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Steady state testing of shallow alluvial Christchurch silty soils

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ABSTRACT: State-of-practice liquefaction assessment procedures captured the post-earthquake liquefaction observations made in most areas of Christchurch, New Zealand after the 2010-2011 Canterbury earthquake sequence. However, there were also cases where state-of-practice procedures indicated that significant liquefaction would occur, yet no surface manifestations were observed. These no-liquefaction case histories are concentrated primarily in southwest Christchurch, an area characterized predominantly by alluvial silty soils. Cyclic triaxial testing was performed on high-quality ("undisturbed") specimens retrieved from several sites to evaluate the cyclic response of these soils. Steady state testing was then conducted on reconstituted specimens prepared using soil from the cyclic test specimens. This paper presents laboratory data and steady state lines for three silty soil units. Steady state response findings are compared with cyclic response observations. The in-situ states of the tested soils are estimated, and the use of reconstituted specimens to represent layered alluvial soils is critiqued.

1 INTRODUCTION

1.1 *Background and motivation*

Liquefaction devastated major parts of the city of Christchurch, New Zealand during the 2010-2011 Canterbury earthquake sequence. There were many sites where state-of-practice liquefaction triggering procedures indicated liquefaction would be expected and liquefaction manifestations were observed during post-earthquake reconnaissance. However, there were also many sites, which were predominantly silty soil sites, where state-of-practice methods indicated liquefaction would be expected, but no surface manifestations of liquefaction were observed (e.g. no ejecta, no cracking, nor differential ground settlement). This discrepancy between state-of-practice liquefaction estimations and post-earthquake liquefaction observations led to the development of a research program aimed at investigating the liquefaction response and characteristics of the silty soil sites that did not manifest liquefaction at the ground surface.

Initial research efforts at the silty soil sites consisted of field investigations to characterize subsurface conditions and laboratory testing to estimate cyclic resistance of the potentially liquefiable soils. The field investigations, cyclic testing, and liquefaction assessments are presented in previous publications (Beyzaei et al. 2018a, b, Stringer et al. 2015) and are summarized briefly in the following section. Following the field investigations and cyclic testing program, steady state testing was performed to supplement the cyclic response characterization of potentially liquefiable layers at selected sites. This paper presents results from the steady state testing program and provides relevant details from the field investigation and cyclic testing which inform the interpretation of the results of the steady state testing program. Observations from the earlier field and laboratory investigations are also compared with the steady state response findings.

1.2 *Field investigation and cyclic testing program*

Subsurface investigations at the silty soil sites consisted of cone penetration tests (CPTs), sonic borings, piezometer installation, and rotary-wash borings with high-quality ("undisturbed") sampling (Beyzaei et al. 2018a). Figure 1 presents the CPT profiles for tip resistance (q_t) and soil behavior type index (I_c) at the sites selected for steady state testing (referred to as Sites 2, 14, and 33).

The high-quality ("undisturbed") samples obtained at each site were used for cyclic triaxial (CTX) testing performed at the University of Canterbury Geomechanics Laboratory (Beyzaei et al. 2018b, Stringer et al. 2015). Conventional laboratory testing was also performed for the CTX specimens (e.g. particle size distributions, index testing, minimum/maximum void ratio, and specific gravity tests). The cyclic testing program established the laboratory-based characterization of cyclic response and cyclic resistance curves for soils at the silty soil sites. Results of the CTX testing program demonstrated cyclic resistance consistent with state-of-practice method estimates, axial strain development corresponding to cyclic mobility with limited strain response, and time-dependent post-liquefaction reconsolidation.

2 STEADY STATE TESTING PROGRAM

2.1 *Representative soil mixtures*

Soil layers assessed to have liquefied during the Canterbury earthquake sequence using state-of-practice procedures (Beyzaei et al. 2018a) were selected for characterization through steady state testing. The tested soils were primarily non-plastic silty sands from layers with minimal stratification observed in the in-situ deposit, so that the homogeneous laboratory-prepared specimens would more closely approximate the in-situ deposit. Similar CTX specimens were combined to create representative soil mixtures for liquefiable soil layers at sites 2, 14, and 33. Particle size distributions, index parameters, CPT data, and observed cyclic response were considered in selecting the CTX specimens to be combined in representative soil mixtures. Steady state testing was then conducted on reconstituted specimens prepared using the representative soil mixtures. The specimens combined for the soil mixture at each site are:

– Site 2 soil mixture: specimens S2-DM1-7U-A and S2-DM1-7U-B
– Site 14 soil mixture: specimens S14-DM2-5U-A and S14-DM2-5U-B
– Site 33 soil mixture: specimens S33-DM1-4U-A and S33-DM1-5U-A

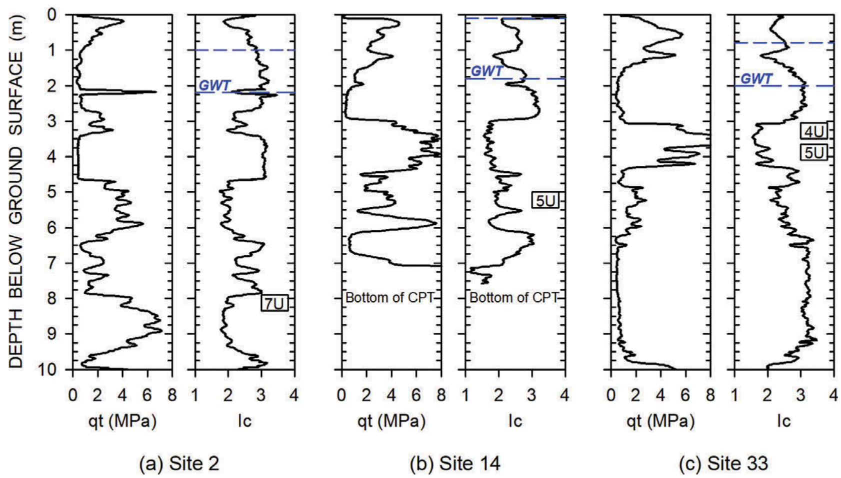

Figure 1. CPT Profiles: Site 2 (CPT_36417), Site 14 (CPT_37818), Site 33 (CPT_36421). CPT numbers refer to New Zealand Geotechnical Database identification numbers (www.nzgd.org.nz). Soil samples used in testing are indicated at the depths retrieved. Estimated groundwater level ranges also shown.

The samples from which CTX specimens were obtained are shown on the CPT profiles in Figure 1 (specimen naming convention is “Site-Boring-Sample-Specimen”). Properties of the individual CTX specimens and resulting soil mixtures are discussed next.

2.2 *Specimen properties: Fines content and void ratio*

Table 1 provides the CTX specimen properties and characteristics of the resulting soil mixtures tested in this study. Nominal CTX specimen height and diameter were 140 mm and 62 mm, respectively. Figure 2 provides particle size distributions for the CTX specimens and a scanning electron microscope (SEM) photograph of Site 33 soil showing the angularity of the sand and silt particles. Fines content (FC) for the CTX specimens was determined primarily using laser diffraction analysis; for the Site 2 specimens, sieve/hydrometer analysis was also performed. Differences in FC estimates from laser vs. sieve analysis were observed for several CTX specimens in the cyclic testing program (Beyzaei et al. 2018b), with the sieve typically providing higher estimates of FC but with no clearly quantifiable trend. For the Site 2 specimen data in Table 1 the variation in FC from laser vs. sieve analysis is approximately 10%. Christchurch silty soils are characterized by angular particles with relatively uniform particle size distributions. Figure 2 illustrates the borderline nature of these angular-shaped fine sand and coarse silt soils, which possess variable estimates of FC and are not likely well-characterized by FC.

After the soils were mixed, a #200 sieve wash was performed to confirm the FC estimate for each mixture. The Site 2 and Site 14 soil mixtures contain sufficient fines such that the fines control behavior of the soil matrix (FC > approximately 35%). The Site 33 soil mixture is in the transitional zone where fines begin controlling response (i.e. FC of 24 to 30%). The nuances of FC estimation for silty soils composed of uniformly graded angular particles should be considered before using a single fines content estimate as representative of a soil mixture.

Minimum (e_{min}) and maximum (e_{max}) void ratio tests were conducted in accordance with the Japanese Standard (JIS, 2015). The standard specifies that the tests are applicable for soils with at least 95% sand (particle diameter = 75 μm to 2 mm), although Cubrinovski & Ishihara (2002) found that the tests were applicable for sands with up to about 30% fines. For comparison, the ASTM Standards for maximum density (ASTM D4253) and minimum density (ASTM D4254) specify that the tests are applicable for soils with up to 15% soil particles passing the #200 sieve (75 μm). Several e_{min} and e_{max} tests were conducted on the selected CTX specimens following the Japanese Standard prior to mixing soils. After combining the CTX specimen soil, additional

Table 1. Cyclic triaxial specimen properties and characteristics of the resulting soil mixtures.

Specimen	Mid-Depth (m)	FC, laser (%)	FC, sieve (%)	FC^* mix, sieve (%)	G_s	e_{min}^{**}	e_{max}^{**}	e_{min}^*, mix	e_{max}^*, mix
S2-DM1-7U-A	8.07	30	41	39	—	0.61	1.30	0.59	1.31
S2-DM1-7U-B	8.22	43	54		2.69	0.60, 0.667	1.34, 1.371		
S14-DM2-5U-A	5.17	47	—	44	—	0.60, 0.634	1.37 1.395	0.60	1.34
S14-DM2-5U-B	5.32	36	—		—	0.62	1.33		
S33-DM1-4U-A	3.22	11	—	24,30	2.68	0.596	1.298	0.63	1.26
S33-DM1-5U-A	3.82	10	—		2.69	0.69, 0.684	1.25, 1.321		

Notes: * Value listed for the soil mixture.
** Specimens with two values listed had two sets of testing conducted by different testers.

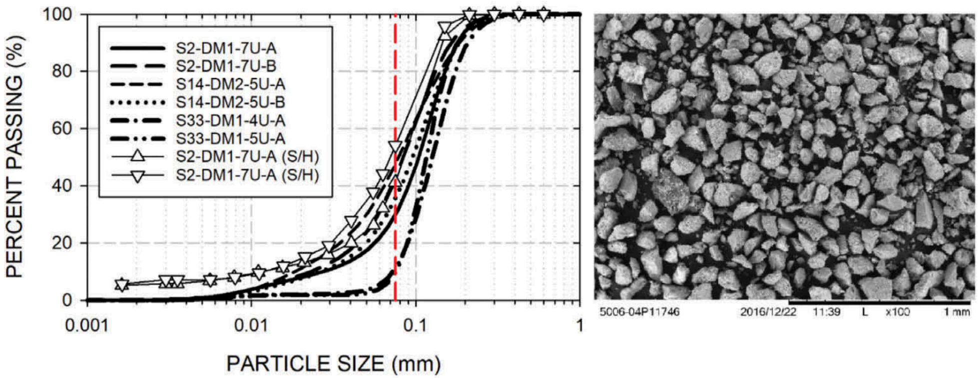

Figure 2. Left: Particle size distributions for CTX specimens (“S/H” indicates sieve/hydrometer analysis, other curves obtained using laser diffraction analysis). Right: SEM photograph illustrating particle angularity.

testing was performed for the resulting soil mixtures. The measured values of e_{min} and e_{max} for these silty sands with high FC enable relative densities of the test specimens to be estimated (in an approximate manner) and compared with the results of the steady state testing and CTX testing. They are also used to estimate in-situ relative densities of the soils presented later in the paper. Void ratio test data are summarized in Table 1.

2.3 *Specimen preparation and testing procedure*

Reconstituted specimens were prepared using moist tamping (with compaction water content of 8%) at a nominal height and diameter of 150 mm and 70.9 mm, respectively. Lubricated end caps were used for all specimens. Membranes were applied to the lucite end caps with a thin layer of vacuum grease between the membrane and face of the end cap. The end caps contained brass-sintered coarse porous stones. For some tests, wetted filter paper was placed over the porous stones to prevent fines loss during specimen preparation, saturation, and consolidation. Specimens were sheared using a Wykeham Farrance testing apparatus at a rate of 1% axial strain per minute. All tests were performed on saturated specimens in an undrained loading.

The Site 2 and Site 14 soil mixtures contain significant amounts of fines and the reconstituted specimens proved difficult to saturate using the procedures developed for reconstituted specimens of sand (e.g. flushing with deaired water followed by backpressure saturation). System compliance issues may have limited the maximum B-value that could be achieved during testing. B-values were observed to reach maximum values of B ~ 0.94-0.95, possibly due to chamber pressure leaking through the piston seal and the length of tubing between the transducer and the drain line ports on the triaxial testing cell. Site 2 - Test 3 reached a B-value of only ~0.76 before reaching maximum chamber and back pressures; however, it is possible that specimen saturation was higher than the B-value indicated, due to issues with system compliance and considering that typical house pressure fluctuations were not observed in the effective stress recorded during undrained loading. Additionally, Tsukamoto (2017) presents triaxial compression data with results that were relatively insensitive to B-values from ~0.7-1.0. Based on these considerations, the test was included in the overall results.

3 RESULTS AND DISCUSSION

3.1 *Testing data and interpretation to develop steady state lines*

Steady state testing data are summarized in Table 2. Figure 3 presents representative steady state testing data demonstrating the undrained response of laboratory-prepared specimens using soil mixtures from the silty soil sites. Data were corrected for membrane effects and area effects (Duncan & Seed 1967), assuming the specimen deforms as a right cylinder during shearing. All specimens tested approached steady state from the contractive side. For the three sites considered in this paper, steady state effective stress ratios (σ'_1/σ'_3) range from 3.2-3.3, corresponding to a friction angle of 32 degrees. Figure 4 presents the steady state lines (SSL) developed for the Site 2, Site 14, and Site 33 soils. The Cubrinovski & Ishihara (2000) form of the steady state line was used to define the fitting parameters shown in Equation 1:

$$e_{ss} = e_{10-ss} + \lambda(1 - \log_{10} p'_{ss}) \quad (1)$$

where e_{ss} = steady state void ratio; p'_{ss} = steady state mean effective stress; and fitting parameters $e_{10\text{-}ss}$ = steady state void ratio at p'_{ss} = 10 kPa, and λ = slope of the fitted line in semi-log space. Equation 1 is valid for $p'_{ss} \geq 10$ kPa. Cubrinovski & Ishihara (2000) found that the SSL becomes relatively flat for $p'_{ss} \approx 0$-10 kPa so the value of e_{ss} at $p'_{ss} \approx 0$ kPa is estimated on the e vs. p' arithmetic plot. Cubrinovski & Ishihara (1998) showed that at void ratios above or equal to the void ratio on the SSL at $p'_{ss} \approx 0$ kPa, the effective stress and shear strength drop to zero on undrained shearing, defining the initial horizontal part of the "UR-line". Specimens with initial states at or above the "UR-line" exhibit zero residual strength. The Site 33 – Test 1 data point is estimated to be approximately at the UR-line; and projecting a horizontal line from the data point at p'_{ss} = 0 kPa to p' = 10 kPa aligns well with the Site 33 SSL for p'_{ss} > 10 kPa.

The steady state testing of Christchurch sands by Rees (2010), Taylor (2015), and Markham (2015) provides context for the data presented in Figures 3 and 4. The soils tested in those studies were sands with fines contents typically ranging from about 0% to 30%, as compared with the higher FC soils tested in this study. As noted previously (e.g. Markham et al. 2018), the vertical position of the SSL in Figure 4 shifts downwards with increasing FC. The slope of the SSL also

Table 2. Summary of steady state testing on reconstituted specimens.

Test	B-value	e_0	p'_0 (kPa)	e_{ss}	p'_{ss} (kPa)	Estimated D_r (%)
Site 2 – Test 1	0.96	0.95	170	0.95	7	50
Site 2 – Test 2	0.90	0.89	142	0.89	40	58
Site 2 – Test 3	0.76	0.86	143	0.86	86	63
Site 14 – Test 1	0.95	0.93	167	0.93	13	55
Site 14 – Test 2	0.95	0.87	169	0.87	63	64
Site 14 – Test 3	0.95	0.84	150	0.84	105	68
Site 33 – Test 1	0.94	1.03	171	1.03	1	37
Site 33 – Test 2	0.94	0.99	173	0.99	45	43
Site 33 – Test 3	0.94	0.95	220	0.95	148	49

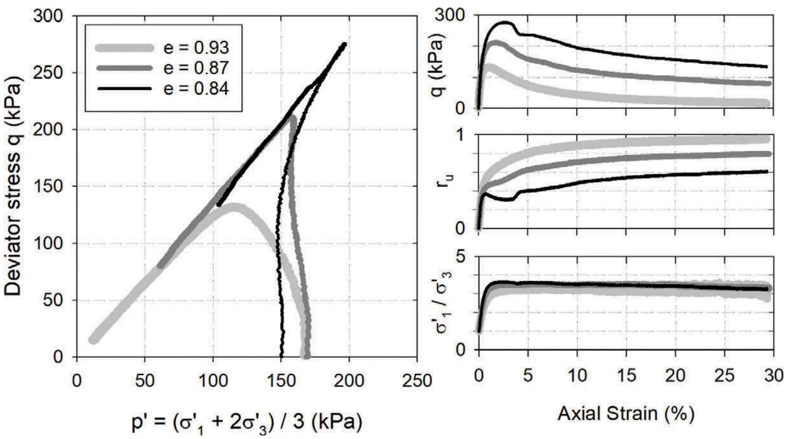

Figure 3. Representative steady state testing data (Site 14 data shown).

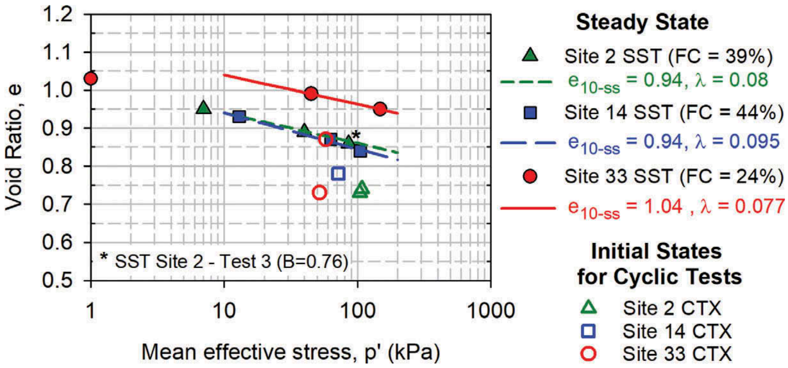

Figure 4. Reconstituted specimen steady-state data (void ratio and mean effective stress at steady state) and SSL, with parameters for the Cubrinovski & Ishihara (2000) form of the SSL. Initial states for the cyclic tests on high-quality "undisturbed" specimens are shown for comparison with the respective SSL. Markham et al. (2018) SSL parameters for select Christchurch sands are: "FTG-7 Lower Sand" (FC = 6%), $e_{10\text{-}ss}$ = 0.97, λ = 0.08; "FTG-7 Upper SM" (FC = 38%), $e_{10\text{-}ss}$ = 0.97, λ = 0.19; "CTH Lower Sand" (FC = 6%), $e_{10\text{-}ss}$ = 1.11, λ = 0.15.

increases slightly with increasing fines, as noted in the literature and Taylor (2015). The lowering of the vertical position with increasing fines content is more apparent than the change in slope. Markham et al. (2018) developed SSL parameters using the same soil mixture approach and testing apparatus as in this study; selected results from his study are provided herein for comparison with the SSL in Figure 4: "FTG-7 Lower Sand" soil (FC = 6%), $e_{10\text{-}ss}$ = 0.97, λ = 0.08; "FTG-7 Upper SM" soil (FC = 38%), $e_{10\text{-}ss}$ = 0.97, λ = 0.19; "CTH Lower Sand" soil (FC = 6%), $e_{10\text{-}ss}$ = 1.11, λ = 0.15. Results from the testing of the Christchurch silty soils compare well with the trends observed in steady state testing of Christchurch sands by Rees (2010), Taylor (2015), and Markham et al. (2018).

3.2 *Insights from comparison with the cyclic triaxial testing program*

Development of the steady state lines allows for further evaluation of the high-quality ("undisturbed") cyclic triaxial testing specimens to supplement the cyclic response observations. The CTX specimen's void ratio and effective stress achieved at the end of consolidation in each cyclic test (i.e. initial states) can be compared with the steady state line to evaluate the dilative or contractive tendency of the specimen that would be expected under monotonic loading. The initial states of the cyclic tests are shown in Figure 4, relative to the steady state lines. State parameters ($\psi = e - e_{ss}$) for the CTX specimens are as follows: Site 2, S2-DM1-7U-A (ψ = -0.13), S2-DM1-7U-B (ψ = -0.12); Site 14, S14-DM2-5U-A (ψ = -0.08), and Site 33, S33-DM1-4U-A (ψ = -0.25), S33-DM1-5U-A (ψ = -0.11). The state parameters indicate that the initial states for the cyclic tests of the CTX specimens are on the dense side of the respective steady state lines and would be expected to show eventually strongly dilative tendencies in monotonic shearing. The cyclic response of these tests reflects their relative state as above and is characterized by axial strain development with limited strain potential. The interpretation of the observed cyclic response of the "undisturbed" test specimens, described in Section 1.2, can be enhanced by the steady state characterization of the soils presented in this paper.

3.3 *Estimated relative density and in-situ conditions*

Steady state testing often refers to laboratory-prepared specimens according to relative density (D_r). However, using D_r for soils with significant fines is problematic because of the segregation of the coarse and fine fractions and differing fine/coarse particle interaction and void space distribution for wet vs. dry soil conditions (i.e. e_{min} and e_{max} testing is conducted on dry soils whereas reconstituted specimens are prepared via moist tamping). Therefore, relative density and void ratio are better used as indices for comparison among tests from each soil mixture, rather than between the soil mixtures from different sites or to in-situ conditions. Despite the drawbacks for soils with appreciable fines, D_r remains a useful parameter that can be estimated without advanced laboratory testing, which allows for preliminary interpretations prior to developing a steady state line and being able to use the state parameter.

Estimated relative densities for the reconstituted steady state testing specimens are given in Table 2. The relative density estimates for the high-quality ("undisturbed") CTX specimens are: Site 2, $D_r \approx 81\%$, 79%; Site 14, $D_r \approx 76\%$; and Site 33, $D_r \approx 84\%$, 62%. The cyclic responses of the CTX specimens suggested dilative tendencies (Beyzaei et al. 2018b), so in order to approach the SSL from the contractive side, reconstituted specimens were prepared to a higher void ratio. Initial tests using reconstituted specimens prepared to lower D_r (higher void ratio) than those shown in Table 2 resulted in steady state tests where the specimen exhibited static liquefaction. Taylor (2015) and Markham (2015) note the potential effects of soil fabric and structure on laboratory-prepared specimens as compared with "undisturbed" specimens. Taylor (2015) observes that for soils with high FC (> 30%), the projected end points for "undisturbed" specimens were typically positioned above the SSL established for homogeneous (reconstituted) specimens. This agrees with general understanding of natural vs. laboratory-prepared soils.

In-situ relative density estimates can be obtained using CPT correlations (e.g. Kulhawy & Mayne 1990), although these correlations are truly only applicable for clean sands. Estimates of in-situ D_r at the Site 2, 14, and 33 CTX specimen depths range from approximately 35-40%, 50-60%, and 50-60%, respectively. Converting the D_r estimates to void ratio and comparing with the SSL and CTX specimens, the in-situ estimates appear to underestimate relative density. For Site 2 and Site 14, the in-situ estimates fall on the contractive side of the SSL. This discrepancy may be attributable to the correlation applicability itself, difficulties estimating D_r for soils with appreciable fines, or possible disturbance of the CTX specimens, although the specimens were assessed to be relatively undisturbed (Beyzaei et al. 2018b). Consideration of state parameter ($\psi = e - e_{ss}$) allows for an alternative interpretation of the in-situ estimates and laboratory data. In contrast with relative density, CPT correlations (e.g. Robertson 2010) estimate in-situ state parameter $\psi \leq -0.05$ at the depths of the CTX soil specimens tested in this study, indicating the potential for dilative tendencies at large strains (Jefferies & Been 2006), which corresponds to the observed CTX specimen cyclic response.

4 CONCLUSION

This paper presents steady state characterization for shallow alluvial Christchurch silty soils. Steady state lines are developed for non-plastic silty soil layers at three sites. The trends observed are consistent with those observed in testing of Christchurch sands. The results of the steady state testing program support and enhance the interpretation of specimen response characterized through cyclic triaxial testing, providing important insights regarding the dilative and contractive tendencies of the specimens used for cyclic testing. The collective insight gained from the cyclic and steady state testing programs demonstrates that both should be considered in the overall evaluation and characterization of these non-plastic and low-plasticity silty soils.

Fines content, void ratio, and relative density are discussed, highlighting the difficulty in establishing these parameters to characterize intermediate silty soils. There is a clear need to consider the nuances of fines content, void ratio, and relative density parameters for characterizing these transitional intermediate soils, including particle shape and particle size distribution. Relative density is an imprecise parameter for silty soils and yet is often used in practical applications, including modeling and settlement estimates. Without in-situ void ratio measurements at a site it is difficult to use D_r. Alternatively, the state parameter (ψ) estimates from CPT correlations indicate dilative tendencies that were observed in the CTX specimen cyclic response. Thus, the CPT-based estimates of ψ provide valuable insight in analysis of these silty soil sites.

Subsurface deposits at the three silty soil sites were formed in the alluvial Christchurch environment where layered subsurface conditions lead to in-situ variations in density and fines sequencing that are not replicated with homogeneous laboratory-prepared specimens. The steady state testing program focused on layers with minimal stratification so that the reconstituted specimens would more closely approximate the in-situ deposit, but it is difficult to relate laboratory testing of such soils to a field equivalent. Consideration of fines sequencing and in-situ layering of Christchurch silty soils is critical for improving understanding of those soil deposits. In high-quality ("undisturbed") specimens that maintain in-situ layering, soils within each layer may move toward their own critical state, whereas mixed, homogeneous specimens may move to another critical state for the soil mixture. Minor variations in soil layering may affect the response of silty soils in a way that is not observed in clean sands and reconstituted specimens.

Additional testing for these soils could evaluate the effects of specimen preparation method (e.g. moist tamping vs. pluviation). It would be beneficial to perform steady state testing on high-quality ("undisturbed") specimens obtained from the same layers at these silty soil sites to evaluate the differences in response for naturally deposited vs. reconstituted silty soil specimens and compare steady state lines for undisturbed specimens with those presented in this paper.

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