnitude, so that the average permeability for the liquefiable layer had a similar value to that reported by Stuedlein & Gianella (2016).

All other layers in model were assumed to have uniform properties in order to isolate the effects of spatial variability in the liquefiable layer. The properties for the uniform layers are shown in Table 1. Most of these properties were provided by Stuedlein & Gianella (2016) and typical values were assumed for those that weren't available. The clay layer was modeled using an elastic-plastic-Mohr-Coulomb constitutive model. The layer was assigned an undrained strength with a friction angle of zero and a shear wave velocity of 168 m/s. The fill and dense sand layers were modeled using PM4Sand. Uniform relative densities of 80% and 90% were used for the fill and dense sand layers, respectively, based on the CPT results in these layers. G_o for these layers was estimated

Table 1. Soil properties for the uniform layers.

Layer	Input properties Cohesion (kPa)	Friction angle (deg)	Dry density (kg/m^3)	D_r	q_{c1Ncs}	Maximum shear modulus (kPa)	Permeability (cm/s)
Fill	-	-	1712	0.8	174	1.1x10^5	0.01
Clay	38	0	1800	-	-	5x10^4	0.0001
Dense sand	-	-	1745	0.9	211	1.2x10^5	0.01

Table 2. Characteristics of recorded earthquake motions.

Earthquake Motion	Year	M_w	Duration (s)	CAV$_5$ (m/s) PGA = 0.15g	PGA = 0.25g
1. Imperial Valley-06	1979	6.53	63.7	15.03	25.34
2. Duzce, Turkey	1999	7.14	41.5	5.46	9.28
3. Taiwan SMART1(45)	1986	7.30	32.9	5.89	10.02
4. Helena, Montana-01	1935	6.00	40.0	0.93	1.70
5. Victoria, Mexico	1980	6.33	24.4	2.07	3.68
6. Duzce, Turkey	1999	7.14	28.83	4.89	8.36
7. Sitka, Alaska	1972	7.68	55.0	5.87	10.23

using correlations from Boulanger & Ziotopoulou (2015). The permeability of the sandy layers was equal to the liquefiable layer while the permeability of the clay layer was 100 times less.

3.2 *Input motions*

Seven input motions were selected for use in this study. These seven motions were chosen from the suite of motions recommended by Jayaram et al. (2011) to approximate a strike-slip event with a moment magnitude (M_w) of 7 at distance of 10 km. These seven motions were recorded at rock sites and have a range of spectral shape. Table 2 lists some of the important characteristics of the selected records. Each motion was linearly scaled to match the desired PGA.

3.3 *Calibration for reconsolidation settlements*

Reconsolidation strains following liquefaction are difficult to model with traditional elasto-plastic constitutive models as a large portion of the volumetric strains is due to sedimentation effects which are not captured in these models. PM4Sand approximates the reconsolidation process using a pragmatic approach in which the elastic moduli are reduced after shaking to compensate for the fact that sedimentation is not modeled (Ziotopoulou & Boulanger 2013). This process is controlled by two dimensionless parameters, $f_{sed,min}$ and $p'_{sed,o}$, which control the magnitude of modulus reduction and the range of mean effective stresses over which the reduction is active, respectively. An increase in $p'_{sed,o}$ (while keeping $f_{sed,min}$ constant) decreases the settlement, whereas a decrease in $f_{sed,min}$ (at constant $p'_{sed,o}$) increases the settlement. During early analyses, it was observed that the default calibration produced numerical settlement values that were about 10 times smaller than those calculated using the empirical models (Figure 5). Thus, to obtain a better match between the numerical and empirical settlements the numerical model was recalibrated. It is noted that the observed post-liquefaction deformations induced by controlled blasting ranged from 15 to 20 cm per liquefaction event (Gianella & Stuedlein 2017), which is of the same order as those computed using empirical methods.

The reconsolidation parameters were recalibrated by simulating the response of 1D soil columns with uniform layer properties. The soil columns consisted of two layers: a liquefiable sand layer overlain by a sandy crust having a high relative density (non-liquefiable). The water table depth was 2 m below the ground surface, implying a fully saturated liquefiable deposit. The simulations were performed for five D_r values ranging from 40% to 80%. These simulations were performed using the default calibration first and with both the fault normal and fault parallel components of the seven ground motions listed in Table 2. These initial analyses showed that the

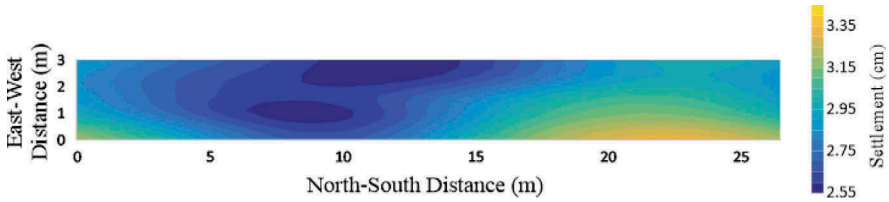

Figure 5. Contours of ground surface settlement for Hollywood site estimated numerically using default PM4Sand parameters with analysis performed using the Motion 6 having a *PGA* of 0.15g.

fault normal component of Motion 6 produced approximately median reconsolidation settlements, so this motion was used in subsequent calibrations. Each of the reconsolidation parameters was adjusted until the simulations gave approximately similar results to the empirical model across the full range of D_r for *PGA*s of 0.15g and 0.3g. The value of $p'_{sed,o}$ was modified to $-P_a$ *2 (P_a = 101.3kPa, atmospheric pressure), while $f_{sed,min}$ was found to depend on D_r (Equation 3).

$$f_{sed,\min} = 3 \times 10^{-5} (D_r)^{1.4262} \tag{3}$$

4 NUMERICAL SIMULATION RESULTS

4.1 *Effect of spatially-variable soil on settlement*

FLAC analysis were performed for each of the 2D soil sections of Hollywood using Motion 6 with a *PGA* of 0.15g. The adjusted calibration for the reconsolidation parameters was used. Figure 6 shows the settlement at ground surface for the recalibrated numerical model. Although the numerically-simulated settlements are smaller than the empirical settlements at all locations (comparing Figures 4 and 6), the average settlement is in greater agreement with the empirical results as compared to the default calibration (Figure 5). This agreement could be likely be improved with further calibration of the reconsolidation parameters. One significant difference between the empirical model and the numerical results is in terms of differential settlement. The numerical results indicate that the site is expected to experience a maximum and minimum settlement of 23.3 cm and 19.8 cm, respectively. This corresponds to a maximum differential settlement of 3.5 cm. This differential settlement is less than half of that predicted by the empirical model. This difference is attributed primarily to diffusion of excess pore pressures between the looser and denser elements in the numerical model. Diffusion is not accounted for in empirical models, which considers settlement potential of each sub-layer independently.

4.2 *Effect of ground motion variability on settlement*

The analysis described above was repeated for the section shown in Figure 3 with all of the ground motions considered (Table 2), and PGAs of 0.15g and 0.25g. Figure 7 compares the settlement profiles for each of the motions to the empirical estimates. A large variation in settlement can be observed between the different motions, which is much larger than differences due to differential settlement of the spatially-variable soil. The variation in settlement due to the ground motion is due to the large variation in motion characteristics, such as the moment magnitude, duration, and frequency content. For example, the duration of the Motion 4 is too short to generate significant shear strains or excess pore pressures, thereby limiting the loss of stiffness and reconsolidation strain (Lee & Albaisa 1974). The empirical model only considers the scalar quantities *PGA* and M_w, and therefore cannot account for the differences in other motion characteristics.

For almost all of the motions, increasing the motion intensity results in an increase in the average magnitude of settlement. Motion 2 shows a slight decrease in settlement for the higher intensity, which is attributed to negative excess pore pressures that developed in some of the zones at the end of shaking for the higher intensity motion. At the lower intensities, the numerical simulations under-predict the empirical estimates for all of the motions considered. At higher intensities,

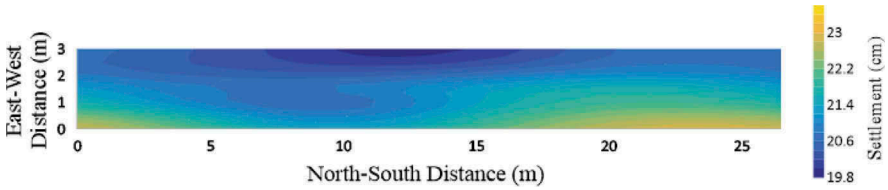

Figure 6. Contours of numerically-simulated ground surface settlement for the Hollywood site using the modified PM4Sand reconsolidation parameters andMotion 6scaled to *PGA* of 0.15g.

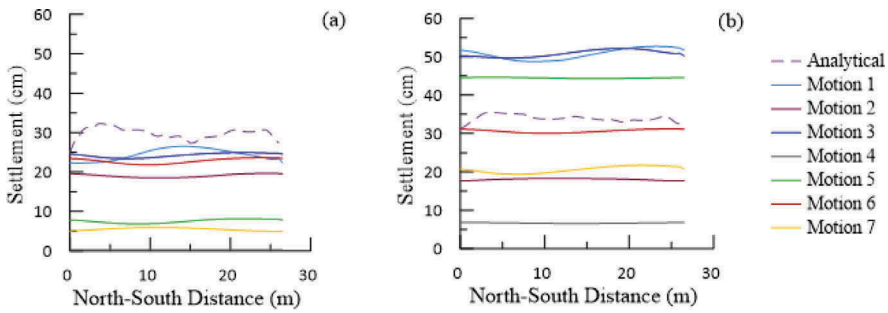

Figure 7. Numerical settlementsand analytical settlements at ground surface for soil section at 0.25 m distance in E-W direction (2nd section) for motions with *PGA* of (a) 0.15gand (b) 0.25g.

three of the input motions produce higher settlement estimates than the empirical models. The empirical settlements at the two intensities are similar because the model has a limiting shear strain magnitude (i.e., 8%) which acts as a cap on the maximum settlement. The numerical simulations and empirical model tend to show less differential settlement at higher intensities due to more uniform liquefaction across the model. For the numerical models, this results in relatively uniform settlements. Similar observations were made by Montgomery & Boulanger (2017) and Bong & Stuedlein (2018), who found that variability in volumetric strains across a spatially variable deposit tended to increase with decreasing input motion intensity due to the likelihood of local liquefaction in looser zones.

5 DISCUSSION AND CONCLUSIONS

A study to estimate post-liquefaction settlement for a test site in Hollywood, SC using numerical and empirical models was presented in this paper. The reconsolidation behavior of the constitutive model PM4Sand (Boulanger & Ziotopoulou 2015) was recalibrated to obtain a better match between the numerical simulations and the empirical models by Ishihara & Yoshimine (1992) and Yoshimine et al. (2006). The results from the recalibrated model are in better agreement with the empirical models and the observations of Gianella & Stuedlein (2017), but this agreement depends on the characteristics of the input motion. The numerical results exhibit less differential settlement than the empirical models, which is attributed primarily to diffusion of excess pore pressures between looser and denser elements within the model. Increasing the ground motion intensity increases the magnitude of settlement, but generally reduces the amount of differential settlement as the generation of excess pore pressure becomes more uniform across the model.

The results presented in this study highlight two important considerations when using empirical models to estimate liquefaction-induced settlements. The empirical model used in this study uses only the *PGA* and magnitude of the earthquake to characterize the shaking intensity. This ignores other important ground motion characteristics, which may affect the numerical results. The second consideration is that the inherent 1D assumptions in empirical models may ignore the effects of pore pressure diffusion leading to estimates of differential settlement that exceed those

produced in consideration of redistributed excess pore pressures. While this may be conservative for the purposes of design, it is important to recognize this potential limitation when comparing the results of empirical models to case histories or when using these models in risk assessments where best estimates of deformations are needed. Additional work is needed to identify case histories or develop physical models where spatial variability in properties may have affected the response of the soil in order to validate simulations such as those described herein.

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