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Pile group lateral efficiency under static, impact and seismic conditions in clays

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ABSTRACT: The scope of this article is to numerically investigate the pile group effect in soft to medium clays under static, impact and seismic conditions, with the help of fully coupled 3D numerical analyses. The numerical model was firstly calibrated with the help of the commonly used P-y curves proposed by API (2002) and Kim et al. (2009) for a single pile in soft to medium clay, in order to create a base case set of soil parameters and numerical p-y curves, which will be used for the parametric expansion of the results in pile groups. In total twenty-eight (28) parametrical numerical analyses have been performed in order to examine the influence of pile diameter D, normalized pile-to-pile distance d/D, undrained shear strength C_u, ground depth z and loading rate V. Finally, a systematic review of the calculated P-y curves and p-multipliers under static, impact and seismic conditions was performed.

1 INTRODUCTION

It is widely acknowledged that the efficiency of pile groups under lateral loading deteriorates rapidly as the pile-to-pile distance becomes less than about five pile diameters (5D), due to the "shading" effect of the leading piles shown in Figure 1a. Most researchers estimate this effect in each pile row (or distinct pile) through the ratio of the maximum measured pile lateral resistance to the resistance of an equivalent single pile, often referred to as p-multiplier or group efficiency factor (Figure 1b). This methodology is in agreement with common engineering practice, which dictates the use of "winkler" type springs following a simple load-deflection (P-y) law for the lateral response of piles. However, the evaluation of p-multiplier is a very difficult process since the factors affecting it are not yet fully researched. Also the range of the calculated p-multiplier from the relations proposed in the literature, either from individual researchers (e.g. Reese and Van Impe 2001, Mokwa & Duncan 2001, Rollins et al. 2003) or from codes and guidelines (US Army 1993, AASHTO 2000, WSDOT 2000), is significant. For example, for the second row of a group and for distance between piles equal to three pile diameters, the various relations give p-multipliers that range from 0.25 to 0.85.

In the last three decades a number of large-scale and centrifuge experiments have been performed in order to determine the p-multiplier value. In Table 1 the p-multipliers measured from several researchers in clays, along with the basic parameters of the relevant experiments, are being presented. As it can be seen there is significant variation between the reported p-multipliers for similar problem parameters. In addition, based on the fact that such experiments are quite demanding and expensive, most researchers examine a very limited number of "cases", thus making their findings difficult to extend for other cases, even if the problem parameters are similar.

Even more limited are the results for dynamic loading (impact, seismic or cyclic). Early enough Brown et al. (1987, 1988) made large scale tests in order to demonstrate the effect of pile

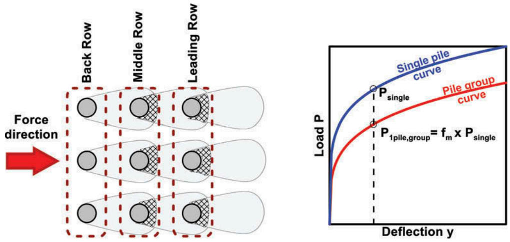

Figure 1. (a) Pile row definition and shading effect between rows. (b) Definition of p-multiplier f_m

Table 1. List of recent full scale experiments, with basic parameters and the evaluated P-multipliers.

Author	Soil type and strength	Load type	Size of group	d/D	Lead row	2nd row	3rd row	4th row	5th row
Brown et al. 1987	OC clay, C_u=70~180kPa	Static	3x3	3.0	0.70	0.50	0.40		
		Cyclic	3x3	3.0	*up to 40% strength degradation after 100 cycles*				
Meimom et al. 1986	Silty clay, C_u=25kPa	Static	2x2	3.0	0.90	0.50			
Rollins et al. 1998	Clayey silt, C_u=50~75kPa	Static	3x3	3.0	0.60	0.38	0.43		
Ilyas et al. 2004	NC clay, C_u=0~20kPa	Static	2x1	3.0	0.80	0.63			
		Static	2x2	3.0	0.96	0.78			
		Static	3x3	3.0	0.65	0.50	0.48		
		Static	4x4	3.0	0.65	0.49	0.42	0.46	
Rollins et al. 2003a, 2003b, 2006a, 2006b	Sandy clay, C_u=35~100kPa	Static	3x3	5.65	0.95	0.88	0.77		
			3x4	4.40	0.90	0.80	0.69	0.73	
			3x5	3.30	0.82	0.61	0.45	0.45	0.46
		Cyclic	3x3	5.65	*20% strength degradation after 15 cycles*				
			3x4	4.40	*17% strength degradation after 15 cycles*				
			3x5	3.30	*15% strength degradation after 15 cycles*				
		Dynamic	all	-	20~35% higher than static lateral resistance				

The numbers in italic have not been reported by the authors and were estimated based on the reported P-y curves

group (of nine piles) in both monotonic and cyclic lateral loading (total of 100 cycles), concluding that the soil resistance during cyclic loading was about 60% of the initial value. More recently, Rollins et al. (2003, 2006a, 2006b) showed similar results in piles under cyclic loading.

Nevertheless, these studies refer to cyclic loading of pile groups, i.e. several cycles of loading with small loading rate, rather than seismic or impact loading which involves large loading rates. Thus, they report only the cyclic reduction of soil resistance due to clay softening, or the developing gaps between pile and soil (Zarin et al. 2018), and not the opposite mechanism that dictates the increase of the lateral resistance of soils due to the fast loading rate and the resulting inertia effects (increase of pile-soil stiffness). The beneficiary effect of loading rate on soil strength has been shown in laboratory experiments performed by several researchers in the past (e.g. Litkouhi & Poskitt 1980, Randolph & Deeks 1992) and in full-scale experiments of vertical loading in piles (e.g. Audibert and Dover 1982, Wang & Sitar 2006). For lateral loading of piles, there are more limited references concerning the effect of loading rate, due to the complexity of the phenomenon. Brown et al. (2001) performed a series of full scale experiments that also showed increase in lateral resistance of piles in sands and clays due to loading rate, concluding that the foundation stiffness under dynamic loading ranges from 100% to 130% of the stiffness under static loading, for loading rates ranging from 0 to 0.8m/s. Rollins et al. (2003a) presented P-y curves from static, cyclic (15 cycles of static loads) and dynamic loading from full-scale experiments on a 5 x 3 pile group. From the reported P-y curves it derives that the cyclic lateral resistance of the group was about 83% of the static, while the dynamic resistance was 20~35% higher than static lateral resistance. In the past, code DIN 4014 (1990) also proposed three times increase of the coefficient of lateral subgrade reaction for impact loading.

The existing limited results from centrifuge or full-scale experiments, can be extended with the use of fully coupled 2D or 3D numerical analyses, properly calibrated. Several researchers have used numerical methodologies in the past to study the lateral loading of pile groups (e.g. Brown & Shie 1990, Yang & Jeremic 2002, Fayyazi 2015, Hazzar et al. 2017). In this article a 3D numerical methodology will be employed with two different goals:

a. to independently examine the various parameters affecting the P-y curves and the corresponding p-multipliers.
b. to examine the effect of the loading rate to the calculated P-y curves and p-multipliers. This article focuses on loading rates which are encountered during monotonic impact loading and seismic loading (small number of main cycles) and thus the degradation of P-y curves in clays due to softening is not examined, since it concerns large number of loading cycles, usually encountered in piles near water bodies (rivers, lakes, sea shores).

2 NUMERICAL ANALYSES

The numerical analyses were performed with the commercial code FLAC3D (Itasca 2012) which is capable of performing fully-coupled static and dynamic numerical analyses based on an explicit finite difference solution scheme. As it can be seen from Table 1 reporting the results from various experiments, the soil resistance for all pile rows after the second is more or less the same. The same can be said also for all middle piles in each row. Thus, the simple 3x3 pile group contains all the important parameters of the problem and thus, all performed numerical analyses refer either to a single pile or a 3x3 pile group. The typical grid used for the simulation of the 3x3 pile group used in this research comprises of 24510 zones and is shown in Figure 2. In brief, the following assumptions were made for all numerical analyses:

a. The lateral loading of the piles was simulated by applying a constant movement ratio (velocity) on all grid points of the top of the piles (pile head). This loading resembles more closely the loading on real-conditions, from all examined cases: (a) static loads from the superstructure, (b) impact loading at the superstructure (e.g. ship crashing to a bridge abutment) and (c) seismic loading resulting in lateral movement or rocking of the superstructure. Practically this means that the pile movement and soil reaction is concentrated at the top 5.0~6.0m.
b. The difference between the computed p-y curves and resulting p-multipliers under static, seismic and impact loading conditions, was examined by differentiating the loading rate (movement rate) ranging from 10^{-6} to 10^{-3} meters per second. This way the inertia effect can be studied, without interference from hysteretic damping or clay softening.
c. In pile groups the same movement rate was applied in all pile heads, simulating the effect of a very stiff pile cap moving laterally.
d. The pile was considered elastic with the following parameters for all numerical analyses: unit weight $\gamma_{concrete}$=25kN/m^3, Bulk modulus: K=7.5GPa, Shear modulus G=5.59GPa, total pile length L = 15m (so as to avoid any limit effect).

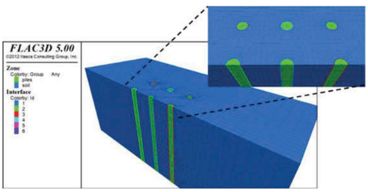

Figure 2. Typical grid used for the numerical analyses.

e. The saturated clay layer has been simulated with a Mohr-Coulomb model with constant undrained shear strength C_u (undrained conditions). Although this is an unrealistic assumption for typical clays where C_u is expected to increase with depth, this way the produced P-y can be compared directly for all depths. In addition, the following parameters where considered for the saturated clay layer: dry unit weight γ_{dry}=15.1kN/m^3, porosity: n=0.411, friction angle: φ=0°, Bulk modulus: K=150MPa, Shear modulus: G=32MPa, coefficient at rest K_0=0.5. More sophisticated soil models (e.g. capable of hysteretic damping, etc.) were not employed due to the monotonic nature of the applied loading.

f. Between the soil and pile zones, interface elements were used that allow slippage or/and detachment between the soil and the pile. These elements had the following characteristics: Normal stiffness: k_n=7.5GPa, Shear stiffness: k_s=5.59GPa, friction angle: δ=0°, cohesion: c=2/3*C_u.

g. Since the problem is axisymmetric only half of the grid was simulated.

2.1 *Calibration of the model*

The numerical simulation was firstly calibrated with the help of two commonly used P-y curves for piles in soft clay, i.e. the API (2002) and Kim et al. (2009) P-y curves, and not with any single centrifuge or large-scale experiment, because these curves are based on extensive field experience and experimental data and at the same time are widely accepted and used in practice worldwide. The goal was to create a single set for the parameters of the interface elements (cohesion, friction, normal & shear stiffness), which will be subsequently used in all performed analyses, irrespective of the variation of soil-strength parameters, pile and group geometry and loading velocity. Note that, this article studies the pile group behavior in comparison with those of single piles, and this calibration only serves to provide realistic starting values for the single pile lateral resistance.

Figure 3 shows the comparison between the static p-y curves proposed from API (2002) [for J=0.35, ε_{50}=0.5%], Kim et al. (2009) [for k_C=450 which refers to OCR=1.0 and PI≈40%] and those derived from the numerical simulation with the assumptions discussed previously. As it can be seen there are significant differences both for the p-y shape and ultimate lateral resistance deriving from the two analytical solutions and thus, no attempt for "1-1" calibration was made. The interface parameters where chosen so as the whole range of calculated P-y curves to be in agreement with the same range from both API (2002) and Kim et al. (2009) predictions, for all examined depths, pile diameters and undrained shear strengths.

2.2 *Parametric numerical analyses*

Based on the afore mentioned assumptions for the numerical methodology, a series of parametrical analyses were performed in order to examine the effect of the following problem parameters:

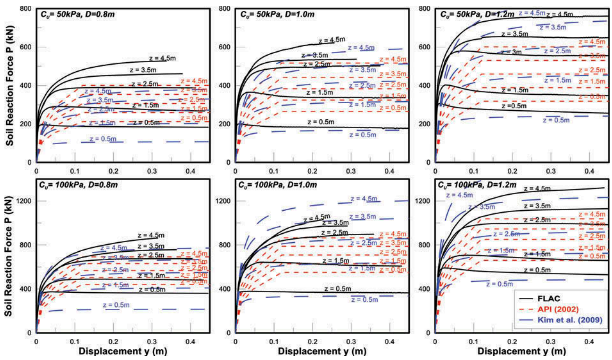

Figure 3. Comparison between P-y curves computed from the numerical analyses and those proposed by API (2002) and Kim et al. (2009).

Table 2. Set of parameters for each numerical analysis.

No	D (m)	C_u (kPa)	d/D	Vel (m/sec)	No	D (m)	C_u (kPa)	d/D	Vel (m/sec)
1	1.0	20	-	$1e^{-6}$	15	0.8	100	3.0	$1e^{-6}$
2	1.0	50	-	$1e^{-6}$	16	1.2	100	3.0	$1e^{-6}$
3	1.0	80	-	$1e^{-6}$	17	1.0	50	2.5	$1e^{-6}$
4	1.0	100	-	$1e^{-6}$	18	1.0	100	2.5	$1e^{-6}$
5	0.8	50	-	$1e^{-6}$	19	1.0	50	4.0	$1e^{-6}$
6	1.2	50	-	$1e^{-6}$	20	1.0	100	4.0	$1e^{-6}$
7	0.8	100	-	$1e^{-6}$	21	1.0	50	5.0	$1e^{-6}$
8	1.2	100	-	$1e^{-6}$	22	1.0	100	5.0	$1e^{-6}$
9	1.0	80	3.0	$1e^{-6}$	23	1.0	50	3.0	$1e^{-5}$
10	1.0	100	3.0	$1e^{-6}$	24	1.0	100	3.0	$1e^{-5}$
11	1.0	20	3.0	$1e^{-6}$	25	1.0	50	3.0	$1e^{-4}$
12	1.0	50	3.0	$1e^{-6}$	26	1.0	100	3.0	$1e^{-4}$
13	0.8	50	3.0	$1e^{-6}$	27	1.0	50	3.0	$1e^{-3}$
14	1.2	50	3.0	$1e^{-6}$	28	1.0	100	3.0	$1e^{-3}$

i. pile diameter (D=0.80/**1.00**/1.20m),
ii. distance between piles to pile diameter ratio (d/D=2.5/**3.0**/4.0/5.0),
iii. undrained shear strength of the clay layer (C_u=20/**50**/80/100 kPa),
iv. depth from ground surface (z=**1.5/2.5/3.5/4.5/5.5m**)
v. the loading ratio in "static" analyses (**10^{-6}**/10^{-5}/10^{-4}/10^{-3} m/sec)

The values denoted with bold characters characterize the basic analysis. In total twenty-eight (28) parametrical numerical analyses have been performed, as shown in Table 2.

In addition to the other parameters affecting the overall behavior of the pile groups, the loading rate has been examined, in order to estimate the group effect under impact or seismic loading. A simple monotonic loading has been applied on the pile heads instead of cyclic loading (sinusoidal or real motion) so as to isolate the effect of the loading rate, by the various other mechanisms that affect the lateral resistance during cyclic loading, such as the softening of the clay layer, the forming of gaps between pile and soil, the rocking of the superstructure or the foundation, etc. Thus, the results of this analysis fit the behavior of lateral loading due to impact, which involves monotonic loading with high loading rates (velocities), and seismic loading, which involves a small number of important cycles with medium to high loading rate.

3 PARAMETERS AFFECTING THE P-Y CURVES OF SOFT CLAY

The results of the parametric analyses performed for the static loading (loading rate 10^{-6} m/sec) can be used to get valuable insight on the various parameters affecting the P-y curves and the p-multiplier. Figure 4 presents typical P-y curves derived for 3x3 pile groups with pile diameter D=1.0m, for undrained shear strengths (C_u=50kPa), for two different depths

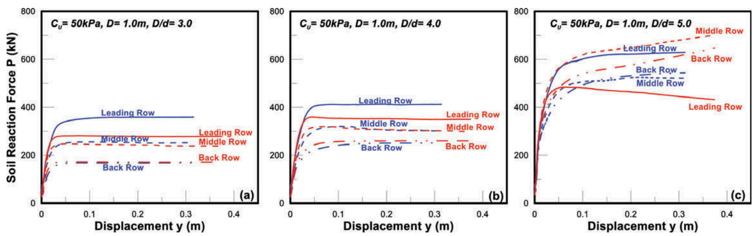

Figure 4. P-y curves for 3x3 pile groups with D=1.0m, normalized pile distance d/D (a) 3.0, (b) 4.0 and (c) 5.0, undrained shear strength C_u =50kPa & two different depths (2.5m red/4.5m blue).

(2.5/4.5m) and for four pile distances (d/D=3.0/4.0/5.0). Note that, only the P-y curve reported for each row (leading, middle and back) is actual the mean value from the P-y curves measured for the center and the two side piles of each row.

The numerical analyses agree with the general concept that 5D is a large enough distance so as to make the pile group effect negligible (the calculated average lateral resistance of the piles of the group is equal to that of an equivalent single pile). Of interest is also that, according to the performed numerical analyses, the group efficiency rapidly decreases for distances less than 5D. Another interesting aspect of the resulting P-y curves is that the pile group effect becomes increasingly important with the depth, with the ratio $P_{ult,back}/P_{ult,lead}$ becoming from an average 60% for 2.5m to 49% for 4.5m below the ground surface.

4 GROUP EFFICIENCY

The pile group efficiency for each performed numerical analyses was estimated by calculating the p-multiplier, i.e. the ratio between the maximum lateral resistance of the piles of the group, to the maximum lateral resistance of a single pile with the same characteristics. Due to the fact that the pile row greatly affects the group efficiency, the p-multipliers were estimated for each pile row (leading, middle, back) and for the total pile group. Figures 5 and 6 show the effect of pile distance d/D, pile diameter D, undrained shear strength C_u and loading rate V to the calculated p-multipliers, for depths varying from 1.5 to 5.5m below ground surface. In each quadruple of figures, the p-multipliers for the leading, middle and back row as well as the overall p-multiplier of the whole group are shown.

It can be seen from Figure 5a that the pile diameter significantly affects the calculated p-multipliers for all performed numerical analyses with d/D=3.0, with the pile group effect reducing with the increase of the pile diameter. On the other hand, as shown in Figure 5b, the undrained shear strength of the clay was not an important factor of the calculated p-multipliers. Figure 6a shows the effect of normalized pile distance d/D to the p-multipliers. As already discussed in previous paragraph for d/D equal to 5.0 the group effect seems not only to eradicate the pile group shading effect, but the calculated p-multipliers are greater than 1.0 (the average group soil resistance is greater than the one exhibited in the single pile) due to the stiffer behavior of the whole piles-soil system which moves as a unity. In addition, the overall p-multiplier is almost constant for smaller normalized distances than 3.0.

Finally, the effect of loading rate is being shown in Figure 6b, where it can be seen that the computed lateral resistance increases by 21% and 35% on the average, for loading velocities $V=10^{-4}$m/sec and 10^{-3}m/sec respectively. These values are in good agreement with an increase of 15.6% and 25% for $V=10^{-4}$m/sec and 10^{-3}m/sec respectively, which can be estimated from

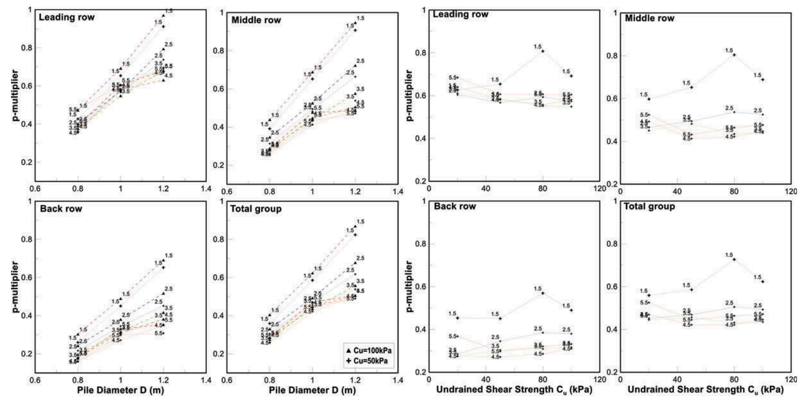

Figure 5. P-multipliers variation for all examined depths and for Leading, Middle and Back row as well as for the Total group, versus: (a) the pile diameter D and (b) the undrained shear strength C_u of the clay.

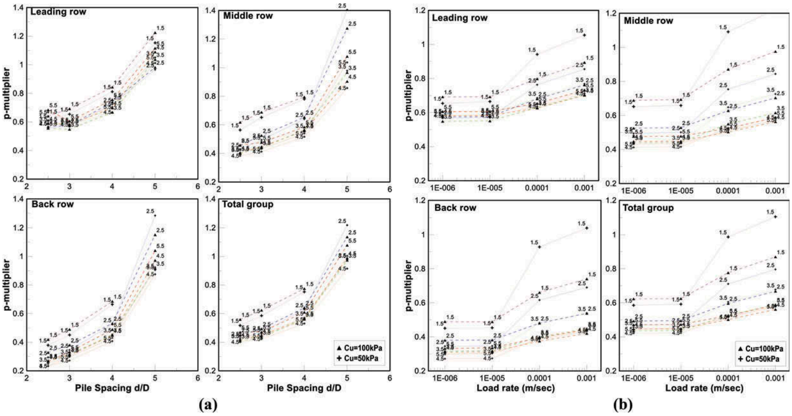

Figure 6. P-multipliers variation for all examined depths and for Leading, Middle and Back row as well as for the Total group, versus: (a) the normalized pile spacing d/D and (b) the loading rate V.

Relation 1 proposed by Litkouhi & Poskitt (1980) for the estimation of the dynamic soil resistance R_D based on the static R_S:

$$R_D = R_S\left(1 + JV^N\right) \tag{1}$$

where V is the loading velocity, J and N are factors depending on the soil nature, with N equal to 0.2, both for point and side resistances, and J equal to 1.0 for clays.

5 DISCUSSION AND CONCLUSIONS

In this paper the pile group effect under lateral loading has been examined for cohesive soils, with the help of 3D numerical analyses. From the calculated p-multipliers it has been shown that the normalized pile distance (d/D), the position of the pile (pile row) and the pile diameter (D) have significant effect on the p-multipliers, with the depth below of the ground surface playing a smaller role. In contrast, the value of the undrained shear strength C_u of the soil does not affect the calculated p-multipliers.

The effect of loading rate has also been examined by increasing the applied velocity at the pile heads and it was shown that the lateral resistance of the pile-soil system increases for velocities larger than 10^{-4}m/sec, which is compatible with the findings of previous researchers. This beneficiary effect is often ignored in design, which may lead to over conservatism especially for larger strain rates 10^{-3}m/sec - 10^{-1}m/sec than those examined in this research, such as those expected from severe impact loading or significant earthquakes.

Finally, it should be mentioned that the effect of each parameter to the p-multipliers shown in this article is somewhat irrelevant of the actual calculated resistance of the piles (either single or group), thus the reported trends can be used independently to extend the results of real-scale or centrifuge experiments or more sophisticated numerical analyses.

REFERENCES

AASHTO (2000), *Bridge Design Specifications*, American Association of State Highway and Transportation Officials, Washington, D.C.

API. (2002). *Recommended Practice for Planning, Design, and Constructing Fixed Offshore Platforms*, API RP 2A-WSD, American Petroleum Institute, Washington, D.C.

Audibert, J.M.E. & A.R. Dover (1982), *Discussion of pile load tests - cyclic loads and varying load rates by kraft et. al.* ASCE J. Geotech. Eng. Div. 108(3), 501–505.

Brown, D. A., Resse, L. C., & O'Neill, M. W. (1987), *Cyclic lateral loading of a large-scale pile group*, J. Geotech. Eng., 113(11), 1326–1343.
Brown, D. A., Morrison, C., & Reese, L. C. (1988), *Lateral load behavior of pile group in sand*, J. Geotech. Eng., 114(11), 1261–1276.
Brown D.A. & Shie Ch-F (1990), *Numerical Experiments into Group Effects on the Response of Piles to Lateral Loading*, Computers and Geotechnics 10, pp. 211–230.
Brown D.A, O'Neil M.W, Hoit M, McVay M, El Nagger M.H. & Chakraborty S. (2001), *Static and dynamic lateral loading of pile groups*, NCHRP 24–29, Highway Research Center, Alabama, January.
DIN 4014 (1990), *Bored cast-in-place piles. Formation, design and bearing capacity*, Deutsche Norm.
Fayyazi M. S. (2015), *Numerical study on the response of pile groups under lateral loading*, PhD Thesis, The University of British Columbia (Vancouver), August.
Hazzar L., Hussien M. N. & Karray M. (2017), *Influence of vertical loads on lateral response of pile foundations in sands and clays*, J. Rock Mechanics and Geotechnical Engineering 9 pp. 291–304.
Ilyas T., Leung C. F., Chow Y. K. & Budi S.S. (2004), *Centrifuge model study of laterally loaded pile groups in clay*, J. Geotech. Geoenviron. Eng.130:274–283.
Itasca (2012), *Fast Lagrangian Analysis of Continua in 3 Dimensions*, Minnesota, Ver. 5.0
Kim, Y., Jeong, S., & Won, J. (2009): *Effect of Lateral Rigidity of Offshore Piles Using Proposed p-y Curves in Marine Clay*, Marine Georesources and Geotechnology, pp. 53–77.
Litkouhi, S. & T.J. Poskitt (1980). *Damping constants for pile driveability calculations*. Geotechnique 30 (1), 77–86.
Meimon, Y., Baguelin, F., & Jezequel, J. F. (1986), *Pile group behavior under long time lateral monotonic and cyclic loading*, Proc. 3rd Int. Conf. on Numerical Methods in Offshore Piling, France, 286–302.
Mokwa, R. L., & Duncan, J. M. (2005). *Discussion of 'Centrifuge model study of laterally loaded pile groups in clay' by T. Ilyas et al*. J. Geotech. Geoenviron. Eng.,131(10), 1305–1308.
Randolph, M.F. & A.J. Deeks (1992), *Keynote lecture: Dynamic and static soil models for axial pile response*. In Frans B.J. Barends (Ed.), Proc. of the 4th Int. Conf. on the Application of Stress-wave Theory to Piles, pp. 3–14. The Hague, the Netherlands.
Reese. L.C. & Van Impe W.F. (2001), *Single piles and pile groups under lateral loading*, A.A. Balkema, Rotterdam, Netherlands.
Rollins K. M, Johnson S. R, Peterson K. T. & Weaver T. J. (2003a), *Static and Dynamic Lateral Load Behavior of Pile Groups Based on Full-Scale Testing*, Proc. of The Thirteenth International Offshore and Polar Engineering Conference, Honolulu, Hawaii, USA, May 25–30
Rollins, K.M., Olsen, R.J., Egbert, J.J., Olsen, K.G., Jensen, D.H. & Garrett, B.H., (2003b). *Response, analysis and design of pile groups subjected to static and dynamic lateral loads*, Report No. UT-03.03.
Rollins, K. M., Olsen, K. G., Jensen, D. H., Garrett, B. H., Olsen, R. J., & Egbert, J. J. (2006a). *Pile spacing effects on lateral pile group behavior: Analysis*, J. Geotech. Geoenviron. Eng., 132(10), 1272–1283.
Rollins, K. M., Olsen, R. J., Elbert, J. J., Jensen, D. H., Olsen, K. G., & Garrett, B. H. (2006b). *Pile spacing effects on lateral pile group behavior: Load tests*, J. Geotech. Geoenviron. Eng., 132(10), 1262–1271.
US Army (1993), *Design of pile foundations*, Technical Engineering and Design Guides No. 1, U.S. Army Corps of Engineers, Washington, D.C.
Wang G. & Sitar N. (2006), *Nonlinear Analysis of a Soil-Drilled Pier System under Static and Dynamic Axial Loading*, PEER Report 2006/06, University of California, Berkeley, November
WSDOT (2000), *Bridge design manual*, Washington State Department of Transportation, Olympia, WA
Zarrin M., Asgarian B., Fulad R. (2017), *A Review on Factors affecting Seismic Pile Response Analysis: A Parametric Study*, Journal of Numerical Methods in Civil Engineering, October.
Yang Z. & Jeremic B. (2002), *Numerical Analysis of Pile Behavior under Lateral Loads in Layered Elastic-Plastic Soils*, Int. J. Numer. Anal. Meth. Geomech. 2002; 02:1-31, John Wiley & Sons, Ltd.