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Fabric evolution in post-liquefaction and re-liquefaction of granular soils using 3D discrete element modelling

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**ABSTRACT:** Liquefaction refers to dramatic loss of soil stiffness and strength during earthquake shaking. Knowing the particle-scale information of granular particles and void space, collectively termed as “fabric”, is crucial for obtaining a fundamental understanding of liquefaction in granular soils. In this study, fabric evolution of granular soils in cyclic liquefaction is investigated using 3D Discrete Element Method (DEM). A set of new fabric indices are developed for quantifying particle-void distribution ($E_d, A_d$), which are found to have a great correlation with the postliquefaction behaviour, as well as irreversible changes in fabrics developed in liquefaction processes, and finally proposed a fabric-based criterion for jamming transition in flow deformation. The descriptor $E_d$ is found to be a good indicator for the compressibility of the sample in the post-liquefaction stage. DEM simulations have been conducted to identify key factors that influence the granular soil’s resistance to multiple liquefactions. It is observed that re-liquefaction resistance is greatly affected by the fabric anisotropy of the sample by surpassing the effect of relative density. Within an unloading cycle in post-liquefaction stage, completely different soil fabric was formed if the liquefied soil is consolidated at different unloading points, resulting in vastly different resistance to re-liquefaction.

1 **INTRODUCTION**

Cyclic liquefaction refers to dramatic loss of stiffness and strength of soils during earthquake shaking, which causes significant damage to infrastructure (e.g. Ye & Wang 2016). Experimental studies to understand cyclic liquefaction are mostly parametric in nature i.e. the effect of each macroscopic soil parameter on the liquefaction potential is well understood. However, it is difficult to interpret the evolution of internal structure i.e. “fabric”, through such tests. The fabric of granular soils refers to the arrangement of particles, particle group and void space distribution (Mitchell & Soga 2005). Soil fabric has a profound influence on the properties of sands such as small-strain stiffness, permeability, peak strength and liquefaction resistance (Sitar 1983; Fonseca *et al.* 2012). Although the geotechnical community has long known that liquefaction results in significant changes in the load-bearing fabric of granular soils, a clear characterization of these changes has been lacking.

Numerical simulations using Discrete Element Method (DEM) provide a more convenient and less costly way to observe the fabric directly (Rothenburg & Bathurst 1989; Thornton 2000). The DEM is capable of simulating the cyclic soil behaviour and cyclic liquefaction (Ng and Doby 1994; Sitharam *et al.* 2009; Wang & Wei 2016; Wei & Wang 2017). Traditional studies on fabric quantification are mostly based on inter-particle contacts (Satake 1992; Thornton 2000), which are easily implemented in DEM
simulations using contact normal vectors or branch vectors. Contact-based fabric has been extensively used in quantifying load-bearing capacity of the soil under monotonic loading (O’Sullivan 2011). However, fabric quantification of liquefied soils is particularly challenging because a liquefied soil loses nearly all of its contact points, so the contact-based fabric may not be a reliable indicator in such a case. Alternatively, a special type of particle-void fabric descriptors was introduced by constructing the “particle-void cells” using weighted Voronoi tessellation on a 2D granular assembly by Wei et al. (2018). No contact information is required to construct the particle-void cells, hence it can easily tessellate the space and quantify the descriptors even in a liquefied state.

In this current study, the particle-void fabrics introduced by Wei et al. (2018) for 2D assembly is extended to 3D, in order to quantify the particle-void distribution. Using the DEM simulations, fabric evolution and its connection to post-liquefaction flow deformation is studied. Irreversible changes in fabrics developed in liquefaction processes is related to post-liquefaction large deformation. A fabric-based criterion for jamming transition in flow deformation is proposed. Also, the compressibility of samples in the post-liquefaction stage has a clear correlation with the particle-void fabrics developed.

Recent 2011 Tohoku earthquake in Japan and Christchurch earthquake highlighted the importance of studying the re-liquefaction phenomenon, as it was observed that liquefaction occurred repeatedly at the same site under a sequence of earthquake events over a short time. Which is contradictory to the common belief of soil becoming denser after each liquefaction and being less prone to future earthquakes. After the first liquefaction, the soil experiences significant change in its internal structure and an irreversible void fabric is attained even after consolidation. The subsequent liquefaction can be more severe or less severe than the precedent event, yet, the fundamental mechanism has not been understood by the scientific community. In this study, a series of DEM simulations have been conducted to identify key factors that influence the granular soil’s resistance to multiple liquefactions. It is observed that completely different soil fabric will be formed if the liquefied soil is consolidated at different states in the post-liquefaction stage, resulting in vastly different resistance to re-liquefaction.

2 CYCLIC LIQUEFACTION SIMULATION USING DEM

Numerical simulations are conducted using an open-source DEM code, Yade (Šmilauer et al. 2015). A three-dimensional (3D) cubic representative volume element (RVE) consisting of 10000 spheres is generated. The radius of the particles ranges from 0.15 mm to 0.45 mm with mean radius ($R_{50}$) being 0.3 mm. Contact interactions are calculated using the Hertz-Mindlin contact model with Young’s modulus of 70 GPa and Poisson’s ratio of 0.3 assigned to all the particles. Rolling resistance proposed by Behelnine et al. (2009) is used at the contact points of the spherical particles to account for surface roughness and particle shapes (Bagi et al. 2004). The dimensionless coefficients for rolling resistance ($\beta$), and for plastic moment ($\eta$) as defined in Behelnine et al. (2009) is set as 0.1 and 0.05 respectively in the simulation. Note that periodic boundary condition is prescribed to the RVE to minimize the boundary effect.

The particle assembly is randomly generated in space with no initial contacts and later is isotropically consolidated to a vertical effective stress ($\sigma_{v0}$) of 100 kPa. To attain different void ratios, the contact friction angle ($\psi$) can be adjusted during consolidation. In the current study, a sample with a void ratio equal to 0.64, i.e. relative density of 60% ($e_{min} = 0.52$, $e_{max} = 0.82$ at 100 kPa) is generated in the initial consolidation process. After initial consolidation, $\psi$ is set as 26° for the later cyclic shearing. Undrained conditions are maintained by keeping the volume of RVE constant. It is important to note that the rate of shear strain loading is maintained to be 0.01/s in order to ensure a quasi-static condition.
3 FABRIC DESCRIPTORS

3.1 Coordination number (Z)
Coordination number is most widely used indicators derived from statistical analysis of inter-particle contacts to characterize the load-bearing structure of the granular materials. Coordination Number \( Z \) measures the average number of inter-particle contacts for each particle in the granular assembly, \( Z = 2N_c/N_p \), where \( N_c \) is the total number of contacts and \( N_p \) is the total number of particles.

3.2 Minkowski tensors for quantification of local particle-void cell
Figure 1a shows the Voronoi tessellation of the 3D granular packing which was performed using an open-source program voro++ (Rycroft 2009). It is to be noted that the number of cells in Voronoi tessellation is same as the number of particles \( (N_p = 10000) \). A random Voronoi cell, \( K \) (particle-void cell) is shown in Figure 1b. The Voronoi cell’s (refer Figure 1c) anisotropy can be quantified from the Minkowski tensors. Minkowski Tensors are a family of six independent tensorial shape measures \( (W_{02}^{20}, W_{12}^{20}, W_{20}^{20}, W_{30}^{20}, W_{01}^{20}, W_{02}^{20}) \) for a convex body such as a Voronoi cell (Turk et al. 2010). In particular, \( W_{02}^{20} \) tensor can be used to analyze the local anisotropy of a particle-void cell, \( K \) and is defined as:

\[
W_{02}^{20}(K) = \int_K r \otimes r \, dV
\]

where \( r \) is the position vector of the points on the surface of the cell with respect to its centroid, and the vector product \( \otimes \) is defined as the dyadic product, \( r \otimes r = r_i r_j \), which is a second order symmetric tensor.

From the definition, \( W_{02}^{20} \) is a measure of the volume mass distribution of the Voronoi cell, \( K \) and is equivalent to the standard moment of inertia tensor for the body \( K \) with unit density about its centroid. The minimum \( (\lambda_{\text{min}}) \) and maximum \( \lambda_{\text{max}} \) eigen values of the tensor \( W_{02}^{20} \) essentially provide an idea of fitting a best ellipsoid to the Voronoi Cell \( K \) as shown in Figure 1c. Hence, the local shape (elongation) factor \( e_d \) can be defined as:

\[
e_d = 1 - \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}}
\]

If \( e_d = 1 \), it represents highly anisotropic shape and \( e_d = 0 \) indicates an isotropic shape (i.e. spherical) of the particle void. We define the vector, \( \mathbf{v} \), as the unit eigen-vector corresponding to \( \lambda_{\text{max}} \) (refer Figure 1c).

3.3 Descriptors for particle-void distribution (\( E_d, A_d \))
The \( e_d \) and \( \mathbf{v} \) described in above section can quantify the local void anisotropy around a single particle. Where \( e_d \) measures the shape elongation and \( \mathbf{v} \) provides the principal orientation of the particle void. By statistical analysis of \( \{e_d, \mathbf{v}\} \) associated with all the particle in the granular assembly, the anisotropy of particle-void distribution for the entire packing can be quantified.

![Figure 1. a) Voronoi Tessellation b) Voronoi Cell c) Ellipsoid fitting](image)
The first descriptor, $E_d$, is the global shape (elongation) descriptor defined as the mean of $e_d$ for all $N_p$ Voronoi cells as:

$$E_d = \frac{1}{N_p} \sum_{i=1}^{N_p} e_d^{(i)}$$

(3)

The second descriptor, $A_d$, is a global descriptor for anisotropy of particle-void orientation, derived from the unit eigen vectors $\{v\}$. First, a fabric tensor $\{V_{ij}\}$ is defined as:

$$V_{ij} = \frac{1}{N_p} \sum_{k=1}^{N_p} v^{(k)}_i v^{(k)}_j = \int_{\Omega} E(\Omega) v_i v_j d\Omega$$

(4)

where $E(\Omega) = \frac{1}{4\pi} \left( 1 + a_{ij} v_i v_j \right)$ is a density function, and $a_{ij} = \frac{15}{2} \left( V^0_{ij} \right)$, with $V^0_{ij} = V_{ij} - \left( \frac{1}{2} \right)$ being the deviatoric part of the fabric tensor. Thus the anisotropy can be quantified using $A_d$ as:

$$A_d = \sqrt{\frac{3}{2} \left( a_{ij} a_{ij} \right)}$$

(5)

Finally, global shape elongation descriptor $E_d$ and global anisotropy descriptor $A_d$ are adopted to quantify the particle-void fabric evolution.

4 POST-LIQUEFACTION BEHAVIOR

It is observed that DEM is capable to qualitatively capture the ideal undrained cyclic simple shear response. Figure 2a,b shows the undrained response of sample for a cyclic stress ratio (CSR) of 0.35. The soil behavior is divided into two stages, pre-liquefaction stage and post-liquefaction stage. Initial liquefaction separates the two stages and is defined as the point at which the vertical effective stress $\sigma_0^v$ state is minimum $\sigma_0^v < 0.5$ kPa. In this study, more focus is put on postliquefaction behavior and its correlation to new fabric indices is emphasized.

4.1 Particle-void evolution in post-liquefaction stage

In the post-liquefaction stage, the stress paths follow the similar butterfly loops every cycle while the shear strain amplitude increases cycle after cycle (refer Figure 2a,b). Within a loading cycle in post-liquefaction stage, the granular assembly attains a flow state and a hardening state. Accordingly, the shear deformation is divided into flow strain ($\gamma_0$) and the hardening strain ($\gamma_d$) as shown in Figure 2c (Shamoto et al. 1997). Flow strain is the double amplitude strain component where the shear stiffness is very low and effective stress is almost zero. Hardening strain is the strain component where both shear stiffness and effective stress have a sharp increase from zero. That is during the flow strain the particle assembly behaves almost like a fluid whereas in hardening stage it behaves like a solid. It is worth noting that the hardening strain amplitude in each cycle is almost the same while the flow strain amplitude increases gradually with every cycle.

Figure 2. a) Stress path b)Shear stress Vs Shear strain curve c) One stress-strain cycle
It is observed that both $E_d$ and $A_d$ have negligible variation before initial-liquefaction (IL) and the variation in $E_d$ and $A_d$ gradually increases in the post-liquefaction stage, as shown in Figure 3b for CSR 0.35. With each loading cycle $E_d$ decreases while the amplitude of $A_d$ increases. After certain cycles, the evolution of both $E_d$ and $A_d$ gets stabilized. It indicates an ultimate state in terms of particle-void distribution has been reached. As discussed earlier, the shear deformation within a loading cycle is divided into flow strain and the hardening strain. The transition points from flow stage to hardening stage can be quantified when coordination number reaches 1.8 or above in the post-liquefaction stage, where the shear stress starts to increase significantly as shown in Figure 3a. These transition points ($Z = 1.8$) are also plotted in Figure 3b, also points corresponding to $Z = 1$ for reference. It is worth mentioning that the transition points clearly define loci of a Hardening State Line (HSL) that separates the flow state and the hardening state in $E_d — A_d$ space, and it is unique for different loading patterns i.e. CSR = 0.3, 0.35 and 0.4 as shown in Figure 4a. Below the HSL, the granular packing behaves like a fluid; Above the HSL, the granular packing has a stable load-bearing structure and behaves like a solid.

Also, the important feature in post-liquefaction stage, i.e. flow strain accumulation can be directly correlated with the descriptor $A_d$. Within a loading cycle the flow strain amplitude is denoted as $\gamma_0$ (refer Figure 2c), which is the shear strain from an unloading point to the next transition point. Whereas $A_d$ is chosen at the unloading point. Figure 4b shows the $A_d - \gamma_0$ relation for the initial sample for different loading patterns. A strong correlation between the flow strain amplitude and $A_d$ indicates that a sample with higher $A_d$ at the unloading point will be followed by a larger flow strain to reach the next transition point. The above findings are very similar to the 2D DEM study done by Wei et al. (2018).

4.2 Volumetric compression in post-liquefaction consolidation

After initial liquefaction, the dissipation of excess pore pressure results in increase in effective stress and volumetric compression of the granular soil. In DEM, the post-liquefaction consolidation is simulated by moving the boundaries of 3D RVE to increase effective stress in a drained condition, until the effective stress reaches its initial hydrostatic state of 100 kPa. Note that the consolidation is conducted when the soil is in the flow state (where $A_d$ reaches

![Figure 3. a) Shear stress Vs Coordination Number b) $E_d$ and $A_d$](image)

![Figure 4. a) $E_d$ and $A_d$ b) $A_d$ and $\gamma_0$](image)
a local minimum). The flow strain ($\gamma_0$) increases with every cycle of loading in the post-liquefaction stage. 16 consolidated samples are prepared at different post-liquefaction cycles corresponding to different maximum flow strains. The volumetric strain ($\epsilon_v$) resulting from the consolidated samples have a clear correlation with the flow strain ($\gamma_0$) experienced by the sample before consolidation as shown in Figure 5a. Sample that experienced higher flow strain before consolidation tends to have higher volumetric compression. Similar phenomenon is observed in experimental studies as reported in Ishihara et al. (2016).

The volumetric strain ($\epsilon_v$) is also well correlated with the particle-void descriptor, $E_d$, and is shown in Figure 5b, where $E_d$ is recorded just before the consolidation. It can be observed that the samples with relatively lower $E_d$ value has higher volumetric compression. A granular packing with relatively low $E_d$ value means that the particle voids are more isotropic (more spherical) and when consolidated hydrostatically results in more uniform compression locally. Hence, giving rise to a higher volumetric compression overall. In other words, $E_d$ is a good indicator for the compressibility of the sample.

4.3 Re-liquefaction behavior

In this study our main focus is on the effect of unloading point in post-liquefaction cycle to the re-liquefaction resistance. The samples at different unloading points give rise to different anisotropic fabrics when consolidated. Recent studies argue that anisotropy is the most influential factor and surpasses the effect of relative density on liquefaction resistance (Yamada et al. 2010). Figure 6a shows the stress-strain response of the 5th cycle in the post-liquefaction stage. Two samples at Point ‘A’ and ‘B’ are consolidated and examined for re-liquefaction strengths. Point A, corresponds to the just unloading point from the positive peak shear stress (refer Figure 6a) where the $A_d$ is at its local maximum and the point B, corresponds to the instant where $A_d$ is at local minimum, this is the same unloading point at which samples are consolidated to calculate volumetric strain in the previous section. These samples are 1-D consolidated with relatively low friction angle till the vertical effective stress reaches 100 kPa. Lower friction angle is used during consolidation in order to generate more contacts internally and later it is adjusted back to 26° during shearing. Figure 6b shows the final stress states after 1-D consolidation.

The re-liquefaction behavior for samples at point A and B for a CSR of 0.15 are shown in Figure 7b,c and e,f respectively. It is observed that sample at point A (1 cycle for initial liquefaction) has relatively low strength compared to the sample at point B (40 cycles for initial liquefaction). The relative densities of samples at A and B after consolidation are almost the same i.e. around 66%, but the variation in strength is quite significant. The contact normal distribution at A and B when projected on X-Y plane are shown in Figure 7a and c respectively. It is clear that sample at point A has preferred orientations of contact normals inclined in between 90° and 180° with almost 1000 contacts in each bin, whereas the contacts in 0° to 90° are relatively much lower. Due to such preferential orientation of contact normal in sample at point A, it happened to be very stiff in one direction and very loose in the other, resulting in overall degradation of liquefaction resistance. At point B, the contact normals distribution is almost the same in 0° to 90° and 90° to 180°, so the sample behaves the same for both positive shear stress and negative shear stress thus resulting in many cycles for complete

Figure 5. a) Volumetric strain Vs Flow strain b) Volumetric strain Vs $E_d$
degradation of stiffness. Hence, different unloading points in post-liquefaction stage results in different fabric after consolidation, which greatly affects the re-liquefaction strength.

5 CONCLUSIONS

In this study, two new particle-void based descriptors, $E_d$ and $A_d$ are developed based on the shape and the anisotropy of the particle-void cells which are generated from the Voronoi tessellation of the granular packing. It is observed that the evolution of these new descriptors primarily occur in the post-liquefaction stage, accompanied by increasing post-liquefaction deformation. After initial liquefaction, the value of $E_d$ decreases and $A_d$ increases gradually cycle after cycle. In the post-liquefaction stage, jamming transition could occur in the granular flow, which transforms the granular assemblage from a “fluid-like” state to a “solid-like” state to arrest the flow deformation. A unique hardening state line (HSL) is defined in the void-fabric space, that delineates the boundary between a flow stage and a jamming/hardening stage when the soil is fully liquefied. The post-liquefaction deformation of the granular packing is strongly correlated to the evolution of the particle-void fabric descriptors, so these fabric indices can be used to quantify the amplitude of flow deformation. Finally, a stable state of particle-void distribution can be achieved in post-liquefaction after sufficient number of loading cycles.

It is observed that the post-liquefaction volumetric strain depends on the magnitude of maximum flow strain that is experienced by the granular packing. Also, it is observed with the decrease in particle-void descriptor, $E_d$, the volumetric strain has increased. This
suggests $E_d$ can be a good indicator for the compressibility of a granular packing. Also, in this study one of the key important factors i.e. the point of unloading in post-liquefaction stage and its effect on re-liquefaction resistance is explored. It is observed that completely different soil fabrics are formed if the liquefied soil is consolidated at different unloading stages, resulting in vastly different resistance to the reliquefaction.

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REFERENCES