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Pseudodynamic seismic stability analysis of cohesive-frictional soil slopes

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ABSTRACT: In this paper, the stability of cohesive-frictional soil slopes has been examined based on the consideration of a rotational log-spiral slip mechanism using the limit equilibrium method with the inclusion of pseudodynamic seismic forces. Based on the one-dimensional wave propagation theory in the soil medium and satisfying stress boundary conditions at the ground surface, a realistic horizontal seismic acceleration distributions and associated seismic forces along the depth are considered in the present analysis. In the proposed analytical formulation, the dynamic characteristics of soil mass such as its shear wave velocity, shear modulus, damping ratio and Poisson's ratio are taken into account for examining the influence of pseudodynamic seismic forces. The results are obtained in terms of stability numbers for varying magnitudes of material properties of soil, angle of slope, and dynamic characteristics of soil mass.

1 INTRODUCTION

Assessment of stability of slopes has been the primary topic of research in the field of the applied geotechnical engineering over the past few decades. Among various methods of analyses of slopes (limit analysis, elasto-plastic finite element analysis, and finite difference method), the limit equilibrium approach of slope stability is one of the oldest approaches, which is widely used because of its relative simplicity. This approach provides acceptable solutions for various engineering problems. Most of the available approaches for the slope stability assessment were performed within the framework of the strength reduction method in which the safety factor (F) is defined as follows:

$$F = \frac{\tan\varphi}{\tan\varphi_m} = \frac{c}{c_m} \tag{1}$$

where ϕ = angle of internal friction of soil mass; ϕ_m = mobilized angle of internal friction of soil mass; c = unit cohesion of soil mass; c_m = mobilized unit cohesion of soil mass. The common notion for this approach is that the reduced (factored) shear strength satisfies the equilibrium requirement of thefailing soil mass. The pioneering work carried out by Taylor (1937) proposed a friction circle method and the stability charts were generated by defining a stability number (N) as follows:

$$N = \frac{c}{\gamma HF} \tag{2}$$

where γ = unit weight of soil mass; H = height of the soil slope. However, the stability charts prepared with the definition of stability number requires iterations to be performed for finding the safety factor of slopes for any given combination of parameters defining the geometry of slopes (that is, the angle of slope (β) and height of slope) and soil strength parameters. Many proposals have earlier been made aiming to introduce an alternative and convenient way for the representation of stability charts that avoids the necessity of performing iterations in assessing the safety factors of slopes (Bishop &Morgenstern 1960; Bell 1966; Singh 1970; Cousins 1978). Among others, Bell (1966) has succeeded by representing stability charts explicitly with the introduction of a modified stability number (N_m) defined as:

$$N_m = \frac{c}{\gamma H \tan\varphi} = \frac{c_m}{\gamma H \tan\varphi_m} \tag{3}$$

The static and pseudostatic seismic slope stability analysis performed by Michalowski (2002) using the kinematic limit analysis approach has pointed out the usefulness of such representation for presenting the stability charts. The analysis performed so far are based on pseudostatic approach for examining the influence of seismic loadings where the magnitude and the phase of acceleration encountered during an earthquake in the soil medium does not change with depth. A simple pseudo-dynamic approach proposed originally by Steedman&-Zeng (1990) and extended later by Choudhury&Nimbalkar (2005) can be used to avoid such unrealistic assumptions employed in the conventional pseudostatic approach. However, the simplest pseudo-dynamic approach violates the ground stress boundary conditions and involves assumptions regarding the variation of the amplification of the vibration.

In the present work, within the framework of limit equilibrium method based on a new pseudo dynamic approach for the inclusion of horizontal seismic forces (Bellezza 2014), the stability of cohesive frictional soil slope has been analyzed. A rotational log-spiral slip mechanism has been adopted in performing the present analysis. The new approachused in this study has an advantage that it overcomes the shortcomings associated with the conventional pseudostatic and pseudodynamic methods. Numerical results of the present analyses have been presented in the form of stability charts from which the safety factor of slopes can be obtained explicitly without performing any iteration. The present analysis is limited to study the influence of horizontal component of ground motion on the stability of slopes; nevertheless, the inclusion of vertical component of the ground motion in the analysis may affect the stability of slopes, which is beyond the scope of the present study.

2 DETAILS OF NEW PSEUDODYNAMIC APPROACH

In the new pseudo-dynamic approach proposed by Bellezza (2014), the horizontal acceleration profiles were obtained considering one-dimensional shear wave propagation in a soil medium modeled as Kelvin-Voigt viscoelastic solid. By satisfying the zero stress boundary condition at the ground surface and the displacement compatibility condition at the toe of slope, for a harmonic vertical vibration of angular frequency ω and period $T = 2\pi/\omega$, the horizontal acceleration $a_h(y,t)$ at any time t and height y from the toe can be obtained as follows:

$$a_h(y,t) = \frac{k_h g}{C_s^2 + S_s^2}\left[\left(C_s C_{sy} + S_s S_{sy}\right)\cos(\omega t) + \left(S_s C_{sy} - C_s S_{sy}\right)\sin(\omega t)\right] \tag{4}$$

where

$$C_{sy} = \cos\left[y_{s1}\left(1-\frac{y}{H}\right)\right]\cosh\left[y_{s2}\left(1-\frac{y}{H}\right)\right] \tag{5a}$$

$$S_{sy} = -\sin\left[y_{s1}\left(1-\frac{y}{H}\right)\right]\sinh\left[y_{s2}\left(1-\frac{y}{H}\right)\right] \tag{5b}$$

$$C_s = \cos y_{s1}\cosh y_{s2} \tag{5c}$$

$$S_s = -\sin y_{s1} \sinh y_{s2} \tag{5d}$$

$$\begin{matrix} y_{s1} \\ y_{s2} \end{matrix} = \pm \frac{\omega H}{V_s} \sqrt{\frac{\sqrt{1+4\xi^2} \pm 1}{2\left(1+4\xi^2\right)}} \tag{5e}$$

where k_h = horizontal acceleration coefficient at the toe level; V_s = average value of vertically propagating shear waves along the slope depth; g = acceleration due to gravity; the dimensionless parameters y_{s1} and y_{s2} are the function of normalized frequency of the vertically propagating shear waves $\omega H/V_s$ and the damping ratio ξ. The applicability of this new pseudo dynamic approach for solving various problems related to active earth pressure and uplift capacity of anchors buried in granular medium has been demonstrated successfully by different researchers (Bellezza 2014, Ganesh &Sahoo 2017, Ganesh et al. 2018). In the present work, this new approach has been applied for the seismic stability assessment of cohesive frictional soil slopes within the framework of limit equilibrium method of analysis considering a rotational log-spiral slip mechanism.

3 ANALYTICAL FORMULATION

3.1 *Geometric details of the collapse mechanism*

The limit equilibrium formulation has been proposed for the stability assessment of cohesive frictional soil slopes with the inclusion of pseudodynamic seismic forces. The soil slope with a height of H inclined at an angle of β with the horizontal plane. The slope material is considered to be dry, homogeneous and satisfy Mohr-Coulomb (MC) failure criterion. MC strength parameters such as the cohesion $c$and angle of internal friction ϕ of slope material are considered unaffected even during the event of an earthquake. The present analysis has been performed assuming a rotational log-spiral slip mechanism as illustrated in Figure 1.Thus, during incipient collapse of slope, the soil mass ABC bounded by the slip surface CB is assumed to rotate as a rigid body about the rotation center O in the direction as shown in Figure 1. The lengths of the initial and final radius of the log-spiral curve are related to the reduced angle of internal friction angle of the MC criterionϕ_m as follows:

$$R_0 = A \exp(-\theta_0 \tan \varphi_m) \tag{6a}$$

$$R_n = A \exp(-\theta_n \tan \varphi_m) \tag{6b}$$

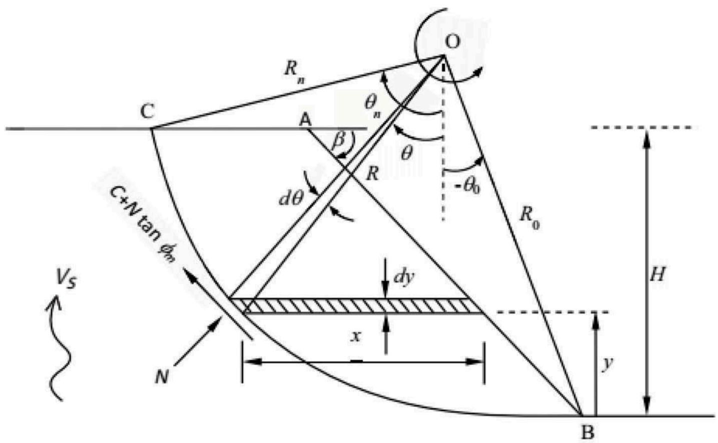

Figure 1. Definition of the problem.

The length of the radius at any angle θ can be defined as:

$$R = A\exp(-\theta\tan\varphi_m) \tag{7}$$

where the constant A is expressed as

$$\frac{A}{H} = \frac{1}{\exp(-\theta_0\tan\varphi_m)\cos\theta_0 - \exp(-\theta_n\tan\varphi_m)\cos\theta_n} \tag{8}$$

For any height y from the toe level, the width x of the differential slice can be expressed by the following relation:

$$\frac{x}{A} = \exp(-\theta\tan\varphi_m)\left[\sin\theta + \frac{\cos\theta}{\tan\beta}\right] - \exp(-\theta_0\tan\varphi_m)\left[\sin\theta_0 + \frac{\cos\theta_0}{\tan\beta}\right] \tag{9}$$

3.2 *Moment due to actuating and resisting forces*

At the failure of slope, the soil mass ABC bounded by the slip surface BC is considered to be rotating about the rotation center O due to actuating forces caused by the seismicity and self-weight of the soil. However, the force C due to cohesion c of the soil mass along the curvilinear slip line BC provides the resistance against the rotational motion. The total resisting moment due to cohesive forces developed along the slip surface can be expressed by the following equation:

$$M_c = \frac{cA^2}{2\tan\varphi_m}[\exp(-2\theta_0\tan\varphi_m) - \exp(-2\theta_n\tan\varphi_m)] \tag{10}$$

It is to be mentioned that the moment due to resultant force (due to normal force N components) along the slip surface is zero because the direction of resultant force passes through thefocus of the log-spiral sector or the rotation center O. The actuating moment due to seismic inertia forces is computed by performing the following integration:

$$M_s = A\int_{\theta_0}^{\theta_n} m(y)a_h(y,t)\exp(-\theta\tan\varphi_m)\cos\theta \tag{11}$$

where $m(y)$ = mass of the differential element. This could be determined from the geometry of the collapse mechanism. Similarly, the actuating moment M_γ due to self-weight of the slope can be obtained as:

$$M_\gamma = A\int_{\theta_0}^{\theta_n} m(y)g\exp(-\theta\tan\varphi_m)\cos\theta \tag{12}$$

3.3 *Equilibrium condition and modified stability number*

The moment due to actuating and resisting forces considered in the analysis must satisfy the moment equilibrium condition, which is written as follows:

$$M_s + M_\gamma = M_c \tag{13}$$

After substituting the respective expressions in Eq. (13), the expression for modified stability number N_m can be written as follows:

$$N_m = \frac{k_hF_4 + F_3}{F_2} \tag{14}$$

In which,

$$F_4 = F_1^3 \int_{\theta_0}^{\theta_n} \left\{ \begin{array}{c} \frac{\cos\theta}{C_s^2+S_s^2}\left[(C_sC_{sy}+S_sS_{sy})\cos(\omega t)+(S_sC_{sy}+C_sS_{sy})\sin(\omega t)\right] \\ \times \exp(-2\theta\tan\varphi_m)(g_1(\theta)-g_2)[\cos\theta\tan\varphi_m+\sin\theta] \end{array} \right\} d\theta \quad (15a)$$

$$F_3 = F_1^3 \int_{\theta_0}^{\theta_n} \left\{ \begin{array}{c} \left[\exp(-\theta\tan\varphi_m)\sin\theta - \frac{(g_1(\theta)-g_2)}{2}\right](g_1(\theta)-g_2) \\ \times \exp(-\theta\tan\varphi_m)[\cos\theta\tan\varphi_m+\sin\theta] \end{array} \right\} d\theta \quad (15b)$$

$$F_2 = \frac{F_1^2}{2}[\exp(-2\theta_0\tan\varphi_m) - \exp(-2\theta_n\tan\varphi_m)] \quad (15c)$$

$$g_1(\theta) = \exp(-\theta\tan\varphi_m)\left[\sin\theta + \frac{\cos\theta}{\tan\beta}\right] \quad (15d)$$

$$g_2 = \exp(-\theta_0\tan\varphi_m)\left[\sin\theta_0 + \frac{\cos\theta_0}{\tan\beta}\right] \quad (15e)$$

3.4 *Solution strategy*

The optimum value of the modified stability number N_m has been obtained by generating a computer programming codes in Matlab 2014 where the parameters defining the geometry of the rotational mechanism (θ_0 and θ_n)and the dimensionless time t/T has been optimized by varying independently with respect to one another in loop. The convergence of the solutions is achieved by stating the condition that the difference between the two successive solutions should be lesser than or at least equal to 10^{-3}. The solution obtained based on the above procedure has been provided in the following sections.

4 RESULTS AND DISCUSSION

4.1 *Variation of modified stability number*

The results of the present study have been presented in the form of stability charts for different combinations of the slope angles and earthquake acceleration coefficients as shown in Figures 2 and 3.

It can be noticed that the stability number N_m increased continuously with an increase in the values of $F/\tan\phi$ ratio. For a given value of $F/\tan\phi$, the magnitude of N_m increases with an increase in the value of (i) slope angle β and (ii) horizontal acceleration coefficient k_h. This result suggests that the value of safety factor of slopes decreases continuously with an increase in the values of (i) coefficient k_h and (ii) slope angle. It is to be mentioned that the results presented in these charts are obtained considering damping ratio ξ = 10% and normalized frequency $\omega H/V_s$ = 1.885. The influence of these factors has been examined in the follow up section.

Further, the results for slopes angle greater than equal to 45° are only presented because slope angles lower than 45° may have possibility to develop deep-seated failure mechanisms. However, such failure mechanisms have not been considered in the present analysis, as this is beyond the scope of this study. Nevertheless, the analysis presented could be easily extended to consider such deep-seated failure mechanisms.

4.2 *Influence of damping ratio and frequency content of input motion*

Figure 4 shows the influence of damping ratio and normalized frequency of the input motion on the value of the stability number N_m for the input parameters: k_h = 0.1, β = 60°

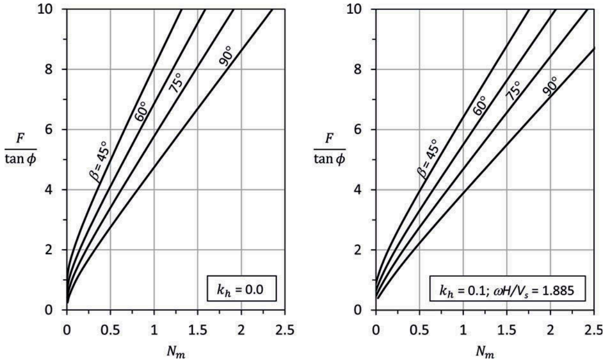

Figure 2. Stability charts for slopes with (a) $k_h = 0$ and (b) $k_h = 0.1$

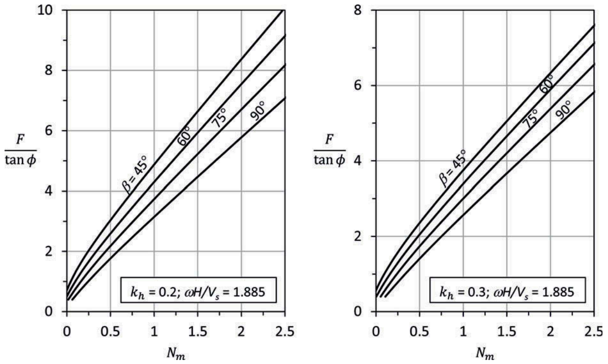

Figure 3. Stability charts for slopes with (a) $k_h = 0.2$ and (b) $k_h = 0.3$

and $F/\tan\phi = 6$. It can be observed from the Figure 4 that the magnitude of N_m increases as the value of damping ratio decreases, particularly frequencies close to natural frequencies, which can be expressed mathematically as follows (Kramer, 1996),

$$\frac{\omega H}{V_s} = i\frac{\pi}{2}; \qquad i = 1, 3, 5, 7, \ldots\ldots \tag{16}$$

For values of frequency larger or smaller than the natural frequencies, the magnitude of N_m is hardly affected by the variation in the values of damping ratio. Within the range of parameters investigated, two peaks are appeared which are corresponding to the frequencies close to natural frequencies. To demonstrate the amplification of acceleration in the soil medium at the incipient collapse of slopes, the normalized acceleration profiles are shown in Figure 5 for threedifferent values of normalized frequencies ($1.2\pi/2$, $\pi/2$ and $0.8\pi/2$) with $k_h = 0.1$, $\beta = 60°$ and $F/\tan\phi = 6$. It can be observed that the amplification of acceleration in the soil medium is maximum when the frequency value close to natural frequency, i.e., $\pi/2$. This shows that for frequencies lower or higher than the natural frequencies, the acceleration distribution throughout the soil medium will not be in phase i.e., some portion of the soil medium is moving in one direction while the remaining portion moving in opposite direction. On the other hand, at frequencies close to natural frequencies, the values of acceleration for the maximum portion of the soil medium are amplified and therefore, the magnitude of N_m is higher despite out of phase acceleration occurred at some small portion of the soil medium.

It is to be mentioned that the approach adopted here predicts a time dependent and nonlinear acceleration profiles depending on the height of the slope, shear wave velocity and

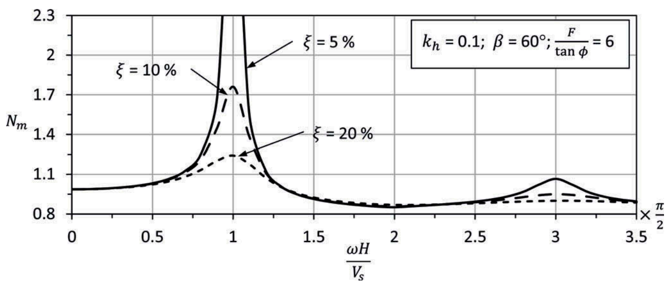

Figure 4. Influence of damping ratio and frequency content of input motion

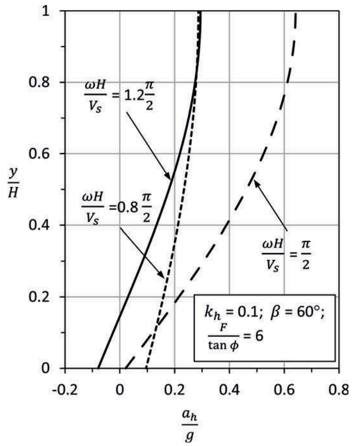

Figure 5. Variation of the normalized acceleration profiles

damping ratio of soil medium, and frequency content of input excitation; whereas, earlier pseudodynamic methods require assumptions regarding the amplification of the input motion. In the present study, the magnitude of acceleration amplifies towards the ground surface with changes in the phase depending on the dynamic properties of the soil medium and frequency content of input excitation. Further, the yield acceleration coefficient required for a performance based analysis of slopes can be estimated from the proposed formulation by imposing $F = 1$.

5 COMPARISON WITH AVAILABLE PSEUDO-STATIC SOLUTIONS

The accuracy of the proposed formulation has been verified by comparing solutions of this study with those results reported by Michalowski (2002) using kinematic limit analysis approach in the presence of pseudostatic seismic forces. The associated comparison has been provided in Table 1. It can be noticed that the present pseudodynamic solutions are very much close to those pseudostatic results reported by Michalowski (2002), especially for larger values of damping ratio. It is attributed to the decrease in the nonlinearity of the acceleration distribution with an increase in the value of damping ratio. At certain large values of damping ratio, the acceleration distribution in the soil medium even becomes uniform as used in the pseudostatic approaches and hence, no difference could be found between the two solutions. The proposed approach considers not only the variation of magnitude of acceleration but also the phase change in the acceleration with depth throughout the soil medium.

Table 1. Comparison of N_m from the present study with those reported by Michalowski (2002) for k_h = 0.1, $F/\tan\phi$ = 6 and $\omega H/V_s$ = 1.885.

Present results					
β	ξ= 10%	ξ = 30%	ξ = 50%	ξ = 70%	Michalowski (2002)
90°	1.658	1.562	1.517	1.498	1.446
75°	1.349	1.266	1.229	1.213	1.183
60°	1.116	1.046	1.016	1.004	0.982
45°	0.916	0.861	0.839	0.831	0.810

6 CONCLUSIONS

In the present study, limit equilibrium method has been applied for the seismic stability assessment of cohesive frictional slopes in the presence of pseudodynamic seismic forces. A realistic nonlinear and time dependent acceleration profiles were considered based on the height of the slope, shear wave velocity and damping ratio of soil medium, and frequency content of input excitation. The influence of damping ratio and normalized frequency of the input motion on the magnitude of modified stability number has been examined. From the proposed formulation, stability charts has been generated for applications in practice, which has the advantage of explicit computation of safety factor of slopes of any given configuration and soil parameters. The proposed analytical formulation found to be in good agreement with pseudostatic solutions reported in the literature. The inclusion of pseudodynamic seismic forces in the analysis of slope stability provides least safety factors than those found from the pseudostatic seismic analysis. This difference is primarily attributed due to consideration of amplification and phase change in the magnitude of accelerations towards the ground in the present analysis.

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