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Model uncertainty of SPT based methods for reliability analysis of soil liquefaction

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ABSTRACT: The potential of liquefaction is evaluated by calculating the reliability index using the first order reliability method (FORM). The probability of liquefaction is then obtained from the reliability index. Two models (the recommended by the NCEER and the one proposed by Boulanger and Idriss) are used to define the limit state function for liquefaction triggering analysis. These models are based on deterministic methods considering SPT tests. The statistical characteristics of the model uncertainty of Boulanger and Idriss model are determined through an extensive reliability analysis using Bayesian mapping function calibrated with a set of quality case histories. Soil from Lebanon corresponding to a site located at the North of Beirut where SPT tests have been carried out is used for comparative analysis between the two models. The impact of the coefficient of variation of the input variables on the probability of liquefaction is examined.

1 INTRODUCTION

The analysis of soil liquefaction potential has been widely based on deterministic methods considering the results of in-situ tests such as the Standard Penetration Test (SPT). The deterministic methods calculate the safety factor against liquefaction. The SPT-based simplified procedure was first established by Seed & Idriss (1971) and was later summarized in a paper published in 2001 by the NCEER Committee (Youd, 2001). This deterministic method is known as the state of art method for liquefaction evaluation. Updates were provided to this method by Idriss & Boulanger (2010 and 2014) and by Cetin et al. (2004, 2016). These methods are based on the cyclic stress approach which evaluates the stress induced by the earthquake loading (cyclic stress ratio CSR) and the resistance of the soil against liquefaction (cyclic resistance ratio CRR).

However, due to uncertainties in defining the soil and earthquake parameters, a safety factor may not be sufficient to determine the potential of soil to liquefy or not. A reliability based method may be used for assessing the potential of liquefaction of soil. The reliability analysis of liquefaction considers both model and parameter uncertainty. Juang et al. (2000) proposed advanced first-order second moment (AFSOM) techniques to calculate the reliability index. They developed the relation between the probability of liquefaction and the reliability index using a Bayesian mapping function based on post-liquefaction data cases. Juang et al. (2009) used the first order reliability method (FORM) along with the Bayesian mapping function approach to determine the probability of liquefaction using SPT data. Rahhal & Zakhem (2011) used the CSR and CRR statistics and the first order second moment to calculate the liquefaction probability and reliability index.

In this study, both SPT-based methods described by Youd et al. (2001) and by Idriss & Boulanger (2014) are adopted as the deterministic models for liquefaction potential evaluation.

In previous study, Juang et al. (2009) undertook a sensitivity analysis in order to determine the model uncertainty associated with the use of the CSR and CRR model as defined by Youd et al. (2001). In a similar way, a reliability analysis associated with the Bayesian mapping approach has been carried out to estimate the uncertainty of the Idriss & Boulanger (2014) model. The model uncertainty as well as the parameters uncertainties are represented by lognormal variables and are characterized by their mean and their coefficient of variation. A case study corresponding to a site located in the coastal area to the North of Beirut Lebanon is considered. An extensive analysis is performed on SPT data from the mentioned site to understand and compare the results in terms of probability derived from the two models. In the aim of studying the impact of the uncertainty of the input parameters on the probability of liquefaction, a sensitivity analysis is performed and discussed.

2 METHODOLOGY

2.1 *Deterministic approach*

In this paper, two deterministic methods are used as limit state model for the reliability analysis: Youd et al. (2001) and Idriss & Boulanger (2014). A brief summary of these two methods is presented below. For both methods, the evaluation of liquefaction resistance of soil in terms of safety factor requires the calculation of (1) the seismic demand on a soil layer expressed in terms of cyclic stress ratio (CSR) and (2) the capacity of the soil to resist liquefaction expressed in terms of cyclic resistance ratio (CRR).

For both methods, the cyclic stress ratio normalized to a reference moment magnitude of 7.5 and effective overburden pressure of σ'_v is given by the Equation 1 below:

$$\mathrm{CSR}_{7.5,\sigma} = \frac{\tau_{av}}{\sigma'_{v0}} = 0.65\left(\frac{a_{max}}{g}\right)\left(\frac{\sigma_{v0}}{\sigma'_{v0}}\right)(r_d)/\mathrm{MSF}/K_\sigma \tag{1}$$

where a_{max} = peak horizontal acceleration at the ground surface generated by the earthquake; g = acceleration of gravity; σ_{v0} and σ'_{v0} = the total and effective vertical overburden stresses respectively; r_d= stress reduction coefficient; MSF = magnitude scaling factors; and K_σ = overburden correction factor.

On the other hand, the two methods differ when estimating the values of r_d, MSF and K_σ. Idriss & Boulanger (2008 and 2014) derived the equation of r_d using the results of numerous site response analysed and updated the equations of MSF and K_σ in order to account for their variation as a function of the soil characteristics.

In the SPT-based model by Youd et al. (2001), the cyclic resistance ratio CRR is calculated from the Equation 2 below:

$$\mathrm{CRR} = \frac{1}{34-(N_1)_{60cs}} + \frac{(N_1)_{60cs}}{135} + \frac{50}{\left[10(N_1)_{60cs}+45^2\right]} - \frac{1}{200} \tag{2}$$

where $(N_1)_{60cs}$ = clean sand equivalence of the overburden stress corrected SPT blow count conducted after the correction for fines content.

Idriss & Boulanger (2004, 2008) developed a correlation between the cyclic ratio (CRR) and the equivalent clean sand $(N_1)_{60cs}$ value. It is expressed as follows:

$$\mathrm{CRR} = \exp\left[\frac{(N_1)_{60,cs}}{14.1} + \left(\frac{(N_1)_{60,cs}}{126}\right)^2 - \left(\frac{(N_1)_{60,cs}}{23.6}\right)^3 + \left(\frac{(N_1)_{60,cs}}{25.4}\right)^4 - 2.8\right] \tag{3}$$

They expressed the correction for the overburden pressure as a function of the relative density of the soil expressed in terms of $(N_1)_{60cs}$ and they defined a new adjustment of the fines content (FC).

Results of the deterministic approaches are presented in terms of the safety factor which is defined as a ratio of CRR/CSR. Practically, a safety factor lower than 1 denotes the occurrence of soil liquefaction. However due to uncertainties in the parameters and in the model, the calculated safety factor by the deterministic methods may not be an adequate indicator of a liquefied case. Efforts are made to take into account the uncertainties by implementing a reliability based probabilistic approach.

2.2 *Reliability based probabilistic approach*

By calculating the probability of liquefaction using the empirical models (Liao et al., 1998; Juang et al., 2000, 2002; Cetin et al. 2004, 2016), only a best estimate of the parameter is used to calculate the probability of liquefaction. Therefore, the calculated probability does not account for the uncertainty of the model and the parameters. To consider these uncertainties a reliability based probabilistic approach is adopted. For the reliability calculation and according to Juang et al. (2006), the limit state function of liquefaction triggering may be expressed as:h(x)= c CRR – CSR. x represents the vector of the input parameters required for calculation of CRR and CSR and c is a variable that represents the model state parameter used to describe the model uncertainty.

Based on the discussions of Juang et al. (2009), a total of six random variables are identified in the Youd et al. (2001) model. These variables are both the components of the determination of CSR and CRR. They consist of $N_{1,60}$, FC, M_w, a_{max}, σ'_v and σ_v.

For the Idriss & Boulanger model (2014), two input parameters $N_{1,60}$ and FC are required for the calculation of CRR. For the determination of the CSR, the parameters required are a_{max}, M_w, σ'_v, σ_v, r_d, MSF and K_σ. Since MSF, K_σ and r_d are function of the moment magnitude M_w, $N_{1,60}$, σ'_v and the depth, therefore those parameters may not be considered as random variable as their calculation uncertainties are taken within the other random variables. On the other hand, the uncertainty in the equations defined to determine these parameters is considered in the uncertainty of the model as these equations are a part of the Idriss & Boulanger (2014) model. Thus, the calibration of the model uncertainty is a must.

Consequently, the limit state can be defined for both models in function of the random variables as $h(x) = h(c, N_{1,60}, \mathrm{FC}, \mathrm{M_W}, \mathrm{a_{max}}, \sigma'_v, \sigma_v = 0$.

The random variables are assumed to follow a lognormal distributed random correlated variables (Juang et al. 2009). The uncertainties of the input parameters are assessed in terms of mean value and standard deviation. The published data on the typical range of the coefficient of variation (COV) for the considered random variable are summarized in Table 1.

The correlation between the six variables representing the parameters can be assumed empirically using statistical methods. The correlation coefficients between the six random variables are shown in Table 2 below (Juang et al. 1999, 2008b). The model parameter is assumed to be independent from the other input variables.

Having the limit state and the correlation coefficients of the input variables, reliability analysis will be done using numerical methods (excel spreadsheet) as the first-order reliability method (FORM). The approach considered by FORM is to transform the correlated lognormal variable into normal independent random variable. The reliability index β as defined by

Table 1. Typical COV of the input variable.

Random variable	Typical Range of COV	References
$N_{1,60}$	0.1-0.4	Harr (1987), Gutierrez et al. (2003), Phoon and Kulhawy (1999)
FC	0.05-0.35	Gutierrez et al. (2003)
σ'_v	0.05-0.2	Juang et al. (2009)
σ_v	0.05-0.2	Juang et al. (2009)
a_{max}	0.1-0.2	Juang et al. (2009)
M_w	0.05-0.1	Juang et al. (2009)

Table 2. Correlation between the input variables.

	$N_{1,60}$	FC	σ'_v	σ_v	a_{max}	M_w	c
$N_{1,60}$	1	0	0.3	0.3	0	0	0
FC	0	1	0	0	0	0	0
σ'_v	0.3	0	1	0.9	0	0	0
σ_v	0.3	0	0.9	1	0	0	0
a_{max}	0	0	0	0	1	0.9	0
M_w	0	0	0	0	0.9	1	0
c	0	0	0	0	0	0	1

Hasofer-Lind is obtained using FORM. The nominal probability of liquefaction is determined as $P_L = 1- \Phi(\beta)$, where Φ is the standard normal cumulative distribution function.

2.3 *Model factor determination*

In order to have a correct value of probability based on the reliability analysis, calibration of the model factor is required. The estimation of the model factor has been described by Juang et al. (2006). A summary is presented herein. The first step of the model uncertainty consists on calculating the reliability index β_1 for the existing case histories without considering the model parameter. By means of Bayes theorem, the probability-reliability index (P_L-β) mapping function may be approximated as follows:

$$P_{L1} = P(L|\beta) = \frac{f_L(\beta)}{f_L(\beta) + f_{NL}(\beta)} \tag{4}$$

where $f_L(\beta)$ and $f_{NL}(\beta)$ are respectively the probability density function of the calculated β of the group of liquefied and non-liquefied cases respectively.

The second step corresponds to recalculate the value of the reliability index β_2 with values of the statistical characteristics of the model parameter. Having β_2 for each case history considered, the probability of liquefaction noted as P_{L2} is calculated as the nominal probability of liquefaction. By means of trial and error process with varying the statistical characteristics of the model parameter, it can be determined by minimizing the root mean square error (RMSE) defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (P_{L2} - P_{L1})^2}{N}} \tag{5}$$

where N = number of cases in the data set.

Juang et al. (2004) found that the effect of the COV of the model parameter is insignificant on the final probability in comparison with the effect of its mean value. Thus for the calibration of the model parameter, the coefficient of variation of c is considered equal to zero. Using Cetin database (Cetin, 2000), Juang et al. (2009) concluded for the Youd et al. model a mean value for the model parameter of 0.96 and a COV of 0. To take into account the uncertainties in the estimation of the model parameter characteristics, the calculated probability is referred to as a mean probability.

Following the procedure described above, the model parameter of Idriss & Boulanger (2014) is calibrated in this study. The database of liquefied and not liquefied case histories used is the one documented by Cetin et al. (2016). The database consists of a total of 202 cases (92 non liquefied cases and 110 liquefied cases). For each case history in the database, the mean value and the coefficient of variation of the input parameters except for the moment magnitude are available. A coefficient of variation of 0.1 for the moment magnitude is assumed for all the cases. The figure below shows the Bayesian mapping P_L-β obtained from the reliability analysis of the Idriss & Boulanger (2014) model.

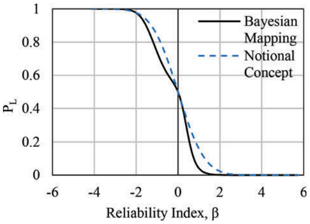

Figure 1. P_L-β Bayesian mapping function.

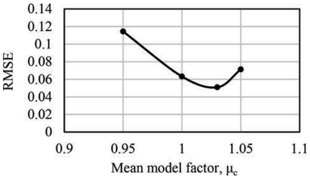

Figure 2. Variation of RMSE function of μ_c.

In the present work, for each of the 202 cases, the FORM analysis are performed assuming different values for the mean value of the model parameter (μ_c= 0.95, μ_c= 1.03 and μ_c= 1.05).

For each value of μ_c, the difference between PL_2 and PL_1 is determined in terms of RMSE. Based on the below chart (Figure 2), the best fit for the mean value of the model parameter is 1.03. It will be used with a null coefficient of variation (μ_c= 1.03 and COV_c=0) for future calculation.

3 CASE STUDY

In this paper, the case study area is located on the southern coast area of Beirut. From a geological perspective, the formation of the considered site is Quaternary formation a (sandy alluvium) overlaying the Cretaceous formation. The Quaternary deposits consist of different soils and material that constitute nonconsolidated layers covering the outcropping rock formations. Within the Beirut area, the different parts of the Quaternary deposits consist, from old to young, of brown reddish soil with cobbles and gravels of limestone origin, yellow sandy soil cover, alluvial quaternary deposits of cobbles and pebbles, sand material called “ramleh” with conglomerates and reddish sandy soil. A groundwater table exists in this area at a shallow depth.

The studied site is located on the coast of Beirut at a distance of ~450m from the sea line. The conducted soil investigation has comprised the execution of six boreholes having variable depth exceeding 20m. These boreholes were executed by continuous coring with Standard Penetration Tests (SPT) performed every 1.5m of depth. Below the fill layer, a sandy layer was recognised over a variable thickness ranging between ~10m and ~24.5m. A total of twenty-three SPT tests were executed in this layer. The lower values of N-SPT are mainly located between the surface and 15m. Sieve analysis tests were conducted on number of samples and gave on average 7% of gravels, 73% of sand and 20% of fines. One of the boreholes has been equipped by piezometer of 45m of length (PVC perforated tubes). The water level measured was recorded at a depth of ~1.77m. This water level corresponds mainly to the sea water level.

Lebanon lie across an estimated 10^6m long fault which extends from the seafloor spreading in the Red Sea to the Taurus in Southern Turkey. This fracture system, known as the Levant or Dead Sea fault system, is an extremely important tectonic feature, which accounts for the bulk of seismic activity in the Eastern Mediterranean. For this calculation, a maximum acceleration of 0.25g and a maximum magnitude of 7 are considered.

Based on the above data, the encountered sand layer is found to be potentially liquefiable. Therefore, a soil profileis used for the comparative study hereafter. The sand layer starts at a depth of 2m.

4 ANALYSIS AND DICUSSIONS

Two methods to evaluate soil liquefaction potential are applied: the method adopted by the NCEER referred as Youd et al. (2001) and the one developed by Idriss and Boulanger (2014) in their deterministic and reliability probabilistic forms. For the probabilistic calculation and for comparison purpose, the mean value of the range of the coefficient of variation for each input parameter is considered for both models.

Figure 3 shows the value (N_{60}) as well as the safety factor and the probability of liquefaction based on the two cited methods as a function of depth. At the depth of 4.5m, N_{60} value of 24 is obtained. After correction for fines content, the obtained $N_{1,60}$ is higher than 30 denoting a sand that is too dense to liquefy. Therefore, this value is not considered for the remaining calculation as it does not serve to any comparison issues.

By observing the results of the calculations, it can be noted that the Idriss & Boulanger (2014) method is more conservative than the Youd et al. (2001) method providing lower safety factor and higher probability of liquefaction.

In the following, the effect of the coefficient of variation of chosen random variable ($N_{1,60}$, FC, a_{max} and M_w) on the response of the soil in terms of probability of liquefaction based on the two presented models is studied. For this purpose, the COV of each parameter is modified within the theoretical range cited in Table 1 while the value of the COV of the other parameters is kept constant. The results of this sensitivity analysis for both limit state models are shown in Figures 4 and 5. For both considered models, the effect of the coefficient of variation

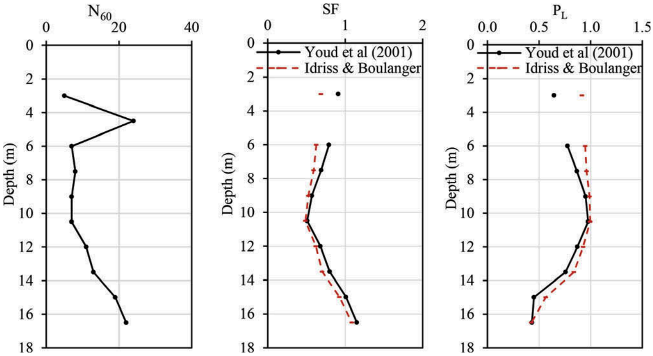

Figure 3. Values of N_{60}, safety factor and probability of liquefaction function of depth.

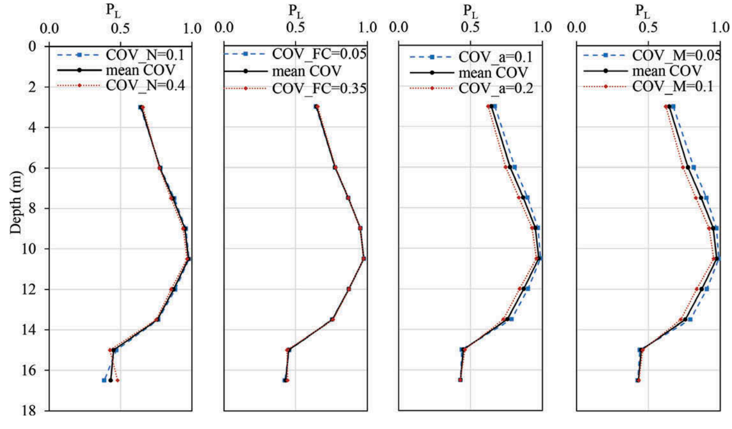

Figure 4. Effect of the variation of the COV on the probability of liquefaction considering Youd et al. model (2001).

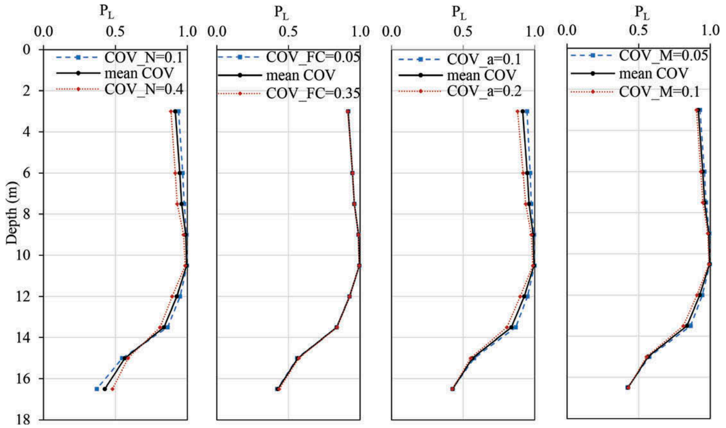

Figure 5. Effect of the variation of the COV on the probability of liquefaction considering Idriss and Boulanger model (2014).

of the fines content seems to be insignificant on the probability of liquefaction; it remains constant along the considered depth with the change of the COV of fines content. The COV of N_{60} appears to affect the probability of liquefaction using Idriss & Boulanger (2014) model more than that based on Youd et al. (2001) model. However, for both models, the effect of the COV of N_{60} is more important when having a safety factor slightly above 1. The effect of the seismic parameters uncertainties is considered within the variation of the coefficient of variation of a_{max} and M_w. For Youd et al. (2001) model, the effect of the coefficient of variation of a_{max} is the same of that of M_w. The probability of liquefaction increases with the increase of the coefficient of variation of a_{max} or M_w. For Idriss and Boulanger model, the variation of the COV of a_{max} has a more important effect on the probability of liquefaction than the variation of the COV of M_w. Finally, for both models, the amplitude of the variation of the probability of liquefaction seems to be less important for safety factors slightly higher than 1.

5 CONCLUSION

Uncertainties in liquefaction analysis deriving from input parameters and model parameter are a common issue that can be targeted by using reliability analysis. In this paper, two deterministic methods have been used as a base to define the limit state function for the reliability analysis: Youd et al (2001) and Idriss & Boulanger (2014).

The model uncertainty of Idriss and Boulanger model has been estimated through an extensive series of sensitivity studies using Bayesian mapping function calibrated with numerous case histories. This calibration leads to define the model uncertainty for future study with a mean value of 1.03 and a coefficient of variation of 0. A comparative analysis shows that Idriss & Boulanger model (2014) is more conservative than Youd et al. (2001) in both deterministic and probabilistic calculation.

Sensitivity analysis have been presented based on the variation of the coefficient of variation of the input parameters. The coefficient of variation of the fines content has no impact on the probability of liquefaction for both models. In consequence for future calculation, the coefficient of variation of the fines content can be defined by its mean value. However, the coefficient of variation of $N_{1,60}$, a_{max} and M_w have an impact on the probability of liquefaction and their values should be investigated.

One case study was used for analysis, further calculations based on other site locations are currently being accomplished. All calculations in this paper have been based on in-situ SPT. Nevertheless, cone penetration test (CPT) is also a commonly used in-situ test for the determination of the potential of liquefaction of soil. A continuity of this work will be adopting reliability analysis based on CPT models. Comparison based on models used will be realized using CPT and SPT data from the same site used herein. More results are forthcoming.

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