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Influence of strain distribution and dynamic response in the prediction of displacements in shallow sloping ground

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ABSTRACT: Earthquake-induced lateral deformations in shallow sloping ground pose significant seismic risk to pile-supported bridge foundations, critical underground infrastructure, and shallow foundations. Historically, such deformations have been difficult to predict due to numerous mechanisms, relating to the characteristics of ground shaking, spatial heterogeneity of soils, and topographic features, which can be difficult to characterize in the field. Current simplified methods for predicting lateral displacements, which consist primarily of empirical models and strain potential-based methods, largely neglect the aforementioned factors. Additionally, the use of Newmark sliding block analyses to predict shallow slope displacements involves several simplifying assumptions, including deformations along a discrete failure surface, rigid perfectly plastic soil behavior, and constant shearing resistance. From a qualitative standpoint, these assumptions are inconsistent with the actual mechanics of the systems they purport to model. In this study, the effects of assumptions of the sliding block model are evaluated in terms of how they might bias sliding block displacement predictions. Using nonlinear dynamic analyses of a soil continuum as a basis for comparison, the sliding block assumptions were shown to result in only a modest unconservative bias in predicting deformations in excess of 50 cm. However, in the range of 5 to 20 cm, the ratios of Newmark- to nonlinear-based predicted displacements ranged from about 80% in profiles with very thin layers of weak material, to less than 50% lower where thicker weak layers in excess of 1.0 m existed, and generally decreased with respect to the thickness of the weak layer. This discrepancy can largely be attributed to the tendency of strains to distribute throughout a weak layer in shallow sloping ground, rather than concentrate along a single failure surface. These results suggest that the use of Newmark-type analyses can result in unconservatively biased displacement predictions in the range of critical limit states for structures at risk for failures in shallow sloping ground.

1 INTRODUCTION

Numerous instances of seismically-induced failure of shallow sloping ground have been observed across the globe in the last half-century or so. Such modes of ground failure, which typically occur in loose sands and silts or soft, sensitive clays, have caused extensive damage to bridges, wharves, embankments, underground pipelines, and other critical infrastructure. In most of these cases, the *in-situ* static shear stresses are smaller than the residual strength of the soil, and permanent lateral deformations are thus driven by transient, dynamic stresses during ground shaking. Such deformations are sensitive to a number of factors that can be difficult to observe in the field or predict in advance of an earthquake. As a result, the process of predicting slope displacements is subject to very high uncertainty and numerous potential sources of bias. In practice, geotechnical engineers often use a range of strategies for estimating lateral displacements. These range in complexity from simplified empirical methods that estimate permanent displacement based on pure statistical regression of case histories, to high-level numerical analyses involving detailed constitutive models that describe the behavior of soils under dynamic loading.

A potentially intriguing class of methods, which lies somewhere in the middle of this range, is based on the sliding block method for predicting permanent displacements. Originally proposed by Newmark in 1965, the sliding block method models a potentially unstable slope as a rigid block resting on a frictional surface, and calculates permanent displacements from an input motion by double-integrating the relative acceleration history between the block and surface. In the half-century since its introduction, numerous sliding block-based methods have been developed to predict displacements in embankment dams (e.g. Makdisi & Seed 1978), landfill systems (Bray & Travasarou 2007, Bray & Repetto 1994), and laterally spreading soils (Olson & Johnson 2008). The sliding block method in its original form is based on three simplifying assumptions: (1) the behavior of the system can be described by simple rigid body motion; (2) permanent deformations manifest along a single, discrete failure plane; and (3) deformations result from rigid-perfectly plastic soil behavior, with a constant shearing resistance.

In this paper, the aforementioned assumptions are assessed with regards to the level of inconsistency with the actual behavior observed at the types of sites in question. A parametric study is presented, in which a series of sliding block-based displacements for a set of shallow sloping sites are compared to those obtained from finite-difference continuum analyses, in order to characterize how the combined effects of the rigid-body and discrete failure plane assumptions may produce biased displacement estimates.

2 BACKGROUND

In contrast to weakening failures, which occur when the strength of a cyclically loaded soil drops below the static shear stress required for equilibrium, deformations associated with inertial instabilities are influenced by a number of factors, many of which are difficult to adequately characterize in the field and to predict *a priori*.

2.1 *Factors affecting slope displacements in the field*

The dynamic response of the site, which depends on the stiffness of the site and the ground shaking characteristics, plays a significant role in both the triggering of strength degradation in weak soils (i.e. liquefaction in sands/silts, cyclic softening or collapse in soft/sensitive clays) and the accumulation of deformations that occur as a result. On a related note, it has been shown that the time at which significant strength loss is triggered relative to the overall duration of shaking significantly influences how post-triggering deformations manifest (Kramer et al. 2016). Spatial variability of soils in the vertical direction affects how strains are distributed with depth and how much they accumulate at ground surface, while horizontal variability will influence how deformation patterns manifest at the surface. Finally, surficial variations in two- or three-dimensional topographic features can result in extremely complex displacement fields that are difficult to observe in post-earthquake reconnaissance and to characterize in forward prediction problems.

2.2 *Assumptions inherent to Newmark sliding block models*

As stated previously, many of the fundamental assumptions that underpin the Newmark model are significant simplifications of the complex mechanisms described above, resulting in inconsistencies that have the potential to produce biased displacement estimates. It is important to understand the implications of these assumptions when using Newmark analyses.

2.2.1 *Rigid block behavior*

Makdisi & Seed (1978) first recognized the value in formulating a method for predicting slope displacements that accounted for the flexibility of the soil mass, introducing a class of sliding block model known as *decoupled* analysis. Kramer & Smith (1997) used analyses of a coupled two-mass system to show that while decoupled analyses tend at least to result in conservative displacement predictions in shallow, stiff sites, they have the potential to be unconservative in

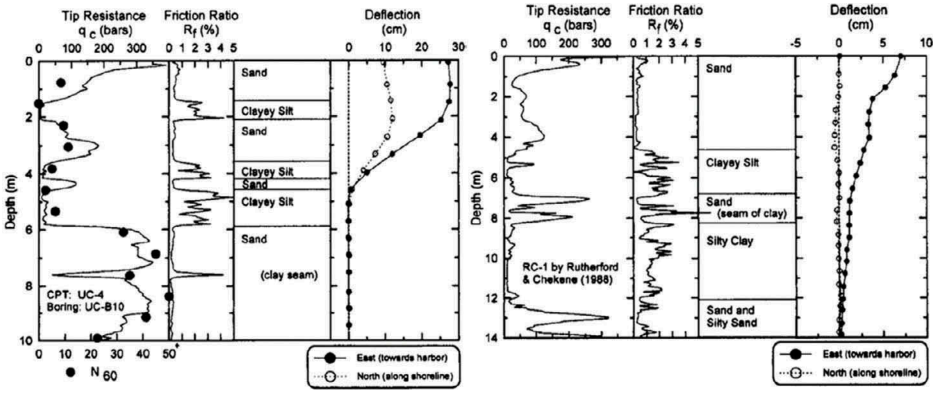

Figure 1. Subsurface conditions and measured lateral displacement profiles from the Moss Landing lateral spreading case history (Boulanger et al. 1995).

softer or deep failure masses such as landfills. Wartman et al. (2003) further emphasized the inconsistencies of the rigid block assumption, showing that rigid block models had the potential to be unconservative in softer sites, particularly when the tuning ratio (the ratio of the natural period of the site to the predominant period of the input motion) is between about 0.2 and 1.3, with significant under-prediction observed at ratios of 0.5 to 0.7.

2.2.2 *Discrete failure surface*

The assumption that deformations are concentrated along a single, discrete sliding surface, while potentially consistent with the mechanics that drive slope failures in slopes comprised of more competent materials with seams of weak material or pronounced bedding planes, is not generally representative of the manner in which slopes with thicker layers of more uniform material deform. Sites with sufficiently thick layers of loose sands or soft clays do not exhibit the kind of single failure surface implied by limit equilibrium stability analyses. Rather, the tendency is for deformations to be distributed throughout the thickness of a weak layer, a classic example of which can be seen in inclinometer data from Moss Landing (Boulanger et al. 1995), where extensive damage to the roadway occurred due to liquefaction-induced lateral spreading during the 1989 Loma Prieta Earthquake (Figure 1).

2.2.3 *Rigid-perfectly plastic behavior*

Finally, the Newmark model assumes that displacements accumulate only when the shear strength of the soil (which is assumed constant) is exceeded by the applied stress generated by the input ground motion. In reality, the constitutive behavior of soils is far more complex; permanent strains tend to develop due to inelastic response at shear stresses lower than the shear strength of the soil. Furthermore, the stiffness and shear strength of some soils is not constant during ground shaking, and generally degrades due to cyclic softening or the generation of excess pore pressure. Qualitatively, all other factors being equal, it is expected that the combination of these two mechanisms (plastic straining and stiffness/strength degradation) would result in larger deformations than those predicted by a simple rigid-perfectly plastic stress behavior. The combination of the aforementioned assumptions has the potential to produce biased sliding block-based estimates of permanent displacements.

3 EVALUATION OF DISCRETE FAILURE PLANE AND RIGID-BODY MOTION ASSUMPTIONS

The objective of the parametric study presented herein was thus to systematically quantify the degree to which these biases may exist, and to identify the conditions in which they may be most prevalent. The study consisted of a comparison between displacements predicted using Newmark analyses and finite-difference analyses of a continuum, for a series of soil profiles containing weak layers of different thicknesses. To simplify and expedite the analyses performed, infinite slopes were modeled using one-dimensional nonlinear analyses. In addition to the weak layer thickness, the soil strength and ground slope inclination angle were varied, and the profiles were subjected to a broad range of ground motion characteristics.

3.1 *Subsurface profiles*

A series of 108 soil profiles was generated for the parametric study, each representing different combinations of weak layer thickness, soil strength, and ground slope inclination angle. A typical profile, consisting of a weak clay layer between two dense sand layers, is shown in Figure 2. The thickness of the weak layer (H), the ground slope inclination angle (β), and the strength of the weak layer were all varied as indicated in Figure 2. The stress-dependent term in the specification of the cohesion was used in order to avoid numerical base-isolation effects observed in profiles of constant cohesion, where failure occurred in the lowest weak sublayer resulted in almost no transmission of stress waves into the overlying sublayers. The constant cohesion term (c_0) was included to ensure a non-uniform pseudostatic factor of safety, and that the minimum yield acceleration for input to the Newmark analyses would be computed in the lowest sublayer.

3.2 *Deformation analyses*

The Newmark analyses were performed using a MATLAB code developed for this specific study, which allowed for sliding in both the downslope and upslope directions. For each profile, the downslope yield acceleration ($a_{y,d}$) was calculated using the static factor of safety FS and the slope angle β (Newmark 1965):

$$a_{y,d} = \left[\frac{c + \sigma'_v \tan\phi \cos^2\beta}{\sigma'_v \cos\beta \sin\beta} - 1\right] \sin\beta \tag{1}$$

The upslope yield acceleration ($a_{y,u}$) was determined via:

$$a_{y,u} = \frac{\tan\theta + \tan\beta}{1 + \tan\theta \tan\beta} \tag{2}$$

where

$$\theta = \tan^{-1}(a_{y,d}) + \beta \tag{3}$$

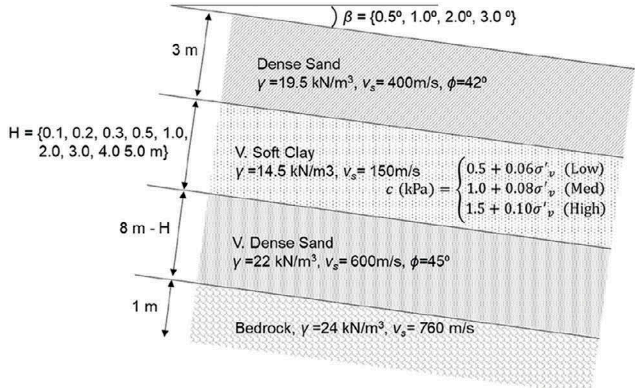

Figure 2. Representative soil profile analyzed in parametric study

The continuum analyses were performed using PSNL (Kramer, personal communication), a finite-difference program for computing the non-linear response of soil continua. In order to assess the discrete failure plane and rigid-body motion assumptions in isolation, the weak layer was simply modeled using a linear elastic-perfectly plastic material model with 1% damping in a total stress analysis. As an explicit finite-difference code, numerical stability requirements control the maximum time step that can be used in an analysis. Because the time step decreases with increasing layer stiffness and decreasing layer thickness, memory limitations largely prevented the use of very stiff or very thin soil layers. The overlying dense sand layers were modeled using the upper-bound curve of the Seed & Idriss (1970) model for sand, and bedrock was modeled as a linear material with 2% damping. For profiles where $H = \{0.1, 0.2, 0.3, 0.5\ \text{m}\}$, the weak layer was divided into 0.1 m-thick sublayers. For profiles where $H = \{1, 2, 3, 4, 5\ \text{m}\}$, memory limitations in the finite-difference code limited the sublayer thicknesses to 0.25 meters.

3.3 *Demonstration case*

The results of a series of analyses for a set of profiles subjected to a single crustal ground motion are shown in Figure 3. The motion was selected and scaled to match a target spectrum generated from ground motion model (GMM)-based estimate of a strike-slip event with M_w=6.75 and R_{jb} = 5 km (see Section 3.4). The selected, scaled motion has a *PGA* of 0.32 g and significant duration of 12.5 seconds. The displacements produced by this motion were generally representative of the median of the overall parametric study. The profiles shown were of the “medium-strength” classification (approximately 6 to 11 kPa, increasing with depth), with a ground slope angle of 2□. In this demonstration case, the shorter duration of the input ground motion allowed for all nine profiles to be modeled using weak sublayer thicknesses of 0.1 m.

For the thinnest-layer case (H=0.1 m), the Newmark- and PSNL-based predictions were within about 2 cm of each other, a difference of about 20%. However, as the thickness of the weak layer increased, the differences between the PSNL- and Newmark-based predictions became more significant. For profiles with thicker weak layers, the combined effects of the compliance of the soil mass, the distribution of strains throughout the weak layer, and the elasto-plastic response (as opposed to rigid-perfectly plastic) of the weak layer resulted in PSNL displacements that were up to twice as large as those predicted by the Newmark analyses.

The time histories in Figure 3 show that the displacements tended to diverge most *not* when the largest displacement pulses occurred (e.g. at approximately 7 seconds), but rather over

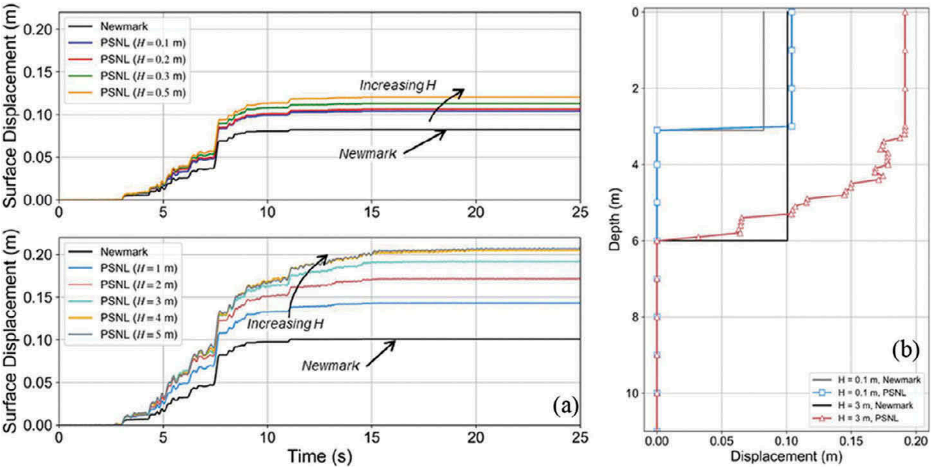

Figure 3. (a) Relative displacement time histories and (b) displacement depth profiles predicted by Newmark sliding block and PSNL analyses for a profile with increasing weak layer thickness.

longer intervals of time, when more moderate displacement increments accumulated over the layer of weak material (e.g. between 11 and 15 seconds). The permanent displacement profiles in Figure 4 show that the largest strains are generally seen in the lowest sublayer, and that they are reasonably well-approximated by the "implied" Newmark displacement profiles. The two analyses tended to diverge in the shallower sublayers, where smaller strains accumulated and produced significant additional displacement at the surface, nearly doubling the displacement from the lowest sublayer to the surface in profiles with thicker weak layers.

3.4 *Parametric analyses*

The overall parametric study consisted of 108 soil profiles, representing all combinations of the variable parameters shown in Figure 2 (weak layer thickness, ground slope angle, and strength classification). Each soil profile was subjected to a total of 90 crustal ground motions (Astaneh-Asl, personal communication) spanning a wide range of magnitudes (6.0 to 8.0) and distances (0 to 80 km). The ground motions were obtained from the NGAWest-2 ground motion database. For each magnitude-distance bin, five ground motions were selected and scaled to match a GMM-generated target spectrum (using the average of the four NGAWest-2 GMMs) for a site with $V_{s,}$=760 m/s and a vertical, surface strike-slip rupture mechanism. Ten additional near-fault motions were also included. The resulting numerical database consisted of 9,720 displacement estimates each obtained from the Newmark and PSNL analyses.

4 ANALYSIS RESULTS AND DISCUSSION

Figure 4 compares the permanent displacements computed by the Newmark and PSNL analyses for the full dataset. The results illustrate some of the systematic differences between the two types of displacement predictions. The plotted lines represent the best linear fit lines to a particular subset of data (hence their curvature at lower displacements in logarithmic space). Figure 5 shows the variation in the ratio of Newmark displacements to PSNL displacements (R_D) with PSNL displacement for several weak layer thicknesses (H). For profiles with $H \leq$ 0.5m, the Newmark model displacements are generally consistent with the PSNL displacements at displacement levels greater than 50 cm. In that range, the Newmark estimated displacements were on average about 85% of those predicted by PSNL. Figure 5 also shows that the PSNL displacements are largely insensitive to weak layer thicknesses less than 0.5 m.

While the Newmark models predicted large displacements in gentle slopes with only modest bias, a more critical design consideration is for smaller lateral displacements, generally in the range of 5 to 20 cm. Displacements at this level tend to cause damage to infrastructure, and to lead to decisions such as whether to pursue ground improvement to mitigate ground

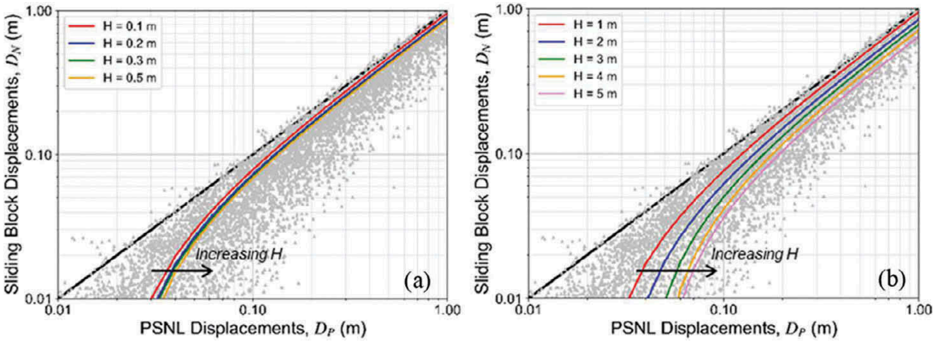

Figure 4. Pairwise displacement plots comparing sliding block- and PSNL-predicted displacements as a function of weak layer thickness for profiles with (a) $H \leq 0.5$ m, and (b) $H \geq 1.0$ m

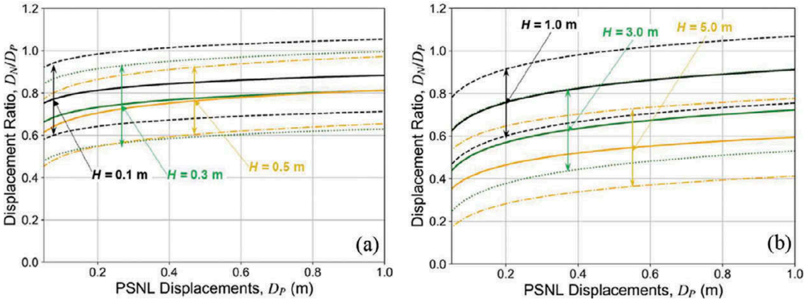

Figure 5. Variation in the ratio of Newmark- to PSNL-based displacements with respect to displacement for profiles with (a) $H \leq 0.5$ m, and (b) $H \geq 1.0$ m

deformation hazards. In this range, Newmark displacements were significantly more biased for profiles where $H \leq 0.5$m, with displacement ratios of about 70% (ranging between 50 and 90%). Additionally, the bias of the Newmark-based predictions was more sensitive to H in this range of displacements; R_D varied from about 65 to 95% for $H = 0.1$ m to 45 to 85% for $H = 0.5$ m.

This trend appears to continue in profiles with $H \geq 1$ m. For displacements greater than 50 cm, Newmark displacements ranged from 55 to 90% of PSNL displacements, with the bias increasing with increasing H. For displacements in the 5 to 20 cm range, the Newmark underpredictions ranged from about 60 to 90% of PSNL displacements at H=1.0 m to only 25 to 65% at $H = 5.0$ m, with much greater sensitivity to increasing layer thickness. Additionally, it is important to note here that the $H \geq 1$ m profiles were modeled with thicker weak sublayers (0.25 m) than the $H < 1$ m profiles (0.1 m). All other factors being equal, it would be expected that a profile with 0.1 m-thick sublayers would produce larger displacement estimates than one with 0.25 m-thick sublayers. For example, in the demonstration case in Section 3.3, where 0.1 m-thick sublayers were used, the PSNL displacements for the $H \geq 1$ m profiles were about 10 to 15% higher than those modeled in the full parametric study using 0.25 m sublayers. As a result, it is likely that the Newmark underpredictions in the 5 to 20 cm range of displacements would be even larger in the $H \geq 1$ m profiles if modeling them with smaller sublayers had been feasible.

Figure 6 illustrates the sensitivity of bias in the Newmark-based predictions to soil strength and ground slope angle. With the exception of very low displacement levels (below about 5–6 cm), the Newmark bias was largely insensitive to the strength level of the soil. There does,

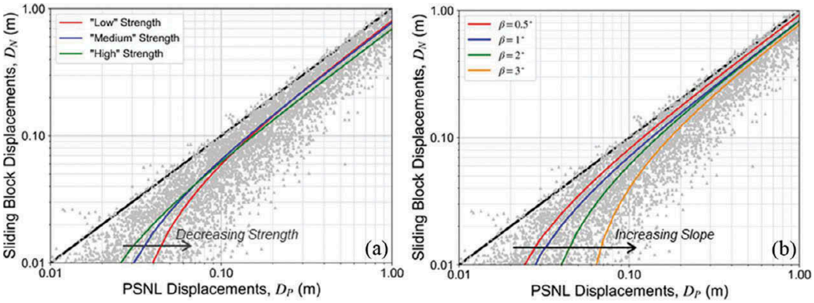

Figure 6. Pairwise displacement plots comparing sliding block- and PSNL-predicted displacements with respect to (a) soil strength and (b) ground slope inclination.

however, appear to be a clear trend of increased bias with increasing ground slope inclination. The sensitivity to ground slope angle appears to be more significant at low-to-moderate displacement levels.

5 SUMMARY AND CONCLUSIONS

Newmark sliding block analyses have been a staple of seismic slope stability evaluation over the past half-century, and appear to reasonably approximate many practical cases of slope response. However, the underlying assumptions of the Newmark method – that a rigid mass of soil slides upon a thin failure surface with constant resistance – are largely inconsistent with the mechanisms that tend to drive deformations in many slopes. In such cases, surface displacements result largely from permanent strains that develop incrementally over the course of ground shaking and are distributed over the thickness of a weak layer, particularly when the stiffness and shearing resistance of that layer varies significantly with time.

To assess the effects of these assumptions on the accuracy of sliding block-based displacement predictions, a parametric study was undertaken in which nearly 10,000 numerical analyses were performed, comparing the displacements predicted by sliding block analyses against those predicted by PSNL, a finite-difference platform capable of representing the nonlinear, inelastic dynamic response of a one-dimensional soil column. The comparison study showed that the assumptions of a discrete failure plane and rigid-body motion produced only a modest unconservative bias at deformations in excess of 50 cm. However, in the range of about 5 to 20 cm, where physical damage occurs and critical design decisions are often made with regards to ground improvement measures, these assumptions led to increased unconservative bias in sliding block predictions, from displacements of about 80% of the PSNL-based predictions in profiles with very thin weak layers, to less than half of the PSNL-predicted displacements where thicker layers of weak materials existed. While the assumptions constant stiffness and shearing resistance are not addressed in this paper, they also have the potential to decrease the accuracy of sliding block predictions even further, particularly in thin-layer cases. The results presented here suggest that, while sliding block analyses may be useful for sites where the soil stratigraphy constrains deformations to thin zones, their applicability to soil profiles with thick layers of weak soils, where strains are likely to be spatially distributed, can produce displacements with considerable unconservative bias.

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