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An insight into the prediction of limiting fines content for mixtures of sand with non-plastic fines based on monotonic and cyclic tests

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ABSTRACT: The paper offers insight into the determination of threshold fines content, FC_{th}, or limiting fines content, based on experimental data from both undrained monotonic and cyclic tests performed on sand-silt mixtures obtained by mixing different amounts of non-plastic fines (in a range 0-40%) to a host silica sand. Different methods were adopted for the identification of FC_{th} such as: variation of index properties (maximum and minimum void ratios) with fines content, change of critical state parameters with fines content (monotonic loading), variation of the cyclic resistance ratio at a reference number of loading cycles with fines content (cyclic loading). The threshold fines content assessed by experimental data through monotonic and cyclic test results were then compared with the values inferred from several analytical relations to verify to what extent the predictions made by these equations could be reliable.

1 INTRODUCTION

Silt-sands mixtures are very common in natural depositional environment or in earth-fill and tailings from man-made activity, hence the knowledge of their behavior is a crucial aspect in many practical applications. Experimental evidence suggests that, if global void ratio is used as the basis for comparison, monotonic and cyclic strength of the mixture decreases as the content of silt increases up to a threshold fines content.

On the other hand, further increase in the silt content beyond the threshold value would increase the strength. Threshold fines content (FC_{th}), also termed “limiting fines content” (Polito 1999, Harzibaba 2005) or “transitional fines content” (Yang et al. 2006) in the literature, is the specific value of the fines content at which the behavioral properties of the mixture is reversed (Mohammadi & Qadimi 2015). The threshold value serves as an indicator for the nature of mixed soils varying from being sand-dominant to fines-dominant (Thevanayagam et al. 2002, Yang et al. 2006). A simple approach to describe the monotonic and cyclic behaviour of sands with non-plastic/low plasticity fines (for $FC < FC_{th}$) is based on the concept of equivalent granular void ratio (Thevanayagam et al. 2002) and it requires just the prediction of FC_{th} of a sand-silt mixture.

In this context, the present paper offers an insight into the prediction of FC_{th} for Ticino sand mixed with non-plastic fines based on both experimental and theoretical approaches. Different methods to evaluate the limiting fines content by experimental data, based on the critical state behaviour from monotonic triaxial tests and the undrained cyclic strength from cyclic simple shear tests, are presented and discussed. Additionally, the reliability of theoretical or semi-empirical relationships proposed in the literature (Polito 1999, Harzibaba 2005, Rahman & Lo 2008) was evaluated through comparing the FC_{th} values predicted by these relations with what can be inferred from the experimental data, also considering additional published data of sand-silt mixtures. This is important especially when experimental data are not available.

2 TESTING PROGRAM AND MATERIALS

The host sand used in this study is Ticino sand, a clean uniform size silica sand with a mean grain size (D_{50}) equal to 0.56 mm. Specific gravity G_s is 2.68 and the tests on the minimum and maximum void ratios according to ASTM D4253 and ASTM D4254 resulted in 0.58 and 0.93, respectively. The fine material (d_{50}=0.025 mm) added to the host sand is a natural non-plastic silt collected from Ticino river bank deposits. Seven different fines contents ranging from 0% to 40% were used in the experimental investigation. The ASTM procedures for determining maximum (e_{max}) and minimum (e_{min}) void ratios adopted in the present study and other similar research (i.e. Yang et al. 2006, Papadopoulou & Tika 2008) are applicable to soils containing up to 15% fines content, and tentatively other methods have been proposed in the literature (Chang et al. 2015). Although the values of e_{max} and e_{min} are somewhat different depending on the method adopted, their trend does not vary with fines content (Chang et al. 2015). Additionally, for a more consistent determination of the minimum void ratio, the Standard Proctor test was used as well. For the tested mixtures of silt and sand, the diameter ratio χ, introduced in the context of the binary packing theory (McGeary 1961) and defined as the ratio between the size of sand at 10% finer (D_{10}) and the size of fines at 50% finer (d_{50}), was equal to 17. The test program comprised undrained cyclic simple shear tests (a total of 60 tests) and undrained monotonic triaxial tests (a total of 57 tests), carried out on specimens of sand-silt mixtures reconstituted by the moist tamping method at different initial (global) void ratios and subjected to different consolidation effective stresses. Cyclic simple shear tests (*CSS*) were performed using a modified *NGI SS* apparatus (specimen diameter of 80 mm and height of 20 mm) (Porcino et al. 2006). In this apparatus, the lateral restraint provided by the reinforced membrane assures a condition of zero horizontal extension at the lateral boundary. Undrained *CSS* tests were carried out under constant volume conditions by an automated control system developed at University Mediterranea of Reggio Calabria (Italy). In the triaxial tests, after saturation, the specimens (diameter of 100 mm and height of 100 mm) were isotropically consolidated at a prefixed effective confining stress, and the void ratio was measured at the end of consolidation (e_o). Undrained shearing phase was carried out at a constant rate equal to 0.104 mm/min. The tests were manually stopped at an average axial strain level equal to 25% in order to capture the behaviour of the specimens at large/ultimate strains (steady state/critical state). All details of test procedure and apparatus can be found in Porcino & Diano (2017) and Porcino et al. (2019).

3 EXPERIMENTAL RESULTS

3.1 *Identification of FC_{th} from index data*

Figure 1 reports the trend of maximum (e_{max}) and minimum (e_{min}) void ratio of the different *TS*-silt mixtures versus fines content in the range 0-40%.

As can be noted in this figure, the maximum void ratio follows a simple pattern similar to that exhibited by the minimum one: both e_{min} and e_{max} decrease with the fines content up to a minimum value after which, as the fines content continues to increase, a reversed trend is observed (i.e., the packing density decreases). A possible explanation of this behavior lies in the fact that, at low fines contents, the fines fill the voids between the sand particles without breaking the sand particle contacts, causing a reduction in the void ratio (Lade et al. 1998). It is assumed in this study that the FC_{th} is reached when the voids of the sand particles are totally filled with fines i.e. the void ratio reaches its lowest point (Lade et al. 1998, Yang et al. 2006). As the fines content increases further, the sand particles start to separate and the void ratio increases.

Figure 1 evidences that due to the flat trend exhibited by the two curves in the transitional zones, the determination of FC_{th} is not straightforward; a likely value within the range between 25% and 27.5% can be assumed for it. The variation of the void ratio with the fines content observed in real soils (Figure 1) becomes "flatter" relative to the ideal *V*-shaped curve of the binary packing density (McGeary 1961). It implies that an increase in fines content increases the participation of the fines in the force chains, consistently with previous research

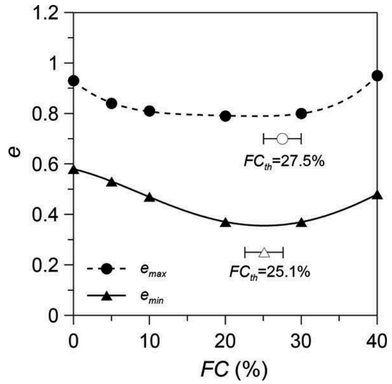

Figure 1. Trend of index properties with fines content of *TS*-silt mixtures.

reported in the literature (Lade et al. 1998, Yang et al. 2006, Papadopoulou & Tika 2008, Fuggle et al. 2014, Zuo & Baudet 2015). Since the determination of FC_{th} from solely void ratio involves uncertainties, the method based on index data of sand-fines mixtures can give only an indication of the possible range in which the true value of FC_{th} may exist (Zuo & Baudet 2015). Accordingly, it is judged preferable to determine FC_{th} by laboratory tests with monotonic or cyclic loading, as will be presented in the following sections.

3.2 *Identification of* FC_{th} *by undrained monotonic triaxial tests*

The determination of FC_{th} was based on monotonic triaxial compression response of *TS*-silt mixtures under undrained conditions (Figure 2). In particular, the figure depicts typical results of triaxial tests at similar (global) void ratio (e_o), in terms of effective stress-path plots. Figure 2 indicates that, as the fines content increases, the behaviour type is more contractive and changes from non-flow to flow types up to 30% fines content. Beyond this value, a reversal response was observed, as it is apparent for FC=40%. he steady state line (*SSL*) for each mixture was assessed and described in the *e* (void ratio)-*p'* (mean effective stress) plane by a power function (Li & Wang 1998, Yang & Li 2004). It was found that the position of the *SSL* in the *e*-p' plane changes with fines content and, in particular, the steady state line moves downwards as the fines content increases from 0% to 30%, and moves up again when the fines content increases from 30% to 40%.

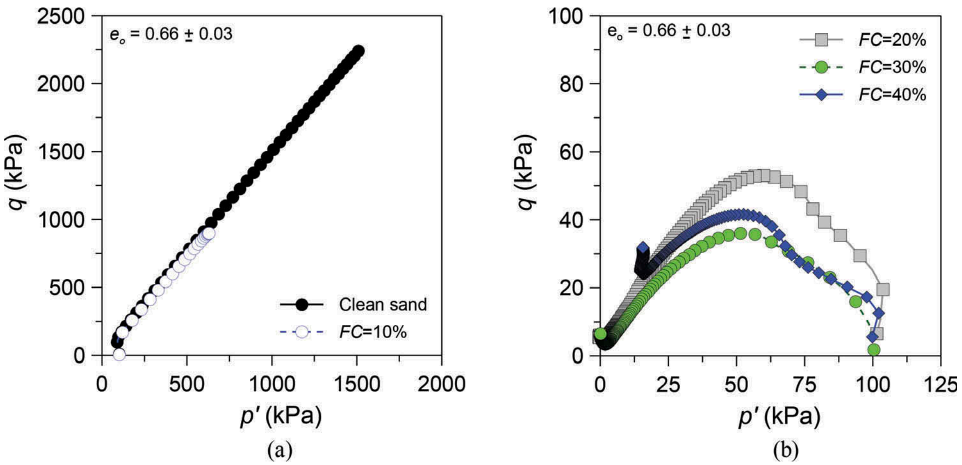

Figure 2. Effect of fines contents on results of undrained monotonic triaxial tests in terms of stress-paths for Ticino sand-silt mixtures.

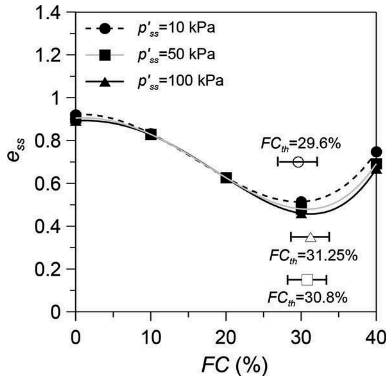

Figure 3. Variation of steady state void ratio with fines content at different mean effective stresses.

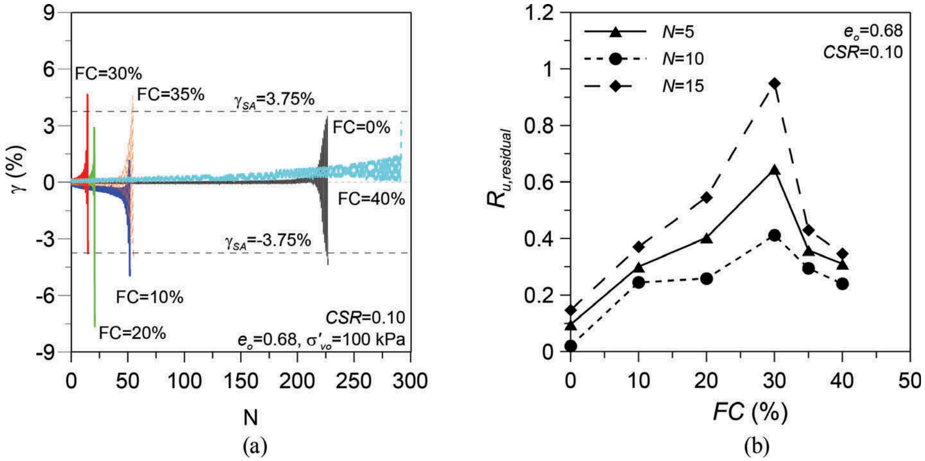

Figure 4. Influence of fines content on (a) cyclic shear strain development and (b) excess pore water pressure generation in undrained simple shear tests for *TS* with non-plastic fines.

To determine the reversal value of *FC* objectively, the void ratio e_{SS} on the *SSL* at p'_{SS} = 100 kPa was plotted against fines content as presented in Figure 3. The reversal point of data defines the FC_{th} which results around 31%. To investigate the sensitivity of FC_{th} on steady state mean effective confining pressure p'_{SS}, the above procedure was repeated using also void ratio data at p'_{SS} of 10 kPa and 50 kPa. Figure 3 shows that the fines content at the reversal point is poorly affected by the selected value of p'_{SS}.

Consistently with the findings reported by other authors (Rahman 2009, Rees 2010), the procedure for the determination of FC_{th} based on steady state data is poorly affected by p'_{SS} in the range of 50 kPa - 300 kPa. Consequently, it can be standardized at p'_{SS} of 100 kPa.

3.3 *Identification of* FC_{th} *by undrained cyclic simple shear tests*

Figure 4a shows the shear strain (γ) vs. number of cycles (N) response of Ticino clean sand with non-plastic silt prepared at (consolidation) void ratio e_o=0.68 in *CSS* tests conducted at a constant cyclic stress ratio $CSR=\tau_{cyc}/\sigma'_{vo}$ =0.10, where τ_{cyc} is the applied horizontal shear stress and σ'_{vo} is the vertical effective stress. It is apparent that the number of cycles needed to reach the failure condition (defined as a single amplitude of shear strain γ_{SA}=3.75%) decreases rapidly with increasing fines content up to a value of *FC*=30%, after which the trend is inverted.

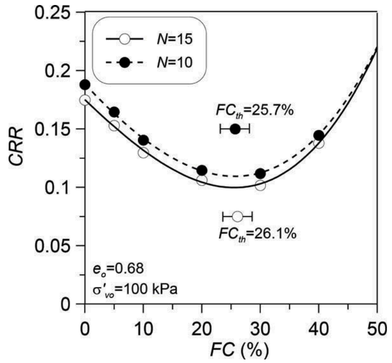

Figure 5. Identification of FC_{th} based on undrained cyclic *SS* tests for *TS*-fines mixtures at the same global void ratio.

The same type of reversal response can be observed in terms of variation of excess pore water pressure ratio ($R_u = \Delta u / \sigma'_{vo}$) with fines content. R_u is defined as the ratio between the excess pore water pressure (Δu) measured at the end of different numbers of loading cycles and the initial vertical effective stress (σ'_{vo}) (Figure 4b).

Based on the cyclic *SS* test results, the relationship between cyclic resistance ratio to liquefaction and the fines content is given in Figure 5. The results refer to *CSS* tests conducted at constant (global) consolidation void ratio e_o=0.68 for two different N_f values, N=10 and N=15. Figure 5 demonstrates that, at a given number of cycles, *CRR* first decreases with increasing *FC* up to a threshold value and, thereafter, increases with further *FC*. The curves appear to change direction at a fines content of about 26%, regardless of N_f.

3.4 *Determination of* FC_{th} *using calculation methods and comparison with the experimental values*

In conjunction with the determination of limiting fines content from experimental data, calculation methods have been proposed in the literature by several authors for predicting FC_{th} based on physical and grading features of sand-silt mixtures.

Polito (1999) proposed the following expression where the transitional fines content is calculated as the ratio of fines solid weight, W_{fines}, to sand solid weight, W_{sand}:

$$FC_{th} = \frac{W_{fines}}{W_{sand}} = \frac{G_f \cdot e_s}{G_s \cdot (1 + e_f)} \tag{1}$$

where G and e are the specific gravity and the maximum void ratio, while the subscripts of f and s denote fines and sand, respectively.

Considering the traditional definition of the fines content as a fraction of the total amount of soil, Harzibaba (2005) modified Eq. (1) and proposed a relation of FC_{th} based on the ratio of the weight of the fines, W_{fines}, to the total weight of the sand and fines, $W_{sand} + W_{fines}$, expressed as:

$$FC_{th} = \frac{W_{fines}}{W_{sand} + W_{fines}} = \frac{G_f \cdot e_s}{G_f \cdot e_s + G_s \cdot (1 + e_f)} \tag{2}$$

A semi-analytical approach was proposed by Rahman & Lo (2008) who carried out back analyses on nine sets of data to predict the value of FC_{th}. They proposed the following expression:

$$FC_{th} = A \cdot \left(\frac{1}{1 + e^{\alpha - \beta \cdot \chi}} + \frac{1}{\chi} \right) \tag{3}$$

where the coefficient A is an asymptotic value equal to 0.40, while α and β are fitting parameters equal to 0.50 and 0.13, respectively.

The reliability of the three equations was evaluated by comparing the predicted FC_{th} values with the corresponding ones inferred from experimental data based on index properties (*IP*), monotonic tests (*M*) and cyclic tests (*C*). This comparison is presented in Table 2 and Figure 6 for various sand-fines mixtures investigated in the literature, including that studied in the present research. Since the values of FC_{th} based on index properties result in most cases lower than those obtained from monotonic and cyclic tests (see Table 2), only the latter data were used for the comparison with those inferred from the calculation methods in Figure 6.

Figure 6a shows how the experimental threshold fines content varies with the particle diameter ratio χ, together with the predicted trend suggested by Rahman & Lo (2008) (Eq. 3). This relationship reveals a distinct pattern with two important features (Rahman 2009): i) FC_{th} is at minimum value when the particle diameter ratio χ is at $(D/d)_{\text{crit}}$ (equal to 7), consistently with binary packing considerations (McGeary 1961); ii) an increase of χ beyond the reversal point leads to a gradual increase of FC_{th} up to an asymptotic value, since more particles can fit in the same void space between the large particles, thus providing a higher FC_{th}. Given the empirical nature of this formulation, it is expected that the calculated FC_{th} using Eq. (3) is close to the values determined from the experimental data, as it can be observed in Figure 6a.

Additionally, it can be noted from Figure 6b that the values predicted by Eq. (2) are significantly lower than the experimental values, in agreement with other authors (i.e. Mohammadi & Qadimi 2015). The predictions made by Eq. (1) and (3) appear to be more consistent with the experimental results, providing acceptable predictions of FC_{th} in most of the cases (Figure 6a, b). It is worth noting that the application of the Eqs. (1) and (2) requires the knowledge of physical and index properties of the fines and the host sand. For this reason, some data points of Figure 6a are not shown in the Figure 6b.

In the present study for *TS*-fines mixtures an average value of FC_{th}=28.5% obtained from monotonic and cyclic tests was applied for the comparison with the calculation methods. Figure 6b evidences that the value for FC_{th} calculated from the relationship by Polito (1999) (FC_{th}=30%) is in good agreement with the experimental average value, while FC_{th} derived from Harzibaba's relation (FC_{th}=23%) tends to underestimate the experimental value. Eq. (3) proposed by Rahman & Lo (2008) provides a value of FC_{th}=36% being substantially higher than the

Table 2. Comparison of the theoretical and experimental threshold fines content of silty sands (data revised by the authors).

Source work	$\chi=D_{10}/d_{50}$	Threshold fines content (%)					
		Theoretical results			Experimental results		
		Eq. (1)	Eq. (2)	Eq. (3)	IP	M	C
Huang et al. (2004)	1.8	34	25	39	-	41	41
Yang & Wei (2012)	2.7	41	29	33	39	-	-
Naeini & Baziar (2004)	2.8	42	29	33	25	37.5	-
Polito (1999)	2.9	36	27	33	37	-	43
Dash & Sitharam (2011)	3	26	21	32	20	21	21
Xenaki & Atanasopoulos (2003)	4	-	-	30	49	-	44
Yang & Wei (2012)	4.6	36	27	30	27	-	-
Karim & Alam (2017)	4.7	36	27	30	-	30	-
Baziar & Sharafi (2011)	6	30	23	29	24	-	30
Yang et al. (2006)	7	40	28	30	25	30	-
Papadopuolou & Tika (2008)	9.6	32	24	31	25	35	35
Polito & Martin (2001)	10	32	24	32	25	-	35
Thevanayagam et al. (2002)	16	-	-	36	20	35	-
Present study	17	30	23	36	26	31	26
Rahman (2009)	40	-	-	41	-	39	-

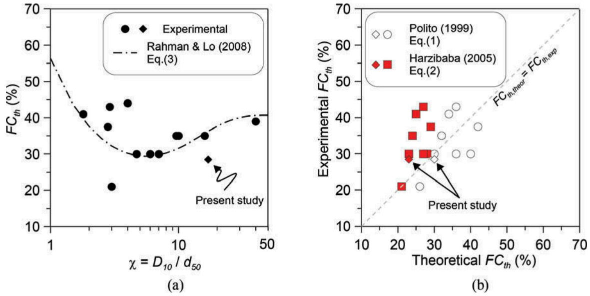

Figure 6. Comparison of the theoretical and experimental threshold fines content of silty sands (data of Table 2.

experimental value. Finally, the transitional fines content obtained from Eqs. (1–3) for *TS*-fines mixtures are in a wide range from 23% to 36%. Other soil characteristics not considered in these equations, such as particle shape and mineralogy, may play a significant role in the behaviour of the sand-fines mixtures, especially for mixtures with intermediate values of the particle size ratio $10 < \chi < 25$ (Zuo & Baudet 2015), such as that tested in the present study.

4 CONCLUSIONS

The transitional fines content is a key parameter for understanding and predicting the behaviuor of sand-silt mixtures since it represents the boundary between sand-dominated and fines-dominated behaviour. In the present study, an experimental investigation was conducted on Ticino sand–non-plastic silt mixtures with different percentage of fines. The mixtures were characterized by an intermediate value of the particle size ratio between the size of the sand and the fines (χ) equal to 17. The threshold fines content of *TS*-fines mixtures was determined experimentally based on the variation with fines content of: 1) maximum and minimum void ratios, 2) steady/critical state parameters (undrained monotonic triaxial tests), 3) cyclic resistance ratio at a given number of loading cycles (undrained cyclic simple shear tests). It was found that:

1. Ticino sand with a certain amount of silt, which is near to the FC_{th}, showed a more pronounced flow type and brittle behaviour in undrained monotonic triaxial tests and a greater susceptibility to cyclic liquefaction in undrained simple shear tests;
2. a range for FC_{th} between 26% and 31% could be identified from experimental test results;
3. the methods based on analytical relationships (Polito 1999, Harzibaba 2005, Rahman & Lo 2008) provided less consistent results among them in terms of FCth varying between 23% and 36%;
4. the comparison of the FC_{th} values predicted by the aforementioned relationships and the corresponding ones determined experimentally highlights that the equations proposed by Rahman & Lo (2008) and Polito (1999) are relatively successful when applied to the majority of datasets published in the literature for various sand–fines mixtures with a scatter that can be probably affected by the different particle size ratios of the investigated sand-fines mixtures. The predictions made by the relationship suggested by Harzibaba (2005) provided an underestimation of the experimental values in most of the cases, in agreement also with the findings reported by previous authors.

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