Earthquake Geotechnical Engineering for Protection and Development of Environment and Constructions – Silvestri & Moraci (Eds)
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Experimental study on fatigue life of limestone in karst tunnel

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ABSTRACT: In order to study the fatigue characteristics of karst limestone under cyclic and seismic loads, low cycle fatigue tests were carried out on karst limestone at the loading frequency of 0.5–3.0 Hz. The fatigue test results show that the fatigue life of saturated rock decreased significantly and increased linearly with the decreased of the upper limit stress. Limestone also has a fatigue threshold, which increased with the increased of loading rate. The stress amplitude plays a key role in fatigue life of the limestone. The fatigue life of the limestone increased with the loading frequency increasing. The relationship between fatigue life and loading frequency is linear in double logarithmic coordinates, which the higher the loading rate, the greater the rock strength. A fatigue life assessment model based on load frequency and stress level was proposed by introducing correction coefficient.

1 INTRODUCTION

With the development of the high-speed and heavy haul railways, the additive effect of low-cycle fatigue damage of rock wall between karst cave and tunnel caused by the combined action of long-term train vibration cyclic load and the seismic dynamic loading is an important problem which restricts the long-term stability of such tunnels.

The fatigue damage law and fatigue life of limestone are the foundation of long-term stability analysis of surrounding rock of karst tunnel under the train vibration cyclic load and the seismic dynamic loading. However, in the previous studies, a lot of research of the fatigue properties and prediction models of metal concrete materials has been done, while the fatigue properties of rocks were rarely studied. Haimson & Kim (1972) proved that the fatigue life of the two marbles is closely related to the rock type and the stress level. When the stress level differs, the fatigue failure cycles of Georgia marble and Tennessee marble are different. Attewell & Farmer (1973) studied the effect of loading frequency (0.3Hz, 2.5Hz, 10Hz, 20Hz) dolomite fatigue life. Ishizuka & Abe (1990) also proved that the fatigue life of rocks increased with the increase of loading frequency, and the fatigue strength of rocks under wet conditions is about 7% lower than that under dry conditions. Singh (1989) proved that the fatigue life of Australian miscellaneous sandstone increased with the decrease of stress amplitude, and the number of loading cycles increased logarithmically with the decrease of stress amplitude. Zhen & Hai (1990) showed that the deformation caused by sinusoidal wave loading was greater than that of triangular wave, and the longer the cyclic amplitude, the shorter the life of rock. Ge & Jiang (2003) found that the rock fatigue failure had a threshold value, which corresponded to the demarcation point of the linear and nonlinear.

Through the low cycle fatigue test of complete limestone under different upper limit stress, stress amplitude, loading frequency and saturation state, the fatigue performance and fatigue life of limestone under train vibration cyclic load and ground vibration dynamic load were studied experimentally, and the fatigue life prediction model of limestone was proposed by fitting test results. It provides a scientific basis for the long-term stability analysis of karst tunnels.

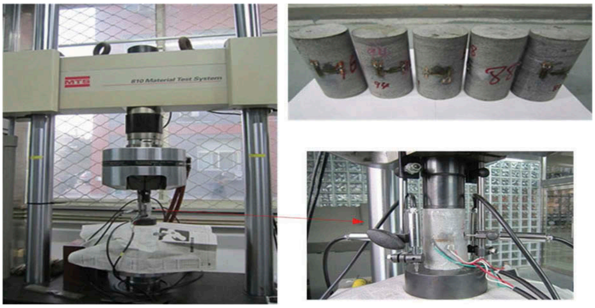

Figure 1. Test equipment and limestone samples

2 ROCK SAMPLES AND TEST SCHEMES

Fatigue tests of limestone samples were taken from Longlingong karst railway tunnel, which is located in Yichang City, Hubei province, China. The specimens were prepared into the standard samples with a diameter 50mm and a height 100mm, as shown in Figure 1. The dynamic uniaxial cyclic fatigue tests of the limestone were carried out by the dynamic test system of MTS 810, as shown in Figure 1.

The deformation monitoring and data acquisition system are composed of two pairs of LVDT sensors (supplemented by strain gauges), as shown in Figure 1. Focus II high-speed synchronous data acquisition instrument and PC system. The fatigue test was divided into two stages by stress control, as shown in Figure 2.

The upper limit stress ratio (S) and lower limit stress ratio ($\bar{S}$) are defined as the ratio of upper limit stress ($\sigma_{\max}$) and lower limit stress ($\sigma_{\min}$) to rock uniaxial compressive strength (σ_c) respectively. In order to obtain the fatigue characteristics of the limestone, a series of fatigue tests were carried out under different upper limit stress, stress amplitude ($\Delta\sigma$=$\sigma_{\max}$-$\sigma_{\min}$) and loading frequency (f) respectively. The dynamic load is compressed by sinusoidal cyclic pressure. The main influence of the vibration load frequency of the train on the surrounding rock at the bottom of the tunnel was the low frequency part. Therefore, the loading frequency of the fatigue test was estimated by referring to the formula for calculating subgrade vibration ($f = v/l$), where v is the driving speed and l is the distance between the wheel pairs, which the Chinese railway train usually takes 23.6m. The experiment was carried out at the loading frequency of 0.5~3.0Hz, which corresponding train operation speed is 40~250 km/h approximately. The vibration frequency of tunnel surrounding rock caused by earthquake is generally lower, so in most cases, this frequency range can cover the seismic response. The upper limit stress ratio, the lower limit stress ratio and the stress amplitude were set to 0.9, 0.3, 0.6σ_c respectively. In addition, tests were conducted on dry as well as saturated samples.

3 TEST RESULTS AND ANALYSES

3.1 *Effect of different water saturation on fatigue life*

According to the loading scheme shown in Table 1, the low cycle fatigue tests of limestone were carried out under dried and saturated conditions at 1Hz, 2Hz, and 3Hz loading

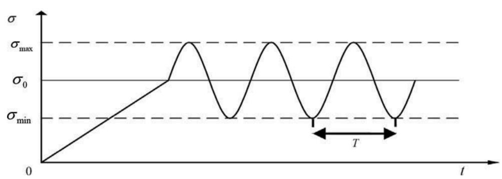

Figure 2. Schematic diagram of two stages loading for cyclic fatigue test

Table 1. Fatigue life statistics of limestone under different saturation state conditions

			Dried			Saturated		
Frequency (Hz)	Upper limit stress ratio	Stress amplitude	Sample number	Fatigue life	Mean fatigue life	Sample number	Fatigue life	Mean fatigue life
1.0	0.9	$0.6\sigma_c$	9	75		93	6	
1.0	0.9	$0.6\sigma_c$	53	102	188	79	18	69
1.0	0.9	$0.6\sigma_c$	82	386		90	184	
2.0	0.9	$0.6\sigma_c$	52	270		42	89	
2.0	0.9	$0.6\sigma_c$	11	371	382	59	281	325
2.0	0.9	$0.6\sigma_c$	94	503		80	604	
3.0	0.9	$0.6\sigma_c$	10	399		64	257	
3.0	0.9	$0.6\sigma_c$	55	954	839	91	499	581
3.0	0.9	$0.6\sigma_c$	76	1164		85	986	

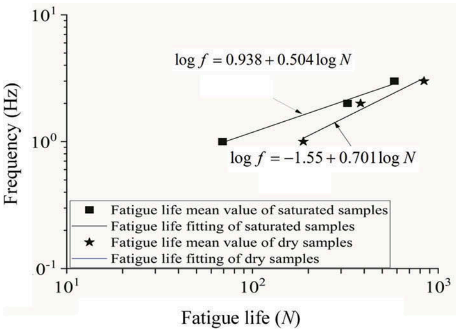

Figure 3. Relationship between fatigue life and loading frequency of drying and saturated specimens

frequencies. The measured fatigue life (N) and mean fatigue life ($\bar{N}$) were summarized in Table 1.

The fatigue life of saturated limestone was lower than dried limestone under the same loading condition and the test results were quite discrete. The test data of the mean fatigue life of dried samples were plotted in double logarithmic coordinates (Figure 3). The fatigue life of saturated specimen increased with the increased of loading frequency, but the growth rate of fatigue life was obviously slower than that of dried specimens. A dual logarithmic estimation model for saturated fatigue life was obtained by linear fitting (as followed in Equation 1).

$$\log f = -0.938 + 0.504 \log N \tag{1}$$

The results of uniaxial compressive test before fatigue test show that the compressive strength of the limestone decreased by about 6.4% compared with that of dried limestone. Under the same cyclic stress, the softening effect of water on rock will aggravate the fatigue damage speed of the limestone, reduce the fatigue resistance of rock and short the life span. Therefore, the softening effect of water on the limestone was the main reason why the fatigue life of saturated limestone is less than that of dry limestone.

3.2 *Influence of the upper limit stress on fatigue life*

The fatigue test results of the limestone under different upper limit stress were shown in Table 2. The relationship between fatigue life and stress level was in accordance with exponential function (Equation 2). Equation 3 is obtained by fitting the linearized measured data.

Table 2. Fatigue life statistics of the limestone under different upper limit stress

Sample number	Upper limit stress ratio	Lower limit stress ratio	Stress amplitude	Fatigue life
35	0.75	0.15	$0.6\sigma_c$	No destruction (1022022)
19	0.80	0.20	$0.6\sigma_c$	2782
31	0.80	0.20	0.6σc	4420
47	0.80	0.20	$0.6\sigma_c$	No destruction (15628)
70	0.85	0.25	$0.6\sigma_c$	752
62	0.85	0.25	$0.6\sigma_c$	1010
17	0.85	0.25	$0.6\sigma_c$	1569
52	0.90	0.30	$0.6\sigma_c$	270
11	0.90	0.30	$0.6\sigma_c$	371
94	0.90	0.30	$0.6\sigma_c$	503
49	0.95	0.35	$0.6\sigma_c$	21
67	0.95	0.35	$0.6\sigma_c$	34
15	0.95	0.35	$0.6\sigma_c$	97

* Numbers in parentheses indicate the number of periodic load cycles.

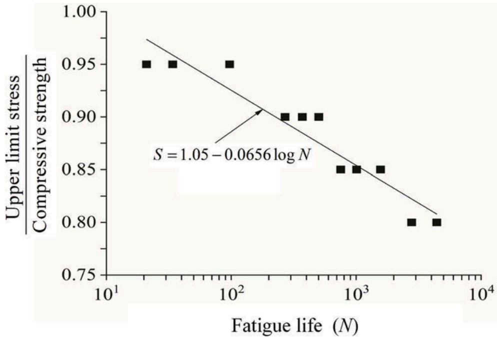

Figure 4. The relationship between the fatigue life and the upper stress limit.

$$N \cdot e^{15.24S} = 1.088 \times 10^{16} \tag{2}$$

$$S = 1.052 - 0.0656 \log N \tag{3}$$

As shown in Figure 4, the fatigue life of the limestone increased with the decreased of the upper limit stress. Many fatigue tests of metal and concrete materials have proved that when the stress amplitude is constant, the lower the upper limit stress level of the material, the longer the fatigue life, but the stress level has a certain critical value. When the stress is lower than this critical value, the fatigue life of the material tends to infinite, which is the same as the Ge & Jiang (2003) proposed by the rock fatigue. According to Ge's point of view, the fatigue threshold corresponds to the boundary stress level between the linear segment and the nonlinear segment of the complete stress and strain relation curve of the rock, and this stress level is called the initiation stress (σ_{ci}). If the upper limit stress is lower than the initial stress, the rock will not suffer fatigue damage even if the cycle is millions.

The threshold value of different rocks fatigue obtained by Chinese scholars are different, ranging from 60% to 85% of σ_c. The initiation stress and compressive strength of the limestone were not constants, but increased with the increased of loading strain rate ($\dot{\varepsilon}$). The ratio of initiation stress to compressive strength satisfied the relationship shown in Equation 4 within the loading strain rate (2.44×10–^{6} ~ 1.92 ×10–^{4}).

$$\sigma_{ci}/\sigma_c = 0.102 log\dot{\varepsilon} + 1.06 \tag{4}$$

Table 3. Fatigue threshold value of the limestone under different loading rates

Loading frequency (Hz)	0.5	1.0	1.5	2.0	2.5	3.0
Mean strain rate	1.50×10^{-3}	2.91×10^{-3}	4.51×10^{-3}	5.75×10^{-3}	7.25×10^{-3}	8.60×10^{-3}
Threshold value / Compressive strength (%)	77.2	80.1	82.1	83.2	84.2	84.9

Table 4. Statistics of fatigue life of the limestone under different stress amplitudes

Sample number	Stress amplitude	Upper limit stress ratio	Lower limit stress ratio	Fatigue life
52	$0.60\sigma_c$	0.9	0.3	270
11	$0.60\sigma_c$	0.9	0.3	371
94	$0.60\sigma_c$	0.9	0.3	503
32	$0.50\sigma_c$	0.9	0.4	549
29	$0.50\sigma_c$	0.9	0.4	878
5	$0.50\sigma_c$	0.9	0.4	1228
27	$0.40\sigma_c$	0.9	0.5	735
65	$0.40\sigma_c$	0.9	0.5	3141
60	$0.40\sigma_c$	0.9	0.5	4846

When the loading frequency is 2.0Hz, the average strain rate is 5.75×10^{-3} during the whole cyclic process. The fatigue threshold values of the limestone are shown in Table 3, and the threshold values of fatigue increased with the increased of loading rate.

3.3 *Effect of the stress amplitude on fatigue life*

The fatigue life test results of the limestone under different stress amplitudes are shown in Table 4. The loading frequency is 2 Hz and the upper stress limit ratio is kept at 0.9.

The fatigue life of the limestone decreased with the increased of stress amplitude. The test data of 9 samples were linearly fitted, and the fitting results were shown in Figure 5. Xiao (2009) carried out an experimental study on the relationship between fatigue life and stress amplitude of granite. Compared with limestone, the stress amplitude has a greater impact on the fatigue life of granite. However, all studies show that the fatigue life decreased with the increased of stress range.

It can be seen from Figure 5 that the effect of stress amplitude on fatigue life obviously does not fully reflect the effect of stress level on the fatigue life of rock, and the upper limit stress, stress amplitude and average stress should be considered at the same time. Based on the modification of the fatigue life model of concrete proposed by Aas-jakobsen (1970), a prediction model for the fatigue life of limestone was proposed, as shown in Equation 5.

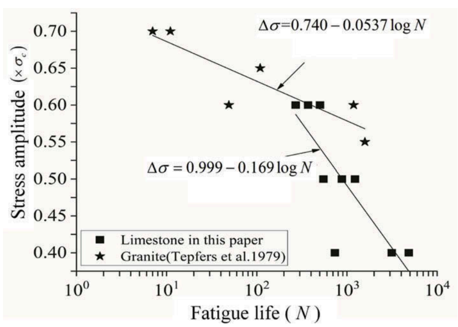

Figure 5. The relationship between fatigue life and stress amplitude

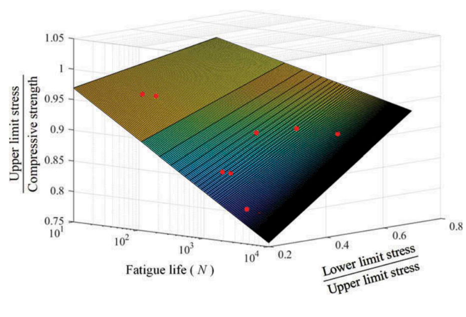

Figure 6. 3D figure of multi-parameter model for fatigue life estimation

$$S = 1.04 - 0.00887(1 - R)\log N \tag{5}$$

Where $S = \sigma_{max}/\sigma_c$; $R = \sigma_{min}/\sigma_{max}$.

As shown in Figure 6, Equation 5 can well fit the fatigue test data including Tables 2 and 4.

3.4 *Effect of the loading frequency on fatigue life*

At present, the relationship between rock fatigue life and loading frequency was not clear, and the relationship between loading frequency and fatigue life obtained by different researchers was even the opposite. Jiang (2003) and Xiao (2009) believed that the loading frequency is actually the effect of loading rate. When loading at high frequency, the loading rate is faster, so the failure rate of rock is faster and the fatigue cycle is shorter. The fatigue test results of sandstone under three different loading strain rates given by (Ray et al. 1999) showed that the fatigue life of sandstone decreased at high strain rate (high loading frequency). The experimental results of Atewell & Farmer (1973) showed that the fatigue life of dolomite increased with the increased of loading frequency. The uniaxial and triaxial fatigue tests conducted by Ishizuka et al. (1990) showed that the loading frequency of granite was linearly related to its fatigue life in the double logarithmic coordinate system.

In order to study the effect of loading frequency on the fatigue life of limestone, fatigue tests were carried out with stress control method and sinusoidal wave loading method at 0.5Hz, 1.0Hz, 1.5Hz, 2.0Hz, 2.5Hz and 3.0Hz loading frequencies, respectively. The tests were divided into six groups, three in each group. The test results are shown in Table 5. The relationship between the mean fatigue life and the loading frequency is linear in the double logarithmic coordinates.

$$logf = -1.55 + 0.701\log\bar{N} \tag{6}$$

The test results of the average strain rate of the limestone at different loading frequencies are shown in Table 6, and the strain rate effect of the limestone is very obvious. In the 0.5~3.0Hz frequency range, the average strain rate of the limestone is at 10^{-3} magnitude, and the dynamic strength (σ_d) of the limestone at this loading rate will be significantly higher than

Table 5. Statistics of fatigue life of the limestone under different loading frequencies

Sample number	Frequency (Hz)	Upper limit stress ratio	Stress amplitude	Fatigue life	Mean fatigue life
40	0.5	0.9	$0.6\sigma_c$	12	
6	0.5	0.9	$0.6\sigma_c$	34	59
13	0.5	0.9	$0.6\sigma_c$	125	
9	1.0	0.9	$0.6\sigma_c$	75	
53	1.0	0.9	$0.6\sigma_c$	102	188
82	1.0	0.9	$0.6\sigma_c$	386	
45	1.5	0.9	$0.6\sigma_c$	115	
57	1.5	0.9	$0.6\sigma_c$	219	344
63	1.5	0.9	$0.6\sigma_c$	697	
52	2.0	0.9	$0.6\sigma_c$	270	
11	2.0	0.9	$0.6\sigma_c$	371	382
94	2.0	0.9	$0.6\sigma_c$	503	
43	2.5	0.9	$0.6\sigma_c$	288	
75	2.5	0.9	$0.6\sigma_c$	422	493
95	2.5	0.9	$0.6\sigma_c$	769	
10	3.0	0.9	$0.6\sigma_c$	399	
55	3.0	0.9	$0.6\sigma_c$	954	839
76	3.0	0.9	$0.6\sigma_c$	1164	

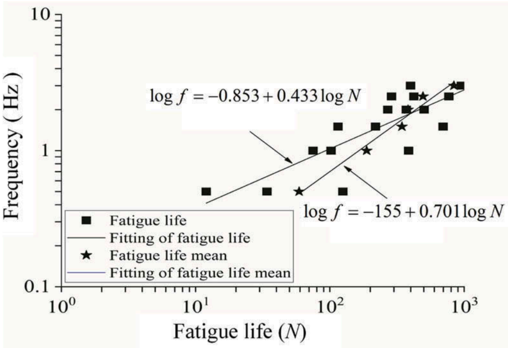

Figure 7. The relationship between fatigue life and loading frequency

Table 6. Mean strain rate of the limestone under different loading frequencies

Frequency (Hz)	Mean strain rate (s^{-1})	Coefficient variation (%)
0.5	1.500×10^{-3}	5.04
1.0	2.908×10^{-3}	4.08
1.5	4.509×10^{-3}	4.43
2.0	5.749×10^{-3}	2.97
2.5	7.249×10^{-3}	4.41
3.0	8.601×10^{-3}	7.49

its static strength (σ_c). However, the static strength was used as the reference value when determining the upper and lower limit stress ratios of the fatigue test, which means that the upper limit stress and stress amplitude of the fatigue tests are reduced. As mentioned earlier, the lower the stress level, the longer the fatigue life of the rock. Therefore, it is easy to explain the test results of fatigue life increasing with the increase of loading frequency.

In order to consider the effect of loading frequency on fatigue life, the stress level needs to be corrected to the true value, and the stress correction factor (M_r) shown in Equation 7 is defined.

$$M_r = \frac{\sigma_d}{\sigma_c} = c_1 + c_2 \log f \tag{7}$$

Where c_1 = material parameter; c_2 = material parameter.

The relationship between the upper limit stress ratio and the modified upper limit stress ratio (S') can be established by M_r.

$$S' = \frac{\sigma_{\max}}{\sigma_d} = \frac{\sigma_{\max}}{\sigma_c M_r} = \frac{S}{M_r} \tag{8}$$

The Equation 7 was fitted by using the test data of Zhao (2014), and the c_1=1.039 and c_2=0.0802 were obtained. By introducing the stress correction coefficient, a calculation model for predicting the fatigue life of the limestone based on the loading frequency and upper limit stress ratio was established, as shown in Equation 9.

$$S = M_r(A - B\log N) = (c_1 + c_2 \log f)(A - B\log N) \tag{9}$$

Where A = material parameters; B = material parameter; for the limestone, A=1.05, B=0.0746. Substituting these two parameters into Equation 9, the following empirical equation was obtained.

$$S = (1.024 + 0.0802 \log f)(1.05 - 0.0746 \log N) \tag{10}$$

In Equation 10, two most important factors affecting the fatigue life of limestone are considered, i.e. stress level and loading frequency.

4 CONCLUSIONS

Through the low cycle fatigue test of karst limestone under different conditions, the results and conclusions obtained from this study can be summarized as follows:

1. The fatigue life of limestone increased with the decreased of the upper limit of loading stress, and the relationship was in accordance with the exponential relationship. The fatigue life of the limestone decreased with the increased of stress amplitude and the relationship is linear in semi-logarithmic space.
2. A model for estimating the fatigue life of the limestone with multi-stress parameters was proposed.
3. The fatigue life of the limestone increased with the increased of the loading frequency. In the double logarithmic coordinate system, there is a good linear relationship between the average fatigue life and the loading frequency.
4. A fatigue life assessment model based on load frequency and stress level was proposed by introducing correction coefficient.

ACKNOWLEDGEMENTS

This work was financially supported by the China Natural Science Foundation (No. 50978018 and No. 51478031).

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