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Using OpenQuake for probabilistic hazard and risk assessment of seismically-induced ground displacements: Lessons learned from implementation and considerations for application

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ABSTRACT: The scientific framework for probabilistic seismic risk assessment due to ground shaking has become well-established, and open, practical tools that serve the risk modelling community are widely available. Nevertheless, the modelling of secondary hazards remains further behind, despite a growing body of literature presenting a practical framework for such assessments, and a widespread recognition of their significant contribution to the losses in recent earthquakes. To address this, efforts have been made to introduce a probabilistic analysis module for secondary hazards (landsliding and liquefaction) inside OpenQuake, an open-source software for seismic hazard and risk assessment. The theoretical framework and implementation of these calculators are described alongside a discussion of the relevance of aleatory and epistemic uncertainties, and of spatial correlations. A controlled hazard example, extending a PEER software verification test to landsliding hazard, illustrates some of the modelling possibilities that these tools offer, and the considerations for real-world applications.

1 INTRODUCTION

The effective mitigation of risk from earthquakes has, as its foundation, the need to not only understand the earthquake process and resulting phenomena, but to transform this understanding into quantitative, practical information that can be utilised by scientists, engineers, insurers and policy makers alike. In the case of direct seismic ground shaking, the probabilistic seismic hazard assessment (PSHA) methodology provides a practical framework for implementing this transformation. Losses from earthquakes, however, are seldom limited to just those resulting from the direct ground shaking, but encompass secondary phenomena induced by the shaking, such as landsliding, liquefaction and tsunamis. The means by which these phenomena can be into a probabilistic hazard assessment framework have been eloquently stated in the scientific literature (Rathje & Saygili, 2008; Goda *et al.*, 2011; Du & Wang, 2014), yet such extensions have not yet found their way into widespread practice. Probabilistic assessment of secondary hazards is hampered firstly by a paucity of practical empirical models relating the degree of seismically induced displacement to the strength of shaking and local geotechnical conditions of a site, and, secondly, a lack of tools for scientists and practitioners to undertake such analysis. To attempt to address the latter, a new calculation module has been developed to extend the functionalities of OpenQuake, an open source software for probabilistic seismic hazard and risk assessment (Pagani *et al.* 2014a), to allow for fully probabilistic modelling of co-seismic landslide and liquefaction. This brief paper outlines the key features of this module, highlighting the scientific basis and the challenges associated with

deploying such tools on an operational platform. OpenQuake stands out as an ideal platform for such a development owing to its fully open code base and its modular structure. It is establishing a growing user community across the globe, and, with the release of its global hazard model database at the end of 2018, provides both a standard format and a comprehensive set of probabilistic seismic hazard models for every region of the world.

2 PROBABILISTIC ANALYSIS OF SEISMICALLY-INDUCED GEOTECHNICAL HAZARDS

2.1 Theoretical overview

The “classical” PSHA formulation describes the rate of exceeding a given level of ground motion (z) at a site located at a distance from a given source, dependent upon the distribution of magnitudes ($f_M(m)$) and closest rupture distances ($f_R(r|m)$) of earthquakes that the source is capable of generating:

$$\lambda_{GM}(z) = \lambda_0 \int_m \int_r P[GM > z|m, r] \cdot f_M(m) \cdot f_R(r|m) \cdot dm dr \quad (1)$$

where $\lambda_{GM}(z)$ is the annual rate of exceeding ground motion level z , λ_0 the annual rate of earthquakes greater than the minimum magnitude of the source, and $P[GM > z|m, r]$ the probability that a ground motion exceeds the specified level given an earthquake of magnitude m occurring at a distance r from the site. This latter term is described by a standard lognormal probability density function, $f_{GM}(z|m, r, \theta)$ with a mean ground motion, $\mu_{\ln GM}$ and standard deviation $\sigma_{\ln GM}$ in natural log units. θ describes the set of additional source and site attributes utilised by a given ground motion model (hereafter referred to as a *Ground Shaking Intensity Model*, or GSIM).

Conceptually, extension of this approach to geotechnical hazard may be quite simple, and is generalisable to any co-seismic phenomenon giving rise to the ground failure. Defining co-seismic geotechnical hazard (referred to subsequently as just geotechnical hazard) as the probability of exceeding a given level of permanent ground deformation d given a strength of shaking z at a site, the classical PSHA formulation is extended according to Rathje & Saygili (2008):

$$\lambda_D(d) = \lambda_0 \int_z \int_m \int_r P[D > d|GM = z] f_{GM}(z|m, r) f_M(m) \cdot f_R(r|m) \cdot dm dr dz \quad (2)$$

Thus the additional component to extend classical PSHA to consider geotechnical hazard is $P[D > d|GM = z]$, an empirical model relating the level of permanent ground deformation resulting from the input ground shaking to the strength of the ground shaking, hereafter referred to as *Ground Deformation Estimation Model* (GDEM). Though not nearly as abundant as empirical GSIMs, several practical GDEMs can be found in the scientific literature (Jibson, 2007; Saygili & Rathje, 2008; Fotopoulou & Pitilakis, 2015).

2.2 The vector approach

The geotechnical hazard formulation outlined in equation 2 implies that the GDEM is itself a scalar model, dependent on only one single measure of ground shaking. A more *efficient* prediction of displacement can be obtained when conditional the model upon multiple measures of ground motion (intensity measures, or IMs, hereafter), including peak ground acceleration (PGA), Peak Ground Velocity (PGV), Arias Intensity (I_A) and ground motion duration T_D . The use of vector GDEMs adds additional complications, requiring that one must characterise not only the marginal distributions of each IMT, but their joint distribution. A framework extending the classical PSHA model to this case is that of vector PSHA (VPSHA), proposed by Bazurro & Cornell (2002). In their adaptation, the ground motion probability density function ($f_{GM}(z|m, r, \theta)$) is adapted to describe the joint probability density of two intensity measures (GM1 and GM2):

$$f_{GM1,GM2}(z, y|m, r, \theta) = f_{GM1}(z|m, r) \cdot f_{GM2|GM1}(y|z, m, r, \theta) \quad (3)$$

where $f_{GM2|GM1}(y|z, m, r, \theta)$ is a lognormal distribution with its mean and standard deviation conditional upon the level of ground motion z of GM1 described respectively by:

$$\mu_{\ln GM2|z,m,r} = \mu_{\ln GM2|m,r} + \rho \frac{\sigma_{\ln GM2|m,r}}{\sigma_{\ln GM1|m,r}} \cdot (\ln z - \mu_{\ln GM1|m,r}) \quad (4)$$

$$\sigma_{\ln GM2|z,m,r} = \sigma_{\ln GM2|m,r} \cdot \sqrt{1 - \rho^2} \quad (5)$$

where ρ is the correlation coefficient between GM1 and GM2. Together with equation 2 we arrive at the vector hazard formulation proposed by Rathje & Saygili (2008).

$$\lambda_D(d) = \lambda_0 \cdot \int_y \int_z \int_m \int_r P[D > d | GM1 = z, GM2 = y] \cdot f_{GM1,GM2}(z, y|m, r, \theta) \cdot f_M(m) \cdot f_R(r|m) \cdot dm \cdot dr \cdot dz \cdot dy \quad (6)$$

2.3 Event-based geotechnical hazard and risk modelling

For estimating permanent ground deformation at a single location, or for producing a map of geotechnical hazard of uniform probability of exceedance, both the classical scalar and vectorial approaches are feasible. The modelling of seismic risk, however, may not consider sites to be treated independently, but may instead need to calculate losses across a spatially extended area for each given event in order to determine the total distribution of losses. If considering seismic risk for networks or lifelines, where connectivity is a critical measure of performance, it becomes imperative to model the spatially extended ground deformation fields. By introducing the spatial element into the modelling process it is necessary to account for the spatial correlation in the residual distributions of the ground motion model and, when multiple IMTs may be considered, the spatial cross-correlation of these residuals for the selected IMTs should also be integrated into this analysis (e.g. Loth & Baker, 2013; Weatherill *et al.*, 2015).

When analysing risk to a geographically distributed portfolio of structures, or when considering the connectivity of infrastructures, it becomes far more appropriate to characterise the seismic hazard using a set of ground motion and displacement fields generated by scenarios. In probabilistic seismic risk analysis, an event-based (or Monte Carlo) approach is more widely used. Here the distributions of $f_M(m)$ and $f_R(r|m)$ are sampled to create synthetic catalogues of earthquake ruptures representing potential realisations of seismicity in a specified time period. For each rupture the corresponding fields of ground motions can be generated, taking the median ground motion from the GSIM and sampling the aleatory variability at each site. For shaking hazard analysis these fields can be sampled from a multivariate normal distribution, with a covariance matrix describing the spatial correlation and/or cross-correlation constructed using empirical spatial correlation models available in the scientific literature. To extend this to displacement it is necessary to generate one or more samples of the displacement fields (sampling their aleatory uncertainty) conditioned upon the ground motions at each site (Wang & Du, 2014).

Seismic hazard curves can be obtained by summing the number of exceedances of given levels of ground motion or displacement at the sites of interest and dividing by the effective duration of the synthetic catalogues. With multiple synthetic catalogues of sufficient duration, stability should be achieved in order to determine the long-term rate of exceedance of the ground motion and/or displacements that is equivalent to the classical formulation. In the risk domain, such results are obtained by aggregating losses (or calculating a network performance metric) for each field and summing then normalizing the exceedances to give the resulting annual rates of exceedance of given loss levels. By creating synthetic catalogues of realisations of seismicity the event-based approach can cleanly accommodate the integration of additional probability distributions to capture uncertainties in both the ground motion and displacement. The obvious cost of doing so, however, is that longer catalogues (more samples) are then needed before stability can be obtained.

3 OPENQUAKE IMPLEMENTATION

3.1 The core calculation workflows

OpenQuake provides the ideal platform for developing geotechnical hazard calculation models, not only because it provides an open-source, fully integrated, modular engine for seismic hazard and risk assessment, it already supports the two types of calculation workflows for ground motion hazard (“classical” and “event-based”) described previously. Calculations are built around the *earthquake rupture forecast* (ERF) (Field *et al.*, 2003), which constructs, from a given source model, a complete inventory of potential ruptures for a region *and their associated probability of occurrence*. A full derivation of the formulation of the hazard integral in this format can be found in Field *et al.* (2003) and Pagani *et al.* (2014b).

3.2 The classical geotechnical calculator

The general form of the hazard calculation here defines the probability of a given hazard metric (X) exceeding a specified level (x) in a period of T years, from the product of the probabilities of $X > x$ for each of the j ruptures generated by source i from a set of I sources.

$$P(X > x|T) = 1 - \prod_{i=1}^I \prod_{j=1}^J \left(1 - P_{rup_{ij}}(n1|T)\right)^{P(Xx|rup_{ij})} \quad (7)$$

where n is the number of occurrences of rup_{ij} in T years. For geotechnical hazard, $P(Xx|rup_{ij})$ is replaced by the probability of displacement D exceeding level d given the occurrence of rupture rup_{ij} , according to:

$$P(Dd|rup_{ij}) = \int_z P[D > d|z, rup_{ij}] dz \quad (8)$$

where z is the level of ground motion whose density function described by the GSIM, whilst for the vector case, this requires the evaluation of the double integral:

$$P(Dd|rup_{ij}) = \int_y \int_z P[D > d|z, y, rup_{ij}] dz \cdot dy \quad (9)$$

where $P[D > d|z, y, rup_{ij}]$ is defined equivalently to that in equation 6. In both cases the integral is solved numerically, with both the integration limits and step defined by the user. Integration here is applied to the normal, or multivariate normal, distributions of the ground motion uncertainty; hence the limits refer to the number of standard deviations below and above the median.

For the integral of the ground motion, the shaking, z and y , may be less than the threshold level required for ground failure to occur, i.e. $P_{failure} = 0$. Then, as $P(Dd|z, y, rup_{ij}) = 0$ the secondary integration step is skipped (Rathje & Saygili, 2008). In the case of landsliding, the threshold level of acceleration, (k_y), sets an explicit threshold in terms of PGA such that $P_{failure} = 0$ when $PGA < k_y$. Although a specific calculation workflow to determine $P_{failure}$ is integrated into the package, the same information can be readily retrieved by setting the target displacement level, d , to an extremely small value, meaning that as $d \rightarrow 0$ in the displacement hazard curve, $P(Dd|rup_{ij}) \rightarrow P_{failure}$.

3.3 The event-based geotechnical calculator

The event-based hazard calculator broadly follows the description given in section 2. For each rupture the corresponding ground motion field is generated from the median of the GSIM, and the aleatory uncertainty sampled for each site from a Gaussian distribution with mean of zero and standard deviation according to $\sigma_{ln GM}$. For each site and ground motion field the $P_{failure}$ is calculated from the sampled ground motion, and the occurrence of failure determined from a sample of a uniform distribution in the range $[0, 1]$ such that failure occurs when

$P_{failure}$ exceeds the sampled value. For those sites where failure is determined to occur, the resulting displacement is determined by sampling the GDEM and its uncertainty.

For the event-based approach this same information is derived by repeatedly sampling the number of occurrences of each rupture in T years to create multiple (N) synthetic catalogues of seismicity, each of T duration. In the time-independent case, the total rate of exceedances of ground motion ν is estimated by counting the exceedances from all K ruptures found in all catalogues and normalising by the effective duration ($T_0 = N \times T$).

$$P(X > x|T) = 1 - e^{-\nu T} \quad \text{where} \quad \nu = \frac{\sum_{k=1}^K H(x_k - x)}{T_0} \quad (10)$$

and $H(\cdot)$ is a Heaviside step function.

An additional development here is the introduction of spatial cross-correlation of the ground motion residual fields into the OpenQuake calculation engine. As the entirety of the GDEMs implemented at the time of writing are dependent upon only three different IMTs (PGA, PGV and Arias Intensity), only one empirical model for the construction of the cross-covariance matrix is implemented, and that is the Linear-Model of Co-Regionalisation of Du & Wang (2013).

3.4 HAZUS

Although several published GDEMs are implemented in the geotechnical calculators, particular note is given to the HAZUS methodology, explained in detail in the HAZUS Technical Manual (HAZUS, 2003). This methodology is relevant as it is the only precedent for a comprehensive set of tools to calculate displacement from both landsliding and liquefaction designed for application on a large spatial scale. Though not explicitly created with a complete probabilistic application in mind, its probability models for secondary hazards provides sufficient information for transformation into a probabilistic loss estimation framework.

For the characterisation of the geotechnical conditions of a site, HAZUS adopts *susceptibility categories*, which are semi-qualitative integer scales assigning a site to a given level of susceptibility according to its geological and geotechnical properties. Each level in the scales is associated with its own parameters to quantify: i) $P_{failure}(PGA, M, etc)$, ii) the expected displacement given the occurrence of failure, and the proportion of map unit susceptible to ground failure P_{mu} . This last term recognises that whilst $P_{failure}$ may be a property of the geotechnical unit, failure itself will only be observed for a fraction of the total potential susceptible area. A full recitation of the HAZUS formulation is omitted for brevity, but the key components of each of the liquefaction and slope displacement work in two steps, the first being the determination of $P_{failure}$:

$$P_{failure} = f(P[failure|PGA = a], m, \theta) \cdot P_{mu} \quad (11)$$

where θ may describe additional site properties such as groundwater depth or k_y . In the second step, the expected permanent ground deformation $E[PGD_f]$ (i.e. lateral spread, settlement, landsliding) is usually defined from a simple function of PGA and magnitude. In the case of liquefaction $E[PGD_f]$ lacks any aleatory uncertainty. For landsliding however, $E[PGD_f]$ is presented as a function of the ratio of k_y to PGA and its uncertainty assumed uniformly distributed between an upper and lower bound.

Implementation of the HAZUS methodology in either the classical or event-based calculators still permits integration of the ground motion variability. For the classical method the only difference compared to other GDEMs is that $P[D > d|PGA, rup_{ij}] = H(D - d|PGA, rup_{ij}) \cdot P_{mu}$. For the event-based calculator and additional sampling applied to P_{mu} to determine whether or not failure should occur at a site given that the ground acceleration exceeds the threshold. The HAZUS formulation considers only PGA; hence only the scalar approach is needed here.

3.5 Epistemic uncertainty

Leveraging on top of OpenQuake's existing architecture for PSHA allows us to take advantage of its current tools for constructing and evaluating logic trees for both source and ground

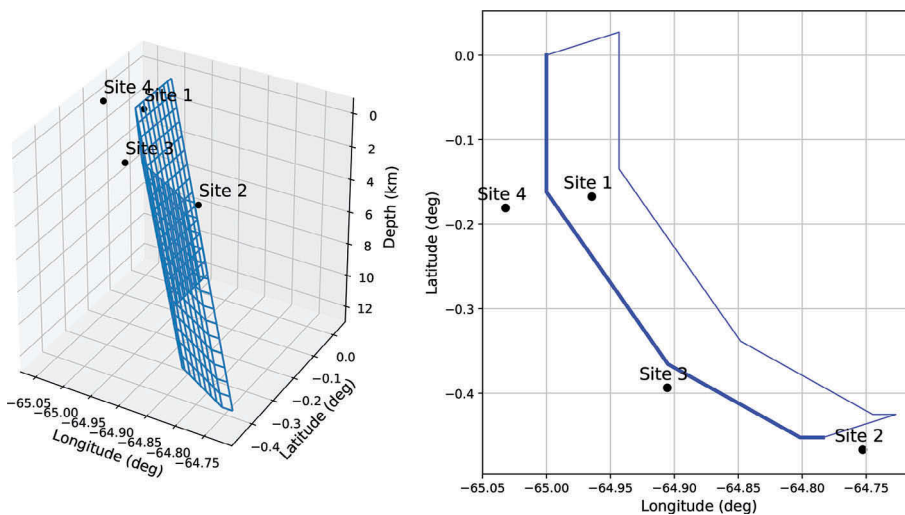


Figure 1. Source and site configuration for the PEER Bending Fault Test Case

motion model epistemic uncertainty. In its current form a GDEM could be considered a special case of a GSIM, returning the probability of exceeding a specific degree of ground displacement given the rupture and site conditions. For each GDEMs, therefore, the user must not only specify the choice of GDEM in the specific branch but also the association of the selected GSIMs to the intensity measures that the GDEM needs. This makes it a relatively simple process to construct logic trees containing not only different GDEMs, but for each GDEM the possibility to take different GSIMs from OpenQuake’s library to define its required ground motion inputs. The full functionality of the source model logic tree is also available to the user.

An exploration of epistemic uncertainty in geotechnical seismic hazard assessment was undertaken by Wang & Rathje (2015) for the case of slope displacement. They demonstrated the potential impact, not only of different GDEMs but of the shear strength and sliding block properties. Their analysis, however, differs from the approach presented here, instead taking a series of PGA (and PGV) hazard values from the mean hazard curve of a regional PSHA and disaggregating each to obtain the joint probabilities of PGA, PGV and magnitude. This is efficient, as it utilises the outputs of a single representative hazard curve, thus permitting exploration of the epistemic uncertainties on displacement. It does not, however, permit the epistemic uncertainties in the seismic shaking hazard to be explored in terms of the slope displacement hazard. The OpenQuake implementation allows complete end-to-end epistemic logic tree analysis, allowing the epistemic uncertainties to be more fully explored.

In spite of the advanced logic tree functionality present within OpenQuake, still missing from the calculation engine is a module for undertaking site model epistemic uncertainty analysis, preventing a complete exploration of the site property uncertainties in the manner undertaken by Wang & Rathje (2015). Work is currently ongoing to add this functionality for the general case of site amplification in PSHA, and when ready, this too can be extended to consider alternative parameterisations of the site. This is a particularly critical area of further development in geotechnical hazard analysis.

4 EXAMPLE APPLICATION: “CLASSICAL PSHA - PEER TEST”

To demonstrate just some of the capabilities of the new calculators we provide an illustration of a “classical” PSHA calculation to produce a slope displacement hazard curve comparing several different GDEMs. A full suite of demonstrations is beyond the scope of this paper, but supporting documentation for the full geotechnical module is currently in preparation. The calculation is

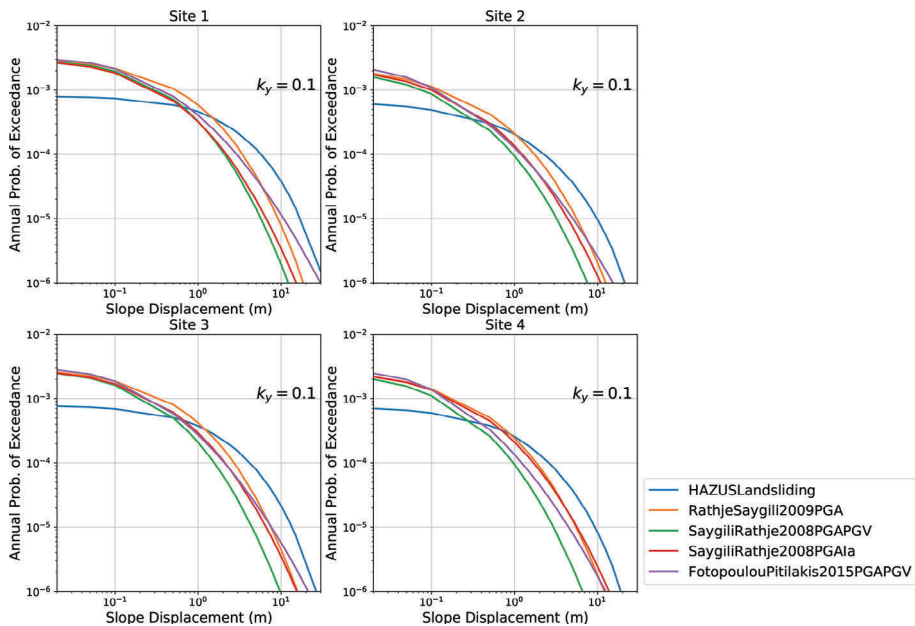


Figure 2. Slope displacement hazard curves for the four target sites, assuming $k_y = 0.1$

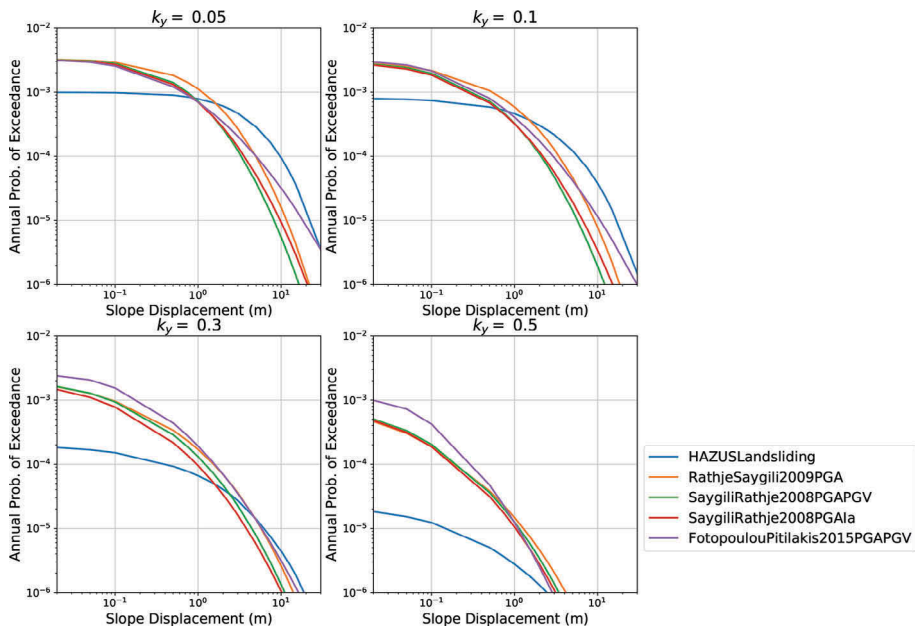


Figure 3. Slope displacement hazard curves for target site 1 assuming k_y of 0.05 g (top left), 0.1 g (top right), 0.3 g (bottom left), and 0.5 g (bottom right)

an extension of a controlled example taken from the 2018 Pacific Earthquake Engineering Research Centre (PEER) seismic hazard code verification tests (Hale *et al.*, 2018). The selected test is that of a bending fault with a length of 60 km and down-dip width of 12 km, dipping 60° to the east. Slip is assumed to be 2 mm/yr and magnitude recurrence is modelled using

characteristic magnitude of M_W 6.75, giving a rate of occurrence of 0.0034 per year. The resulting area of the characteristic magnitude is less than the total fault length, so in this example the rupture is allowed to float (i.e. move around the fault surface to cover all possible locations). Four site locations were considered, to cover both the hanging wall, footwall regions and end points of the fault. The configuration of the source and sites are shown in Figure 1. A simple epistemic uncertainty analysis is undertaken to calculate the slope displacement using five different GDEMs (vector and scalar): i) HAZUS, ii) Rathje & Saygili (2009) PGA only, iii) Saygili & Rathje (2008) PGA and PGV, iv) Saygili & Rathje (2008) PGA and Arias Intensity, and v) Fotopoulou & Pitilakis (2015) PGA and PGV. A correlation coefficient of 0.6 is assumed between PGA and PGV, and 0.8 between PGA and Arias Intensity. Figure 2 shows the resulting seismic hazard curves for the four sites for the case in which $k_y = 0.1$, whilst Figure 3 shows the results for Site 1 assuming different levels of k_y . Although a complete interpretation of these largely illustrative results is not entirely necessary, this demonstration highlights some potentially relevant trends that should be considered carefully in practice. Most notable in the results is the curve for the HAZUS methodology, for which the probability of exceedance is lower at small displacements and higher for larger ones. The lower probability of failure is a consequence of the P_{mu} term, meaning that failure can only occur with an annual probability of P_{mu} multiplied by the annual probability of occurrence of magnitudes greater than M_{min} (about 0.0034 in the present case). However, both the expected displacements predicted by the HAZUS methodology, and the uniform distribution of uncertainty, tend the hazard curve toward higher probabilities of exceedance for large displacements. Interestingly, the three models of Rathje & Saygili (2008) produce similar outcomes, albeit with the vector models tending toward lower hazard than the scalar models as a result of the reduced aleatory variability. The Fotopoulou & Pitilakis (2015) models seem to agree well with those Rathje & Saygili (2008), albeit with a greater sensitivity to k_y , particularly for higher k_y .

5 CONCLUSIONS & CONSIDERATIONS FOR FUTURE APPLICATIONS

The implementation of geotechnical hazard calculators in the OpenQuake-engine has the potential to enhance the practice of seismic risk analysis to account for secondary hazards. It is hoped that upon their full deployment in the OpenQuake package this work will go some way to addressing the barriers in practice arising from a lack of available tools. The careful means of implementation provide a broad suite of tools for a full and detailed application, encompassing full integration of aleatory uncertainties and, where relevant, the spatial correlations and cross-correlations. These modules serve a range of calculation needs, from “classical” slope displacement or liquefaction hazard as in the example shown here, to single scenario *Shakemap*-style applications, through to a fully-probabilistic event based seismic risk analysis of structures and networks.

Of course, the availability of tools now returns the onus to the geotechnical engineering community to develop the databases and models to form the inputs to probabilistic secondary hazard analysis that can capitalise on these developments. For regional assessment the need for large scale characterisation of the geotechnical properties is paramount. The measurement of expected displacements from liquefaction, however, still suffer from a paucity of models that can be applied over an extended spatial scale, and characterisation of epistemic uncertainties in this case remain extremely limited. The assessment of epistemic uncertainties for geotechnical hazard in terms of both the site properties, displacement models and the models of fragility and vulnerability will require extensive study in the future.

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