The paper was published in the proceedings of the 7th International Conference on Earthquake Geotechnical Engineering and was edited by Francesco Silvestri, Nicola Moraci and Susanna Antonielli. The conference was held in Rome, Italy, 17 – 20 June 2019.

Earthquake Geotechnical Engineering for Protection and Development of Environment and Constructions – Silvestri & Moraci (Eds)
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Evaluation of two constitutive models in predicting cyclic behavior of a natural clay

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ABSTRACT: A series of cyclic and monotonic constant-height direct simple shear experiments were conducted on Young San Francisco Bay Mud. Two bounding surface constitutive models have been implemented for numerical simulations of the cyclic behavior of the Young Bay Mud. This paper discusses key differences in predictions of the two constitutive models in simulating the cyclic behavior of the specimens tested in the laboratory. Both models were calibrated using an advanced optimization technique to remove bias introduced by trial and error calibration methods. Comparison of the results of the simulations and laboratory test data show significant differences and indicates that some fundamental features of cyclic behavior of clays cannot be captured by currently available models, suggesting it may be necessary to develop more robust constitutive models that have capability of better capturing pore-pressure response and cyclic strain development during cyclic loading.

1 INTRODUCTION

Constitutive modeling for cyclic behavior of cohesionless soils has had major progress in the last few decades; studies such as Lade and Kim (1988), Elgamal et al. (2003), Dafalias and Manzari (2004), and Taiebat and Dafalias (2008) are good examples among many others. This is perhaps due to devastating effects of soil liquefaction during earthquake events. However, less emphasis has been given to capturing the cyclic response of fine-grained cohesive soils, though, cyclic softening of clays has also played a major role in extensive structural damage and economic loss during earthquakes such as the 1999 Kocaeli event (Bray et al., 2000), 1999 Chi-Chi earthquake (Chu et al., 2004), and the Canterbury earthquake sequence (van Ballegooy et al., 2014).

One of the earliest models developed for clays, the modified Cam-clay (Roscoe and Burland, 1968) is still popular among practitioners perhaps due to its ease of calibration and use in numerical modeling. The model serves reasonably well for monotonic loading conditions, though, may not be suitable for cyclic loading due to its large yield surface and associative flow-rule. Al-Tabbaa and Wood (1989) introduced the Cam-clay bubble model, where a small "bubble" yield surface moves inside a bounding surface, rendering the model more appropriate for cyclic and reverse loading. More recently, the SANICLAY family of bounding surface constitutive models (Dafalias et al., 2006; Taiebat et al., 2010; Seidalinov and Taiebat, 2014) were introduced and are shown to be capable of simulating more accurately the behavior of clays. Seidalinov and Taiebat (2014) present a bounding surface framework for simulating cyclic loading of clays, and later Shi (2016) and Shi et al. (2018) introduce an enhanced flow-rule that is capable of more accurately predicting pore-pressure development upon load reversals in clays.

The predictions of these models have been shown to provide a good match with experimental laboratory results for specific clays and specific loading paths. However, the performance and accuracy of the same models in predicting the behavior of a different type of clay and different load paths remain an obstacle when alternative input parameters are used. This paper aims to address the numerically-simulated cyclic response of San Francisco Young Bay Mud (YBM) by two constitutive models; (1) the Seidalinov and Taiebat (2014) model, hereafter referred to as SANICLAY, and (2) the Shi et al. (2018) model with Hybrid flow-rule, hereafter referred to as SANICLAY-H. Results of monotonic and cyclic laboratory experiments on the YBM is presented first, followed by details on calibration of the two constitutive models. Then the results of the numerical simulations are presented and compared to laboratory results, where key differences and deficiencies of the simulations are highlighted.

2 LABORATORY STUDY

A full suite of laboratory experiments was performed on the San Francisco Young Bay Mud, and the following sections provide details about the outcomes of such experiments. Note that the focus of the below sections is on parameters that will be used in the calibration of the constitutive models for numerical simulations.

2.1 *Atterberg limits and compressibility properties*

Liquid Limit (*LL*) and Plastic limit (*PL*) tests were performed based on methods described in ASTM-D4318; the Casagrande cup was used for running Liquid Limit tests. Results are summarized in Table 1. The YBM classifies as a high plasticity clay (*CH*) based on the Unified Soil Classification System (USCS). Furthermore, an incremental load consolidation test was performed on the Young Bay Mud to evaluate stress history and consolidation properties. The procedures followed were based on the methods described in ASTM-D2435; compressibility properties of the YBM are presented in Table 1.

2.2 *Constant-height monotonic shear response*

Approximately 1-inch segments of thin wall Shelby tube samples of the YBM were cut. Specimens were then trimmed to 2.6 in diameter using a wire cutter and placed carefully into a wire-reinforced rubber membrane for constant-height shear testing using the UCLA Bi-directional Broadband Simple Shear device (Shafiee et al., 2017). The methods described in ASTM-D6528 were followed for the shear testing of the YBM.

Specimens were consolidated under load-controlled conditions to about 1.5 times the anticipated effective stress in the field. The specimens were consolidated to a vertical effective stress of approximately 230 kPa prior to shear testing. Lightly overconsolidated specimens were first consolidated to 230 kPa and unloaded to about 158 kPa, and then sheared under constant-height conditions. Once consolidation to the desired vertical effective stress finished, specimens were sheared under constant-height conditions. Peak monotonic undrained shear strength ratio (s_u/σ'_{vc}) for these specimens as well as void ratios are summarized in Table 2.

Table 1. San Francisco Young Bay Mud Properties

Soil Type	Liquid Limit (*LL*)	Plastic Limit (*PL*)	Plasticity Index (*PI*)	USCS Classification	C_c	C_r
Young San Francisco Bay Mud (YBM)	109	42	67	*CH*	0.625	0.05

Table 2. Undrained Shear Strength Parameters from Monotonic Shear Testing of YBM

Test *ID*	σ'_{vc} (psf)	Void ratio, e^a	Peak Undrained Shear Strength ratio, s_u/σ'_{vc}	OCR^b
Test 1	230	1.64	0.31	1
Test 2	158	1.90	0.44	1.46
Test 12	118	1.57	0.58	1.95

[a] end of consolidation,
[b] $OCR = \sigma'_{vc\text{-}max}/\sigma'_{vc}$

Table 3. Summary of Constant-Height Cyclic Simple Shear Tests on YBM

Test *ID*	σ'_{vc} (kPa)	*OCR*	*e*	*CSR*	$N_{3\%}$
Test 3	230	1	1.68	0.21	31
Test 4	230	1	1.57	0.24	4.8
Test 5	230	1	1.67	0.25	3.8
Test 6	158	1.5	1.61	0.3	5.6
Test 7	158	1.5	1.66	0.34	7.8
Test 8	158	1.5	1.44	0.37	7.6

2.3 *Constant-height cyclic shear response*

Specimens for constant-height cyclic loading were trimmed and consolidated following the same procedures as for the monotonic shear tests. Specimens were cyclically sheared under constant-height stress-controlled conditions following the end of primary consolidation, and the cyclic stresses were imposed by sinusoidal uniform loading paths with a frequency of 0.1 Hz. Table 3 provides summarized data from the cyclic and post-cyclic shear testing of the specimens, and $N_{3\%}$ is the number of uniform loading cycles to reach 3% shear strain.

3 SOIL CONSTITUTIVE MODELS USED IN THIS STUDY

In the present study, two clay models are used to numerically simulate the Bay Mud response under dynamic loading: (1) the Seidalinov and Taiebat (2014) model, hereafter referred to as SANICLAY, and (2) the Shi et al. (2018) model with Hybrid flow-rule, hereafter referred to as SANICLAY-H. The SANICLAY model builds upon two previous versions (Dafalias et al., 2006; Taiebat et al., 2010). Although the previous versions were only limited to the monotonic applications, the SANICLAY model formulation has been modified to upgrade the model's ability for cyclic applications, while preserving the crucial features of the previous versions, namely considering the effect of anisotropy as well as destructuration (or softening).

SANICLAY employs a bounding surface concept and a radial mapping rule to simulate the dynamic response of clays. In other words, the yield surface of the previous versions of the model is substituted with a bounding surface. In the bounding surface plasticity, elastic and plastic strains can occur even if the stress point lies inside the bounding surface, as opposed to classical plasticity in which plastic strains develop only if the stress point goes beyond the yield surface. In this model, plastic strains are computed based on the distance between the current stress state and its projection on the bounding surface by means of a radial mapping rule. The projection of the stress state on the bounding surface is called the image stress. Such algorithm allows for nonlinearities to develop at very low strains, which is consistent with the true soil behavior. However, one of the drawbacks of the SANICLAY model is that after a few cycles, a "lock-up" of the mean effective stress is observed, in which the mean effective stress does not change upon cyclic loading.

SANICLAY-H is the hybrid flow rule bounding surface model. This model was developed by Shi (2016) upon the framework of SANICLAY (Seidalinov and Taiebat, 2014). The main modification in the SANICLAY-H compared to the SANICLAY is the substitution of the image stress flow rule with a hybrid flow rule. Based on the implemented hybrid flow rule in this model, both the current stress state and the image stress state are used to calculate the volumetric plastic strains. The relative contribution of the image and current stress state is controlled by the w parameter, which is a material constant. This modification solves the problem of mean effective stress lock-up and enables the SANICLAY-H model to simulate the excess pore water pressure build-up and simulate the butterfly shapes of the stress path which are expected from clay's behavior under dynamic loading, as seen in various experimental results. The models were calibrated based on the experimental test results and a unique approach that is described in the following section.

4 CALIBRATION OF THE MODELS

Both models combine parameters that are either physically meaningful (such as λ, the slope of the normally consolidated line) or purely model-related. The former is determined either from the laboratory test results or based on common values published in the literature, while the latter is computed with a calibration algorithm presented herein. For instance, λ and κ are taken directly from the results of consolidation tests, M_c from the monotonic tests, and M_e is selected subjectively. The algorithm is based on the work of Liu et. al (2016). This inverse calibration technique aims at finding the set of parameters (θ) of a function, to provide the best match of multiple sets of data. This optimization technique tries to minimize the difference between the results of numerical simulations and laboratory experiments and can mathematically be formulated as:

$$r(\theta) = \left(y(\theta) - y_{exp}\right) \tag{1}$$

Where $r(\theta) \in \mathcal{R}^{s\times 1}$ is a residual vector, while s is the total number of data points, and $y_{exp} \in \mathcal{R}^{s\times 1}$ and $y(\theta) \in \mathcal{R}^{s\times 1}$ are vectors containing the target data to be matched (in the present case a set of laboratory tests) and predictions (calculated from a constitutive model), respectively. These two vectors can be chosen to include different response quantities such as shear strain (γ), pore pressure (u), the ratio of deviatoric stress (q) to mean effective stress (p'), etc.

The calibration algorithm was applied to the SANICLAY and SANICLAY-H models to find the best set of input parameters to match the dataset presented above. For each model, the algorithm is used to calibrate the models based on the strain data from four of the six laboratory tests performed on Bay Mud. Once calibrated, the models are used to simulate the two remaining lab tests to validate the calibration.

The values of all model parameters are presented in Table 4. For SANICLAY, of the 11 model parameters, 5 were included in the calibration process (i.e. $\theta = (h_0, a_d, C, x, N\ ,)^T$). The SANICLAY-H model parameters are calibrated in a similar fashion. The model has 18 parameters, 7 of which are obtained through the calibration process (i.e. $\theta = (h_c, h_e, N_c, N_e, c_d, C, x)^T$. Four of the input parameters of this model are used to better capture the small nonlinearities (i.e. $\theta = \left(e_g, A_g, n_g, \gamma_{0.7}\right)^T$, and the default values are used for these input parameters. More information on the model parameters can be found in Seidalinov and Taiebat (2014) and Shi et al. (2018).

Besides model parameters, the calibration requires initial values of the hardening variables, specific to each model, such as the size of the yield surface and the initial position of the center of the yield surface. The hardening variables were selected to be consistent with the tests' conditions, namely the initial stress conditions, the maximum past pressure, and a coefficient of earth pressure at rest (K_0) of 0.7. Finally, the Poison's ratio is set at a typical value for soils.

Table 4. Model parameters used in numerical simulations

	SANICLAY		SANICLAY-H	
Parameter Category	Parameter	YBM Values	Parameter	YBM Values
Elasticity (small strain)	*NA*		e_a	2.64
			A_a	160
			n_a	0.635
			$\gamma_{0.7}$	0.00016
Elasticity (large strain)	κ	0.022	κ	0.022
	ν	0.2	ν	0.24
Critical state	λ	0.271	λ	0.271
	M_c	1.2	M_c	1.2
	M_e	1.2	M_e	0.86
Bounding surface and plastic modulus	N *	1	N_c*	1.2
			N_e*	0.86
	h_0*	100	h_c*	98
			h_e*	100
	a_d*	40	c_d*	4.72
			ω	2
Rotational Hardening	C*	5	C*	0.06
	x*	0.9	x*	9.6
Destructuration	k_i	1	k_i	1

* *Parameters that were found from the calibration process.*

5 PERFORMANCE OF THE CONSTITUTIVE MODELS

Stress-strain and stress-path response from simulation outcomes of the SANICLAY, SANICLAY-H, and the cyclic simple shear test data for Test 7 (CSR = 0.34) are presented in Figure 1. Strain generation and the equivalent pore pressure ratio (r_u) are plotted for Test 7 in Figure 2. The simulation and lab data are presented up to a shear strain level of 6% for comparison purposes. Both models seem to predict thick hysteresis stress-strain loops as compared to the laboratory data for the Young Bay Mud, indicating that the damping predicted from the numerical simulations are higher than that exhibited by the soil specimens sheared in the simple shear device. Furthermore, it can be inferred from these stress-strain loops that a lower value of modulus reduction is predicted by two models at shear stain levels close to 6%.

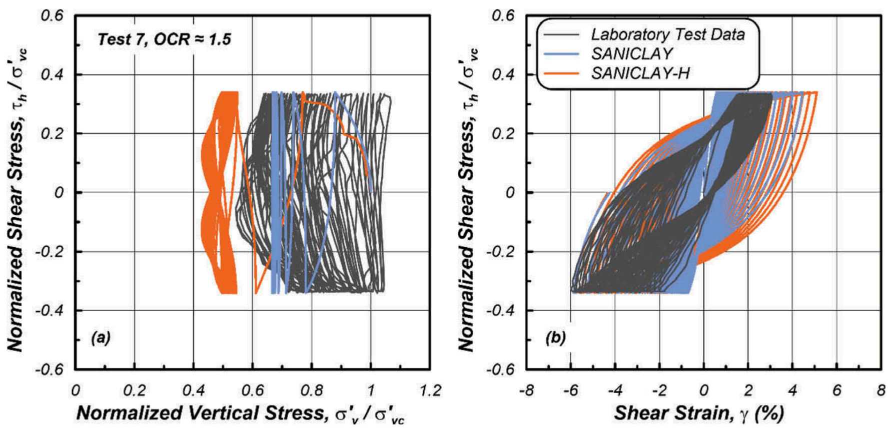

Figure 1. Numerical simulations and experimental laboratory data for Test 7, (a) stress-strain response, (b) stress path response

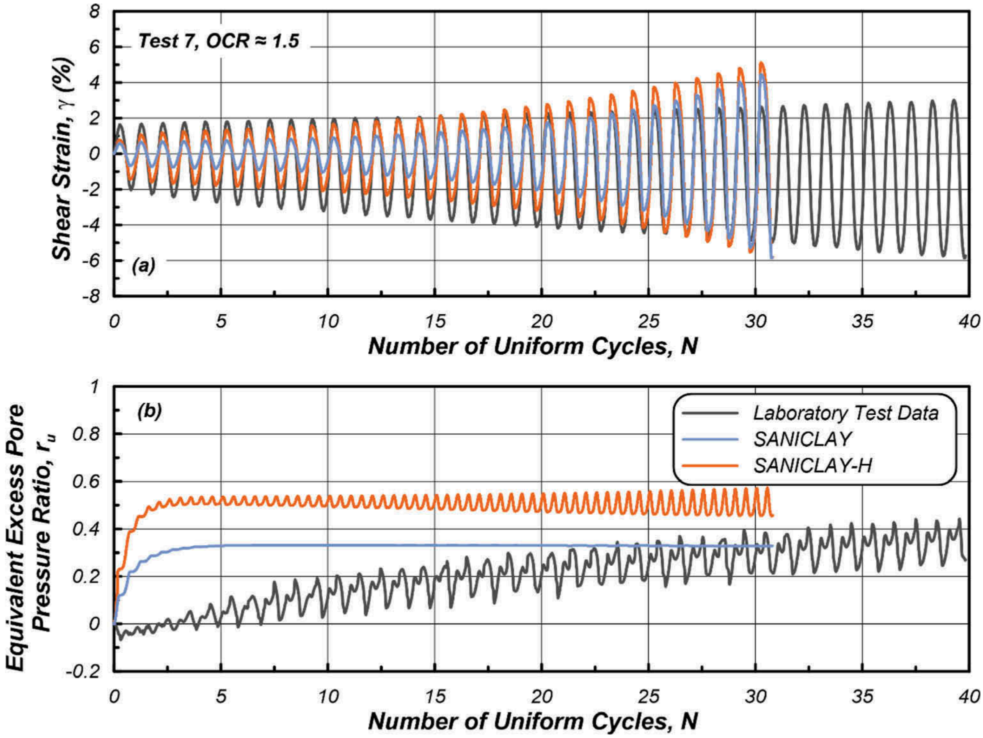

Figure 2. Numerical simulations and experimental laboratory data for Test 7, (a) cyclic shear strain versus the number of loading cycles, (b) excess pore pressure ratio versus the number of uniform loading cycles

Figure 2 shows that both numerical simulations reach the target shear strain of 6% in a lower number of uniform loading cycles compared to the laboratory data. Moreover, it is evident from this figure that the manner which the strain and pore pressure generation is predicted, is different from what is seen from the laboratory results; the lab data shows a relatively gradual increase in excess pore pressure ratios, whereas the numerical simulations both reach to their cap pore pressure ratios quite rapidly in very few numbers of loading cycles. On the other hand, the models predict a rather gradual increase in cyclic shear stain with respect to cycles of loading, which is not exhibited by the soil in the laboratory experiment.

Furthermore, from Figure 1 it can be observed that the SANICLAY model does not accurately simulate dilation-contraction reversals (i.e. negative pore water pressure generation) that occur in each cycle of loading. The SANICLAY-H model can predict the expected switch from contractive to dilative behavior during cyclic load reversals. However, this model does not simulate dilation that occurs at initial cycles of loading due to light over-consolidation of the specimen. In fact, this model only predicts dilative behavior when the stress path reaches the critical state line, where the loops of the stress path start exhibiting a butterfly pattern. The ability of simulating butterfly loops is an advantage of the SANICLAY-H over the SANICLAY model and is closer to real soil behavior during cyclic loading.

Combinations of *CSR* and number of loading cycles (N) to reach a peak shear strain of 3% are presented in Figure 3 for the numerical simulations as well as laboratory data for normally consolidated specimens and specimens that were lightly overconsolidated (i.e. OCR = 1.5). Regressed functional forms relating *CSR* and N are also shown in the figure.

Although the SANICLAY-H curve is relatively closer to the laboratory data, both numerical predictions of the *CSR*-N curve are steeper than the laboratory data for the Young Bay Mud. Note that the less steep curves of the YBM from the laboratory data suggest a more gradual change of N with *CSR*, which is consistent with various experimental test data on

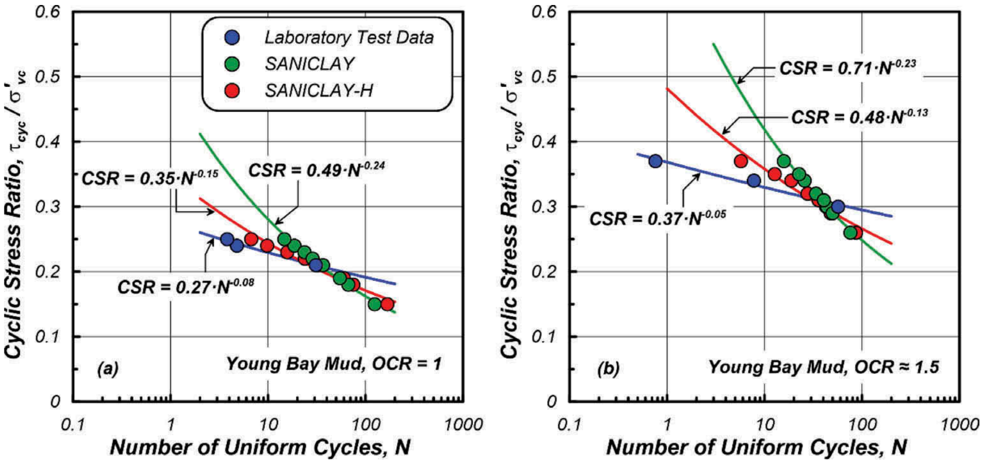

Figure 3. CSR vs. N curves from experimental results and numerical simulations

clays in the literature. Based on this observation, results of the numerical simulations may not adequately represent real cyclic soil response, and therefore, may lead to overestimation of cyclic strength for field conditions. The divergence of the numerical simulations relative to the laboratory data seems more pronounced in the lightly overconsolidated specimens, though, tests results at higher *OCRs* would be necessary to validate such outcome.

6 CONCLUSIONS AND FUTURE WORK

A series of cyclic direct shear tests has been carried out on the Young San Francisco Bay Mud. Two recently-developed clay constitutive models have been implemented to compare numerical simulations of the cyclic response of the Young Bay Mud to that of the experimental laboratory data from running cyclic simple shear experiments. Numerical simulation results from both models show significant differences with the experimental laboratory data. The generation of shear strains with loading cycles, as well as the excess pore pressure generation from the simulations is relatively different from the experimental results. Moreover, it was observed that the relationships of *CSR* vs. *N* are relatively steep from the outcomes of the numerical simulations, whereas the laboratory test data suggest flatter curves. The SANICLAY-H model by Shi et al. (2018) has the benefit of better predicting the pore pressure response upon cyclic shearing due to its hybrid flow rule and butterfly loops, though the issue of the stress lock-up is remains to be solved.

Though the geotechnical engineering profession currently benefits from recently-developed constitutive models with the capability of simulating cyclic response of clays, this study shows compelling differences between laboratory experimental data and numerical simulations. This indicates that some fundamental features of cyclic behavior of clays cannot be adequately captured by the currently available models, and it may be necessary to develop models that provide more realistic predictions of pore-pressure and cyclic shear strain development upon cyclic loading of clays.

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