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Analytic solution for wave propagations in layered unsaturated soil and its application

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ABSTRACT: In most of present analysis for free field seismic response, unsaturated soils are usually regarded as quasi saturated or single-phase media. It is difficult to describe the characteristics of unsaturated soil. In this paper, a free field model for rock-layered unsaturated soil system is established. Based on the theory of wave motion in poroelastic medium, elastic wave propagation under plane P-wave and SV- wave incidence in the rock-layered unsaturated soil system is solved using transfer matrix method. By comparison with the existing solution for an overlying unsaturated soil layer on the bedrock half-space, the solution given here is verified. Assuming the saturation of the soil above the groundwater layer has a gradient variation along the depth, the effects of variation of groundwater level on earthquake ground motions for P-wave incidence are analyzed using this solution.

1 INTRODUCTION

Properties of soil layers cause changes in the amplitudes, spectral content, and duration of strong earthquake ground motion. It's necessary to choose a suitable soil model to analyze seismic ground motion in free-field. During the past few decades, unsaturated soils are usually regarded as quasi saturated or single-phase media in most of analysis for free field seismic response. It's difficult to describe the characteristics of unsaturated soil with replace it by quasi saturated or single-phase media. Wave theory of saturated porous media has been widely studied and applied because it can reflect the mechanical properties of saturated soil very well (Schanz, 2009). However, ideal saturated soil is very rare in reality. Simplifying unsaturated soil into quasi saturated or single-phase media will cause great discrepancy from actual phenomena. Therefore, it is of great significance to establish a model which can reflect the properties of unsaturated soil and study its seismic response.

With the development of porous media theory and mixture theory, some progress has been made in the study of wave propagation in unsaturated porous media in recent years. Researchers (Vardoulakis,1986; Santos, 1990; Tuncay, 1996; Wei, 2002; Lo, 2005; Lu, 2005; Cai, 2006a; Albers, 2009) have established dynamic equations for unsaturated porous media with different conditions based on Biot theory, mixture theory or combination of the two. The propagation characteristics of elastic waves in unsaturated porous media are studied meanwhile. It is found that there are three compressional waves (i.e., P1-, P2-, and P3-waves) and one shear wave (i.e., an S-wave), in which P1- and P2-waves are similar to the fast and slow compressional waves in Biot's theory, and a P3-wave arises owing to the presence of a gas phase. Based on the dynamic equation of unsaturated porous media, the study of bulk wave propagation on the interface between unsaturated porous media and other media has gradually developed. Cai (2006b) studied the reflection and transmission of P1 wave at the interface between saturated porous media and unsaturated sandstone. Lo (2008) analyzed the propagation characteristics of Rayleigh waves in the half space of unsaturated media. Chen (2011, 2012) have studied the propagation of surface wave and P wave at the interface of unsaturated porous media and single-phase elastic media, respectively. Tomar (2006) analyzed the reflection and transmission of plane P-wave and SV-wave from a single-phase elastic medium to the interface of unsaturated porous media. Kumar (2011) have studied the propagation of plane waves at the interface of two different unsaturated media. These studies show that the reflection and transmission waves of each model are significantly affected by the variation of soil saturation, which suggests that we should pay attention to the influence of the variation of soil saturation in the study of site seismic response. However, there are few studies on the ground motion characteristics of unsaturated porous media. Li (2017, 2018) first introduced the dynamic model of unsaturated porous media into the analytic analysis of seismic ground motion in a free field. The elastic wave propagation in an overlying unsaturated soil layer on the bedrock half-space was solved using an analytical approach to study the saturation effects of subsoil on the seismic ground motion of a free-field site. Based on this, a layered site model for overlying unsaturated soils on bedrock is established in this paper. Using transfer matrix method, the analytic solutions of elastic wave propagation under plane P-wave and SV-wave incidence in the rock-layered unsaturated soil system are derived respectively. The effects of the variation of groundwater level on earthquake ground motions are analyzed using this solution.

2 MODEL OF ROCK-LAYERED UNSATURATED SOIL FIELD

The unsaturated soil layered site model is shown in Figure 1. The uniform unsaturated soil layer on the horizontal homogeneous semi-infinite bedrock is divided into n layers with a thickness of (h_i=1, 2, …, n). The bedrock is assumed to be a linear elastic solid. The unsaturated soil is treated as a poroelastic medium, in which the solid matrix is filled with two coupled fluids(liquid and gas).

2.1 *Unsaturated porous media*

Following Wei (2002), a poroelastic model formulated using the theory of mixture with interfaces can be used to analyze the propagation of acoustic waves:

$$n_0^{\mathrm{S}}\rho_0^{\mathrm{S}}\ddot{\boldsymbol{u}}^{\mathrm{S}} = (M_{\mathrm{SS}}+G)\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + G\nabla\cdot\nabla\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{SW}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{SN}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{N}} + \hat{\mu}^{\mathrm{W}}(\dot{\boldsymbol{u}}^{\mathrm{W}}-\dot{\boldsymbol{u}}^{\mathrm{S}}) + \hat{\mu}^{\mathrm{N}}(\dot{\boldsymbol{u}}^{\mathrm{N}}-\dot{\boldsymbol{u}}^{\mathrm{S}}) \tag{1a}$$

$$n_0^{\mathrm{W}}\rho_0^{\mathrm{W}}\ddot{\boldsymbol{u}}^{\mathrm{W}} = M_{\mathrm{SW}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{WW}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{WN}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{N}} - \hat{\mu}^{\mathrm{W}}(\dot{\boldsymbol{u}}^{\mathrm{W}}-\dot{\boldsymbol{u}}^{\mathrm{S}}) \tag{1b}$$

$$n_0^{\mathrm{N}}\rho_0^{\mathrm{N}}\ddot{\boldsymbol{u}}^{\mathrm{N}} = M_{\mathrm{SN}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{WN}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{NN}}\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{N}} - \hat{\mu}^{\mathrm{N}}(\dot{\boldsymbol{u}}^{\mathrm{N}}-\dot{\boldsymbol{u}}^{\mathrm{S}}) \tag{1c}$$

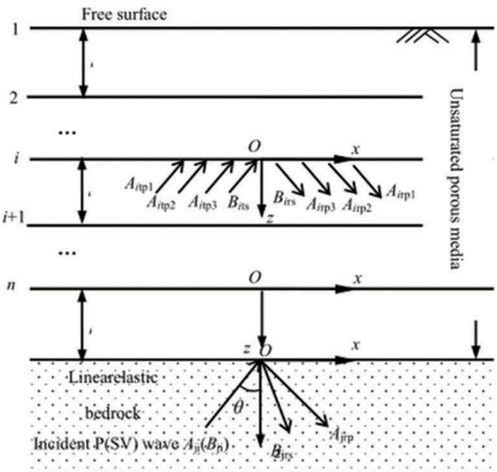

Figure 1. Simplified model of unsturated layered field

where the superscript S denotes the solid component; W and N denote the wetting and non-wetting fluids, respectively; n_0^α,ρ_0^α,and$\boldsymbol{u}^\alpha(\alpha{=}S, W, N)$ are the initial volume fraction, initial density, and displacements of individual components, respectively; M_{SS}, M_{WW}, M_{NN}, M_{SW}, M_{SN}, M_{WN} are elastic coefficients related to bulk modulus K^α of various phases in unsaturated porous media, bulk modulus K of solid skeleton, shear modulus G, effective stress coefficient α_B, and parametersΘ_{W}, Θ_{N}describing capillary equilibrium conditions; $\hat{\mu}^{\mathrm{f}}(\mathrm{f} = \mathrm{W}, \mathrm{N})$ can be related to the intrinsic permeability of a porous medium k, and the dynamic viscosity $\nu^{\mathrm{f}}$of an F-fluid, specific expressions are detailed in the reference Wei (2002). Letting $n_0^{\mathrm{N}}{=}0$, Eq.(1) can be reduced to Biot's poroelastic model (Biot, 1956). Using the reduced form, the saturated condition ($S_r = 1.0$) can be considered.

Based on the poroelastic model proposed by Wei (2002), the stress tensors in an unsaturated porous medium can be expressed as

$$\begin{aligned}\boldsymbol{\sigma}^{\mathrm{S}} &= \left(M_{\mathrm{SS}}\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{SW}}\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{SN}}\nabla\cdot\boldsymbol{u}^{\mathrm{N}}\right)I + 2n_0^{\mathrm{S}}\mu_{\mathrm{S}}\left(\nabla\boldsymbol{u}^{\mathrm{S}} + \left(\nabla\boldsymbol{u}^{\mathrm{S}}\right)^{\mathrm{T}}\right)\\ \boldsymbol{\sigma}^{\mathrm{W}} &= \left(M_{\mathrm{SW}}\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{WW}}\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{WN}}\nabla\cdot\boldsymbol{u}^{\mathrm{N}}\right)I\\ \boldsymbol{\sigma}^{\mathrm{N}} &= \left(M_{\mathrm{SN}}\nabla\cdot\boldsymbol{u}^{\mathrm{S}} + M_{\mathrm{WN}}\nabla\cdot\boldsymbol{u}^{\mathrm{W}} + M_{\mathrm{NN}}\nabla\cdot\boldsymbol{u}^{\mathrm{N}}\right)I\end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}^\alpha(\alpha = \mathrm{S}, \mathrm{W}, \mathrm{N})$ indicates the stress tensors of each phase, $\boldsymbol{I}$ is the unit tensor matrix.

2.2 *Linear elastic bedrock*

The governing equation for linear elastic bedrock is of the following form,

$$\left(\lambda^{\mathrm{R}} + \mu^{\mathrm{R}}\right)\nabla\nabla\cdot\boldsymbol{u}^{\mathrm{R}} + \mu^{\mathrm{R}}\nabla^2\boldsymbol{u}^{\mathrm{R}} = \rho^{\mathrm{r}}\ddot{\boldsymbol{u}}^{\mathrm{R}} \tag{3}$$

where λ^{R} and $\mu^{\mathrm{R}}$are Lame constants, ρ^{r} is the density of solid, and $\boldsymbol{u}^{\mathrm{R}}$ and $\ddot{\boldsymbol{u}}^{\mathrm{R}}$ are the displacement and acceleration vectors, respectively.

3 TRANSITION MATRIX OF WAVE COEFFICIENT IN UNSATURATED SOIL LAYERS

Based on the Helmholtz theorem, the displacement vector $\boldsymbol{u}^\alpha$ is given by

$$\boldsymbol{u}^\alpha = \nabla\phi^\alpha + \nabla\times\boldsymbol{\psi}^\alpha(a = \mathrm{S}, \mathrm{W}, \mathrm{N}) \tag{4}$$

whereϕ^α and $\boldsymbol{\psi}^\alpha$ are the scalar and vector displacement potentials, respectively.

By substituting Eq.(4) into Eq.(1), three compression waves P_1, P_2 and P_3 and a shear wave S can be obtained in unsaturated porous media (Li, 2018).Thus, in the *Oxz* plane, the wave field in the *i*-th unsaturated soil layer can be expressed as,

$$\begin{aligned}\varphi_{i\mathrm{p}}^{\mathrm{S}} =& A_{i\mathrm{tp1}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p1}x}x + k_{i\mathrm{p1}z}z)] + A_{i\mathrm{tp2}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p2}x}x + k_{i\mathrm{p2}z}z)]\\ &+ A_{i\mathrm{tp3}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p3}x}x + k_{i\mathrm{p3}z}z)] + A_{i\mathrm{rp1}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p1}x}x - k_{i\mathrm{p1}z}z)]\\ &+ A_{i\mathrm{rp2}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p2}x}x - k_{i\mathrm{p2}z}z)] + A_{i\mathrm{rp3}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{p3}x}x - k_{i\mathrm{p3}z}z)]\end{aligned} \tag{5a}$$

$$\psi_{i\mathrm{s}}^{\mathrm{S}} = B_{i\mathrm{ts}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{s}x}x + k_{i\mathrm{s}z}z)] + B_{i\mathrm{rs}}\exp[\mathrm{i}(\omega t - k_{i\mathrm{s}x}x - k_{i\mathrm{s}z}z)] \tag{5b}$$

$$\varphi_{i\mathrm{p}}^{\mathrm{f}} = \sum_{j=1}^{3}\delta_{ij}^{\mathrm{f}}\left(\varphi_{i\mathrm{p}j}^{\mathrm{u,S}} + \varphi_{i\mathrm{p}j}^{\mathrm{d,S}}\right)\ \mathrm{f} = \mathrm{W}, \mathrm{N} \tag{5c}$$

$$\psi_{i\mathrm{s}}^{\mathrm{f}} = \delta_{i\mathrm{s}}^{\mathrm{f}}\left(\psi_{i\mathrm{s}}^{\mathrm{u}} + \psi_{i\mathrm{s}}^{\mathrm{d}}\right)\ \mathrm{f} = \mathrm{W}, \mathrm{N} \tag{5d}$$

where,$\omega$is circular frequency, $\mathrm{i} = \sqrt{-1}$; $\delta_{ij}^{\mathrm{f}}(i = 1, 2, 3)$ are the participation coefficient of compression wave, $\delta_{i\mathrm{s}}^{\mathrm{f}}$ are the participation coefficient of shear wave (Li, 2018); $A_{i\mathrm{tp1}}$, $A_{i\mathrm{tp2}}$, $A_{i\mathrm{tp3}}$, $B_{i\mathrm{ts}}$, $A_{i\mathrm{rp1}}$, $A_{i\mathrm{rp2}}$, $A_{i\mathrm{rp3}}$, and $B_{i\mathrm{rs}}$ are the amplitudes; $k_{i\mathrm{p1}x}$, $k_{i\mathrm{p2}x}$, $k_{i\mathrm{p3}x}$, $k_{i\mathrm{s}x}$, $k_{i\mathrm{p1}z}$, $k_{i\mathrm{p2z}}$, $k_{i\mathrm{p3}z}$, and $k_{i\mathrm{s}z}$ are the wave numbers in the x-direction and z-direction respectively. ${k_{i\mathrm{p1}x}}^2 + {k_{i\mathrm{p1}z}}^2 = {k_{i\mathrm{p1}}}^2$, ${k_{i\mathrm{p2}x}}^2 + {k_{i\mathrm{p2}z}}^2 = {k_{i\mathrm{p2}}}^2$, ${k_{i\mathrm{p3}x}}^2 + {k_{i\mathrm{p3}z}}^2 = {k_{i\mathrm{p3}}}^2$, ${k_{i\mathrm{s}x}}^2 + {k_{i\mathrm{s}z}}^2 = {k_{i\mathrm{s}}}^2$; $k_{i\mathrm{p1}}$, $k_{i\mathrm{p2}}$, $k_{i\mathrm{p3}}$, and $k_{i\mathrm{s}}$ are the wave numbers of P_1, P_2, P_3 and SV waves.

The waves mentioned above must have an equal wave number in the x-direction. That is,

$$k_{ip1x} = k_{ip2x} = k_{ip3x} = k_{isx} \overset{\text{Set to}}{\rightarrow} k_x \tag{6}$$

According to Eq. (2) and (4), the stress and displacement components for the *i*-th unsaturated soil layer can be expressed the function of the displacement potentials as follows,

$$u_{xi}^{\alpha} = \frac{\partial \varphi_{ip}^{\alpha}}{\partial x} + \frac{\partial \psi_{is}^{\alpha}}{\partial z},\ u_{zi}^{\alpha} = \frac{\partial \varphi_{ip}^{\alpha}}{\partial z} - \frac{\partial \psi_{is}^{\alpha}}{\partial x}. \tag{7a}$$

$$\sigma_{zzi}^{\mathrm{S}} = M_{\mathrm{SS}i}\nabla^2\varphi_{ip}^{\mathrm{S}} + 2n_{0i}^{\mathrm{S}}\mu_{\mathrm{S}i}\left(\frac{\partial^2\varphi_{ip}^{\mathrm{S}}}{\partial z^2} - \frac{\partial^2\psi_{is}^{\mathrm{S}}}{\partial z\partial x}\right) + M_{\mathrm{SW}i}\nabla^2\varphi_{ip}^{\mathrm{W}} + M_{\mathrm{SN}i}\nabla^2\varphi_{ip}^{\mathrm{N}} \tag{7b}$$

$$\sigma_{xzi}^{\mathrm{S}} = n_{0i}^{\mathrm{S}}\mu_{\mathrm{S}i}\left(2\frac{\partial^2\varphi_{ip}^{\mathrm{S}}}{\partial x\partial z} - \frac{\partial^2\psi_{is}^{\mathrm{S}}}{\partial x^2} + \frac{\partial^2\psi_{is}^{\mathrm{S}}}{\partial z^2}\right) \tag{7c}$$

$$\sigma_{zzi}^{\mathrm{W}} = M_{\mathrm{SW}i}\nabla^2\varphi_{ip}^{\mathrm{S}} + M_{\mathrm{WW}i}\nabla^2\varphi_{ip}^{\mathrm{W}} + M_{\mathrm{WN}i}\nabla^2\varphi_{ip}^{\mathrm{N}} \tag{7d}$$

$$\sigma_{zzi}^{\mathrm{N}} = M_{\mathrm{SN}i}\nabla^2\varphi_{ip}^{\mathrm{S}} + M_{\mathrm{WN}i}\nabla^2\varphi_{ip}^{\mathrm{W}} + M_{\mathrm{NN}i}\nabla^2\varphi_{ip}^{\mathrm{N}} \tag{7e}$$

Substituting the total potential function in Eq. (5) into Eq. (7), the relationship between the stress and displacement components and the amplitudes in the *i*-th unsaturated soil layer can be obtained as,

$$\boldsymbol{S}_i = \boldsymbol{T}_{\mathrm{S}i}\boldsymbol{H}_i \exp[-\mathrm{i}(\omega t - k_x x)] \tag{8}$$

where, $\boldsymbol{S}_i = \left[\sigma_{zzi}^{\mathrm{N}}\ \ \sigma_{zzi}^{\mathrm{W}}\ \ \sigma_{zzi}^{\mathrm{S}}\ \ \sigma_{xzi}^{\mathrm{S}}\ \ u_{xi}^{\mathrm{S}}\ \ u_{zi}^{\mathrm{S}}\ \ u_{zi}^{\mathrm{W}}\ \ u_{zi}^{\mathrm{N}}\right]^{\mathrm{T}}$, $\boldsymbol{H}_i = \left[A_{ip1}^{\mathrm{u}}\ \ A_{ip2}^{\mathrm{u}}\ \ A_{ip3}^{\mathrm{u}}\ \ B_{is}^{\mathrm{u}}\ \ A_{ip1}^{\mathrm{d}}\ \ A_{ip2}^{\mathrm{d}}\ \ A_{ip3}^{\mathrm{d}}\ \ B_{is}^{\mathrm{d}}\right]^{\mathrm{T}}$, and $\boldsymbol{T}_{\mathrm{S}i}$.is a 8×8 matrix. The elements of the matrixes $\boldsymbol{T}_{\mathrm{S}i}$ can be found in the Appendix.

We assume that the two adjacent unsaturated soil layers are in perfect contact at the interface. Therefore, the boundary conditions at the plane interface are the continuity of the stress and displacement components. That is,

$$S_i|_{z=h_i} = S_{i+1}|_{z=0} \tag{9}$$

Substituting Eq. (8) into Eq. (9),

$$\boldsymbol{T}_{\mathrm{S}i}|_{z=h_i}\boldsymbol{H}_i = \boldsymbol{T}_{\mathrm{S}i+1}|_{z=0}\boldsymbol{H}_{i+1} \Rightarrow \boldsymbol{H}_{i+1} = \boldsymbol{T}_{\mathrm{S}i+1}{}^{-1}|_{z=0}\boldsymbol{T}_{\mathrm{S}i}|_{z=h_i}\boldsymbol{H}_i = \boldsymbol{T}_i\boldsymbol{H}_i \tag{10}$$

where, $\boldsymbol{T}_i = \boldsymbol{T}_{\mathrm{S}i+1}{}^{-1}|_{z=0}\boldsymbol{T}_{\mathrm{S}i}|_{z=h_i}$ is the transfer matrix of amplitudes. According to the recurrence relation given by Eq. (10), the wave amplitudes of the *i*-th unsaturated soil layer can be expressed by the wave amplitudes of the first layer,

$$\boldsymbol{H}_n = \boldsymbol{T}_{n1}\boldsymbol{H}_1 \tag{11}$$

in which $\boldsymbol{T}_{n1} = \boldsymbol{T}_{n-1}\boldsymbol{T}_{n-2}\cdots\boldsymbol{T}_1$.

4 SOLUTION OF ROCK-LAYERED UNSATURATED SOIL SYSYEM

Bedrock is regarded as a single-phase elastic medium. Assuming that the excitation froms the bedrock into an unsaturated porous medium as a sinusoidal plane P-wave (or SV-wave) with circular frequency ω, incident angle θ ($\beta$for an SV-wave), and amplitude $A_{\mathrm{p}}^{\mathrm{i}}$ ($B_{\mathrm{s}}^{\mathrm{i}}$for an SV-wave). Then in the bedrock, a reflected P-wave with amplitude $A_{\mathrm{p}}^{\mathrm{r}}$ and an SV-wave with $B_{\mathrm{s}}^{\mathrm{r}}$ are generated. The displacement potentials of all of these waves can be easily defined as follows.

For an incident P-wave,

$$\varphi = A_{\mathrm{p}}^{\mathrm{i}}\exp[\mathrm{i}(\omega t - k_{\mathrm{p}x}x + k_{\mathrm{p}z}z)] + A_{\mathrm{p}}^{\mathrm{r}}\exp[\mathrm{i}(\omega t - k_{\mathrm{p}x}x - k_{\mathrm{p}z}z)],\ \psi = B_{\mathrm{s}}^{\mathrm{r}}\exp[\mathrm{i}(\omega t - k_{\mathrm{s}x}x - k_{\mathrm{s}z}z)], \tag{12a}$$

For an incident SV-wave,

$$\varphi = A_{\mathrm{p}}^{\mathrm{r}}\exp[\mathrm{i}(\omega t - k_{\mathrm{p}x}x - k_{\mathrm{p}z}z)],\ \psi = B_{\mathrm{s}}^{\mathrm{i}}\exp[\mathrm{i}(\omega t - k_{\mathrm{s}x}x + k_{\mathrm{s}z}z)] + B_{\mathrm{s}}^{\mathrm{r}}\exp[\mathrm{i}(\omega t - k_{\mathrm{s}x}x - k_{\mathrm{s}z}z)] \tag{12b}$$

In which,

$$k_{\mathrm{p}x} = k_{\mathrm{s}x} = k_x = k_{\mathrm{p}}\sin\theta(\mathrm{ForPwaveincidence}) = k_{\mathrm{s}}\sin\beta(\mathrm{ForSVwaveincidence}) \tag{13}$$

The relationship between the stress and displacement components and the amplitudes in the bedrock also can be obtained by applying the Helmholtz theorem,

$$\boldsymbol{S}^{\mathrm{R}} = \boldsymbol{T}_S{}^{\mathrm{R}}\boldsymbol{H}^{\mathrm{R}} \exp[-\mathrm{i}(\omega t - k_x x)] \tag{14}$$

where, $\boldsymbol{S}^{\mathrm{R}} = \left[\sigma_{zz}^{\mathrm{R}} \quad \sigma_{xz}^{\mathrm{R}} \quad u_x^{\mathrm{R}} \quad u_z^{\mathrm{R}}\right]^{\mathrm{T}}$, $\mathrm{H}^{\mathrm{R}} = \left[A_{\mathrm{p}}^{\mathrm{i}} \quad B_{\mathrm{s}}^{\mathrm{i}} \quad A_{\mathrm{p}}^{\mathrm{r}} \quad B_{\mathrm{s}}^{\mathrm{r}}\right]^{\mathrm{T}}$, The elements of the matrixes $\boldsymbol{T}_S{}^{\mathrm{R}}$can be found in the Appendix.

All the waves in the rock-layered unsaturated soil field should satisfy the boundary conditions at the ground surface and the interface between different soil layers. The interface between bedrock and unsaturated soil is impermeable and there is no relative motion between them, which can be expressed as,

$$\sigma_{zz}^{\mathrm{R}}|_{z=0} = \sigma_{zzn}^{\mathrm{S}}|_{z=h_n} + \sigma_{zzn}^{\mathrm{W}}|_{z=h_n} + \sigma_{zzn}^{\mathrm{N}}|_{z=h_n},\ \sigma_{xz}^{\mathrm{R}}|_{z=0} = \sigma_{xzn}^{\mathrm{S}}|_{z=h_n}. \tag{15a}$$

$$u_z^{\mathrm{R}}|_{z=0} = u_{zn}^{\mathrm{S}}|_{z=h_n} = u_{zn}^{\mathrm{W}}|_{z=h_n} = u_{zn}^{\mathrm{N}}|_{z=h_n},\ u_x^{\mathrm{R}}|_{z=0} = u_{xn}^{\mathrm{S}}|_{z=h_n}. \tag{15b}$$

On the ground surface, the following stress-free boundary conditions should be satisfied,

$$\sigma_{zz1}^{\mathrm{S}}|_{z=0} = \sigma_{zz1}^{\mathrm{W}}|_{z=0} = \sigma_{zz1}^{\mathrm{N}}|_{z=0} = \sigma_{xz1}^{\mathrm{S}}|_{z=0} = 0 \tag{16}$$

Appling the boundary conditions given by Eqs.(15) and (16) into Eqs.(8), (11) and (14), the unknown amplitudes of all waves in the rock-layered unsaturated soil field are obtained. So all the wave potential functions can be determined, and the stress and displacement components can be defined using Eq. (7). Because of the limitation of length, no more tautology here.

5 APPLICATION

5.1 *Verification*

To check the validity of the solution presented in this paper, two unsaturated soil layers are considered in Figure 1. The material parameters of unsaturated soil in the two layers are chosen to be seem and as in Li (2017). Figure 2 shows the normalized horizontal and vertical displacement amplitudes at the ground surface versus the dimensionless frequency with an angle of incidence θ =20° for P-wave incidence by the present solution and the solution for the single unsaturated soil layer by Li (2017). The comparison shows that both of solutions are in perfect agreement.

5.2 *Influence of groundwater level change on earthquake ground motion*

Suppose the overlying soil of bedrock is the same kind of soil with different saturation. The material parameters of the unsaturated soil and bedrock are given in Li (2017). The analytical model is shown in Figure 3. The soil below the groundwater table is saturated, and the saturation of the soil above the groundwater table varies exponentially with the depth (Zhou, 2013),

$$S_{\mathrm{r}}(z) = S_{\mathrm{r}}(0) + [S_{\mathrm{r}}(H-h) - S_{\mathrm{r}}(0)](\frac{x}{H-h})^{\gamma} \tag{17}$$

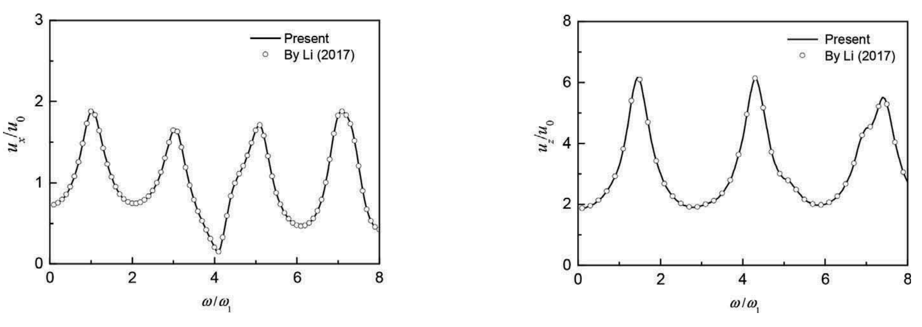

Figure 2. Comparison diagram between the solution in this paper and in Li (2017)

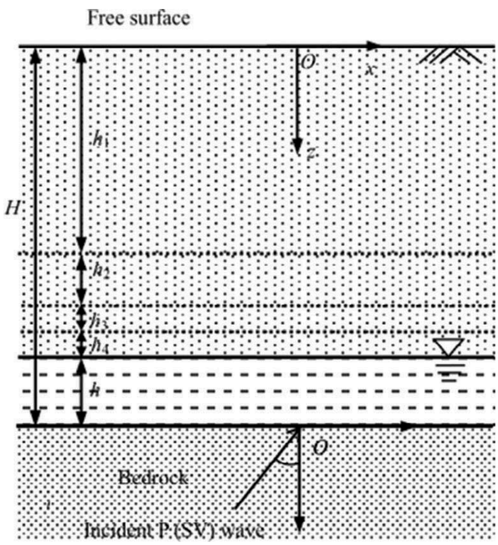

Figure 3. Analysis model for the change of underground water table

To reflect the gradient variation of saturation along the depth, the soil layer above the groundwater table is divided into four layers. The saturation of each layer is 0.3, 0.5, 0.7 and 0.9 from top to bottom, respectively. According to Eq. (17), the thickness ratio of each unsaturated layers is $h_1 : h_2 : h_3 : h_4 = 8 : 2 : 1 : 1$.The dimensionless frequency ω/ω_1 is used in the analysis, where$\omega_1$is the first natural frequency of the upper layer soil with S_r=1.0.

Figure 4 shows the normalized horizontal displacement amplitudes u_x/u_0 and vertical displacement amplitudes u_z/u_0 (where u_0 is the displacement intensity of the incident wave) at the ground surface versus the dimensionless frequency with θ=0° and 30° for P-wave incidence, respectively. In order to reflect the influence of groundwater level, the thickness of saturated soil h is taken as 0.1H, 0.2H, 0.3H, 0.4H and 0.5H respectively, and the increase of h indicates the rise of groundwater level.

It can be seen from the Figure 4 that the variation of groundwater level has a significant effect on the natural frequency and the response peak of the ground motion. When P wave is incident vertically ($\theta = 0°$), the response peak of the vertical displacement decreases and the natural frequency increases with the increase of groundwater level. When P-wave obliquely incident ($\theta = 30°$), the change of groundwater level has relatively little effect on the natural frequency and the response peak of the horizontal displacement. The natural frequency and the response peak of the vertical displacement are greatly affected. With the rise of groundwater level, the response peak of the vertical displacement decreases, and the natural frequency increases gradually.

Considering the case of incident frequency ω/ω_1=1.0, Figure 5 shows the normalized horizontal and vertical displacement amplitudes with the incident angle for different groundwater levels for P-

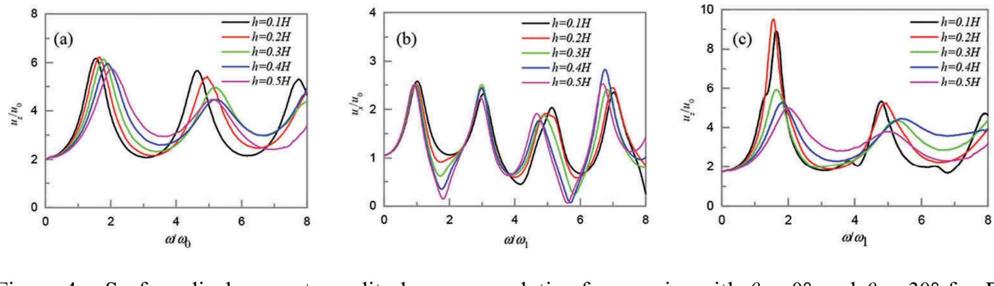

Figure 4. Surface displacement amplitudes versus relative frequencies with θ = 0° and θ = 30° for P-wave incidence (a) vertical displacement with θ = 0°, (b) horizontal displacement with θ = 30°, (c) vertical displacement with θ = 30°

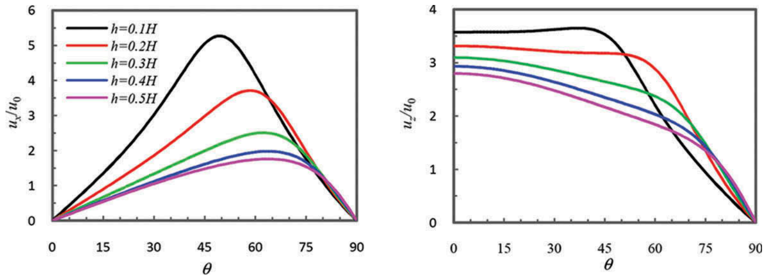

Figure 5. Surface displacement amplitudes versus incident angle θ for P-wave incidence with$\omega/\omega_1 = 1.0$

wave incidence. It can be seen that the horizontal displacement increases first and then decreases with the increase of incident angle. The horizontal displacement is 0 for $\theta = 0°$and$\theta = 90°$. The incident angle at which the horizontal displacement reaches the maximum value is related to the groundwater level. As the groundwater level rises, the angle increases. When$\theta = 0°$, the vertical displacement is the largest. With the increase of the incident angle, the vertical displacement decreases and appears a turning point at a certain incident angle. After the incident angle is greater than the turning point, the vertical displacement decreases sharply, and when $\theta = 90°$, the vertical displacement reaches 0. With the rising of groundwater level, the incident angle at which the vertical displacement appears a turning point increases. It also can be seen from Figure 5 that the influence of the groundwater level on the horizontal displacement first increases and then decreases with the incident angle increasing from 0° to 90°. The horizontal displacement decreases with the rising of groundwater level. When the incident angle is small, the influence of groundwater level on vertical displacement is greater, and the vertical displacement decreases with the rising of groundwater level.

6 CONCLUSION

Based on the theory of wave motion in poroelastic medium, elastic wave propagation under plane P-wave and SV-wave incidence in the rock-layered unsaturated soil system is solved using transfer matrix method. By comparison with the existing solution for an overlying unsaturated soil layer on the bedrock half-space, the solution given here is verified. Assuming the saturation of the soil above the groundwater layer has a gradient variation along the depth, the effects of variation of groundwater level on earthquake ground motions for P-wave incidence are analyzed using this solution. Some preliminary conclusions are obtained: the influence of groundwater level on ground motion is related to the incident wave frequency and incident angle. At the same frequency, when the incident angle is small, the change of groundwater level has a greater impact on the vertical displacement of the surface. The effect of the groundwater level on the horizontal displacement increases first and then decreases with the increase of incident angle. At the same incident angle, the response peak of the vertical displacement decreases and the natural frequency increases with the increase of the groundwater level, but the response peak and the natural frequency of the horizontal displacement change little.

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Appendix

The elements of the matrix $\boldsymbol{T}_{\mathrm{S}i}(T_{\mathrm{S}ij}(i,j=1,2,\cdots,8))$ are as follows:

$$T_{\mathrm{S}i11} = -k_{ip1}^2(\delta_{i1}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i1}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip1z}z),$$

$$T_{\mathrm{S}i12} = -k_{ip2}^2(\delta_{i2}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i2}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip2z}z),$$

$$T_{\mathrm{S}i13} = -k_{ip3}^2(\delta_{i3}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i3}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip3z}z),\ \ T_{\mathrm{S}i14} = 0,$$

$$T_{\mathrm{S}i15} = -k_{ip1}^2(\delta_{i1}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i1}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip1z}z),$$

$$T_{\mathrm{S}i16} = -k_{ip2}^2(\delta_{i2}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i2}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip2z}z),$$

$$T_{\mathrm{S}i17} = -k_{ip3}^2(\delta_{i3}^{\mathrm{N}}M_{\mathrm{NN}i} + M_{\mathrm{SN}i} + \delta_{i3}^{\mathrm{W}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip3z}z),\ \ T_{\mathrm{S}i18} = 0;$$

$$T_{\mathrm{S}i21} = -k_{ip1}^2(M_{\mathrm{SW}i} + \delta_{i1}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i1}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip1z}z),$$

$$T_{\mathrm{S}i22} = -k_{ip2}^2(M_{\mathrm{SW}i} + \delta_{i2}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i2}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip2z}z),$$

$$T_{\mathrm{S}i23} = -k_{ip3}^2(M_{\mathrm{SW}i} + \delta_{i3}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i3}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(\mathrm{i}k_{ip3z}z),\ \ T_{\mathrm{S}i24} = 0,$$

$$T_{\mathrm{S}i25} = -k_{ip1}^2(M_{\mathrm{SW}i} + \delta_{i1}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i1}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip1z}z),$$

$$T_{\mathrm{S}i26} = -k_{ip2}^2(M_{\mathrm{SW}i} + \delta_{i2}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i2}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip2z}z),$$

$$T_{\mathrm{S}i27} = -k_{ip3}^2(M_{\mathrm{SW}i} + \delta_{i3}^{\mathrm{W}}M_{\mathrm{WW}i} + \delta_{i3}^{\mathrm{N}}M_{\mathrm{WN}i})\exp(-\mathrm{i}k_{ip3z}z),\ \ T_{\mathrm{S}i28} = 0;$$

$$T_{\mathrm{S}i31} = -\left[k_{i\mathrm{p}1}^{2}(M_{\mathrm{SS}i} + \delta_{i1}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i1}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}1z}{}^{2}\right] \exp(\mathrm{i}k_{i\mathrm{p}1z}z),$$

$$T_{\mathrm{S}i32} = -\left[k_{i\mathrm{p}2}^{2}(M_{\mathrm{SS}i} + \delta_{i2}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i2}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}2z}{}^{2}\right] \exp(\mathrm{i}k_{i\mathrm{p}2z}z),$$

$$T_{\mathrm{S}i33} = -\left[k_{i\mathrm{p}3}^{2}(M_{\mathrm{SS}i} + \delta_{i3}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i3}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}3z}{}^{2}\right] \exp(\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i34} = -2G_i k_{i\mathrm{s}x} k_{i\mathrm{s}z} \exp(\mathrm{i}k_{i\mathrm{s}z}z),$$

$$T_{\mathrm{S}i35} = -\left[k_{i\mathrm{p}1}^{2}(M_{\mathrm{SS}i} + \delta_{i1}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i1}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}1z}{}^{2}\right] \exp(-\mathrm{i}k_{i\mathrm{p}1z}z),$$

$$T_{\mathrm{S}i36} = -\left[k_{i\mathrm{p}2}^{2}(M_{\mathrm{SS}i} + \delta_{i2}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i2}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}2z}{}^{2}\right] \exp(-\mathrm{i}k_{i\mathrm{p}2z}z),$$

$$T_{\mathrm{S}i37} = -\left[k_{i\mathrm{p}3}^{2}(M_{\mathrm{SS}i} + \delta_{i3}^{\mathrm{W}} M_{\mathrm{SW}i} + \delta_{i3}^{\mathrm{N}} M_{\mathrm{SN}i}) + 2G_i k_{i\mathrm{p}3z}{}^{2}\right] \exp(-\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i38} = 2G_i k_{i\mathrm{s}x} k_{i\mathrm{s}z} \exp(-\mathrm{i}k_{i\mathrm{s}z}z);$$

$$T_{\mathrm{S}i41} = 2G_i k_{i\mathrm{p}1x} k_{i\mathrm{p}1z} \exp(\mathrm{i}k_{i\mathrm{p}1z}z),\ \ T_{\mathrm{S}i42} = 2G_i k_{i\mathrm{p}2x} k_{i\mathrm{p}2z} \exp(\mathrm{i}k_{i\mathrm{p}2z}z),$$

$$T_{\mathrm{S}i43} = 2G_i k_{i\mathrm{p}3x} k_{i\mathrm{p}3z} \exp(\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i44} = G_i\left(k_{i\mathrm{s}x}{}^{2} - k_{i\mathrm{s}z}{}^{2}\right) \exp(\mathrm{i}k_{i\mathrm{s}z}z),$$

$$T_{\mathrm{S}i45} = -2G_i k_{i\mathrm{p}1x} k_{i\mathrm{p}1z} \exp(-\mathrm{i}k_{i\mathrm{p}1z}z),\ \ T_{\mathrm{S}i46} = -2G_i k_{i\mathrm{p}2x} k_{i\mathrm{p}2z} \exp(-\mathrm{i}k_{i\mathrm{p}2z}z),$$

$$T_{\mathrm{S}i47} = -2G_i k_{i\mathrm{p}3x} k_{i\mathrm{p}3z} \exp(-\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i48} = G_i\left(k_{i\mathrm{s}x}{}^{2} - k_{i\mathrm{s}z}{}^{2}\right) \exp(-\mathrm{i}k_{i\mathrm{s}z}z);$$

$$T_{\mathrm{S}i51} = -\mathrm{i}k_{i\mathrm{p}1x} \exp(\mathrm{i}k_{i\mathrm{p}1z}z),\ \ T_{\mathrm{S}i52} = -\mathrm{i}k_{i\mathrm{p}2x} \exp(\mathrm{i}k_{i\mathrm{p}2z}z),\ \ T_{\mathrm{S}i53} = -\mathrm{i}k_{i\mathrm{p}3x} \exp(\mathrm{i}k_{i\mathrm{p}3z}z),$$

$$T_{\mathrm{S}i54} = \mathrm{i}k_{i\mathrm{s}z} \exp(\mathrm{i}k_{i\mathrm{s}z}z),\ \ T_{\mathrm{S}i55} = -\mathrm{i}k_{i\mathrm{p}1x} \exp(-\mathrm{i}k_{i\mathrm{p}1z}z),\ \ T_{\mathrm{S}i56} = -\mathrm{i}k_{i\mathrm{p}2x} \exp(-\mathrm{i}k_{i\mathrm{p}2z}z),$$

$$T_{\mathrm{S}i57} = -\mathrm{i}k_{i\mathrm{p}3x} \exp(-\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i58} = -\mathrm{i}k_{i\mathrm{s}z} \exp(-\mathrm{i}k_{i\mathrm{s}z}z);\ \ T_{\mathrm{S}i61} = \mathrm{i}k_{i\mathrm{p}1z} \exp(\mathrm{i}k_{i\mathrm{p}1z}z),$$

$$T_{\mathrm{S}i62} = \mathrm{i}k_{i\mathrm{p}2z} \exp(\mathrm{i}k_{i\mathrm{p}2z}z),\ \ T_{\mathrm{S}i63} = \mathrm{i}k_{i\mathrm{p}3z} \exp(\mathrm{i}k_{i\mathrm{p}3z}z),\ \ T_{\mathrm{S}i64} = \mathrm{i}k_{i\mathrm{s}x} \exp(\mathrm{i}k_{i\mathrm{s}z}z),$$

$$T_{\mathrm{S}i65} = -\mathrm{i}k_{i\mathrm{p}1z} \exp(-\mathrm{i}k_{i\mathrm{p}1z}z),\ \ T_{\mathrm{S}i66} = -\mathrm{i}k_{i\mathrm{p}2z} \exp(-\mathrm{i}k_{i\mathrm{p}2z}z),\ \ T_{\mathrm{S}i67} = -\mathrm{i}k_{i\mathrm{p}3z} \exp(-\mathrm{i}k_{i\mathrm{p}3z}z),$$

$$T_{\mathrm{S}i68} = \mathrm{i}k_{i\mathrm{s}x} \exp(-\mathrm{i}k_{i\mathrm{s}z}z),\ \ T_{\mathrm{S}i71} = \mathrm{i}k_{i\mathrm{p}1z} \exp(\mathrm{i}k_{i\mathrm{p}1z}z)\delta_{i1}^{\mathrm{W}},\ \ T_{\mathrm{S}i72} = \mathrm{i}k_{i\mathrm{p}2z} \exp(\mathrm{i}k_{i\mathrm{p}2z}z)\delta_{i2}^{\mathrm{W}},$$

$$T_{\mathrm{S}i73} = \mathrm{i}k_{i\mathrm{p}3z} \exp(\mathrm{i}k_{i\mathrm{p}3z}z)\delta_{i3}^{\mathrm{W}},\ \ T_{\mathrm{S}i74} = \mathrm{i}k_{i\mathrm{s}x} \exp(\mathrm{i}k_{i\mathrm{s}z}z)\delta_{i\mathrm{s}}^{\mathrm{W}},\ \ T_{\mathrm{S}i75} = -\mathrm{i}k_{i\mathrm{p}1z} \exp(-\mathrm{i}k_{i\mathrm{p}1z}z)\delta_{i1}^{\mathrm{W}},$$

$$T_{\mathrm{S}i76} = -\mathrm{i}k_{i\mathrm{p}2z} \exp(-\mathrm{i}k_{i\mathrm{p}2z}z)\delta_{i2}^{\mathrm{W}},\ \ T_{\mathrm{S}i77} = -\mathrm{i}k_{i\mathrm{p}3z} \exp(-\mathrm{i}k_{i\mathrm{p}3z}z)\delta_{i3}^{\mathrm{W}},\ \ T_{\mathrm{S}i78} = \mathrm{i}k_{i\mathrm{s}x} \exp(-\mathrm{i}k_{i\mathrm{s}z}z)\delta_{i\mathrm{s}}^{\mathrm{W}};$$

$$T_{\mathrm{S}i81} = \mathrm{i}k_{i\mathrm{p}1z} \exp(\mathrm{i}k_{i\mathrm{p}1z}z)\delta_{i1}^{\mathrm{N}},\ \ T_{\mathrm{S}i82} = \mathrm{i}k_{i\mathrm{p}2z} \exp(\mathrm{i}k_{i\mathrm{p}2z}z)\delta_{i2}^{\mathrm{N}},\ \ T_{\mathrm{S}i83} = \mathrm{i}k_{i\mathrm{p}3z} \exp(\mathrm{i}k_{i\mathrm{p}3z}z)\delta_{i3}^{\mathrm{N}},$$

$$T_{\mathrm{S}i84} = \mathrm{i}k_{i\mathrm{s}x} \exp(\mathrm{i}k_{i\mathrm{s}z}z)\delta_{i\mathrm{s}}^{\mathrm{N}},\ \ T_{\mathrm{S}i85} = -\mathrm{i}k_{i\mathrm{p}1z} \exp(-\mathrm{i}k_{i\mathrm{p}1z}z)\delta_{i1}^{\mathrm{N}},\ \ T_{\mathrm{S}i86} = -\mathrm{i}k_{i\mathrm{p}2z} \exp(-\mathrm{i}k_{i\mathrm{p}2z}z)\delta_{i2}^{\mathrm{N}},$$

$$T_{\mathrm{S}i87} = -\mathrm{i}k_{i\mathrm{p}3z} \exp(-\mathrm{i}k_{i\mathrm{p}3z}z)\delta_{i3}^{\mathrm{N}},\ \ T_{\mathrm{S}i88} = \mathrm{i}k_{i\mathrm{s}x} \exp(-\mathrm{i}k_{i\mathrm{s}z}z)\delta_{i\mathrm{s}}^{\mathrm{N}}.$$

The elements of the matrix $\boldsymbol{T}_S{}^{\mathrm{R}}(T_{\mathrm{S}ij}^{\mathrm{R}}(i,j = 1,2,3,4))$ are as follows:

$$T_{\mathrm{S}11}^{\mathrm{R}} = -\left(\lambda k_{\mathrm{p}}^{2} + 2\mu k_{\mathrm{p}z}{}^{2}\right) \exp\left(\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}12}^{\mathrm{R}} = -2\mu k_{\mathrm{s}x} k_{\mathrm{s}z} \exp(\mathrm{i}k_{\mathrm{s}z}z),$$

$$T_{\mathrm{S}13}^{\mathrm{R}} = -\left(\lambda k_{\mathrm{p}}^{2} + 2\mu k_{\mathrm{p}z}{}^{2}\right) \exp\left(-\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}14}^{\mathrm{R}} = 2\mu k_{\mathrm{s}x} k_{\mathrm{s}z} \exp(-\mathrm{i}k_{\mathrm{s}z}z);$$

$$T_{\mathrm{S}21}^{\mathrm{R}} = 2\mu k_{\mathrm{p}x} k_{\mathrm{p}z} \exp\left(\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}22}^{\mathrm{R}} = \mu\left(k_{\mathrm{s}x}^{2} - k_{\mathrm{s}z}^{2}\right) \exp\left(\mathrm{i}k_{\mathrm{s}z}z\right),\ \ T_{\mathrm{S}23}^{\mathrm{R}} = -2\mu k_{\mathrm{p}x} k_{\mathrm{p}z} \exp\left(-\mathrm{i}k_{\mathrm{p}z}z\right),$$

$$T_{\mathrm{S}24}^{\mathrm{R}} = \mu\left(k_{\mathrm{s}x}^{2} - k_{\mathrm{s}z}^{2}\right) \exp\left(-\mathrm{i}k_{\mathrm{s}z}z\right);\ \ T_{\mathrm{S}31}^{\mathrm{R}} = -\mathrm{i}k_{\mathrm{p}x} \exp\left(\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}32}^{\mathrm{R}} = \mathrm{i}k_{\mathrm{s}z} \exp(\mathrm{i}k_{\mathrm{s}z}z),$$

$$T_{\mathrm{S}31}^{\mathrm{R}} = -\mathrm{i}k_{\mathrm{p}x} \exp\left(-\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}34}^{\mathrm{R}} = -\mathrm{i}k_{\mathrm{s}z} \exp(-\mathrm{i}k_{\mathrm{s}z}z);\ \ T_{\mathrm{S}41}^{\mathrm{R}} = \mathrm{i}k_{\mathrm{p}z} \exp\left(\mathrm{i}k_{\mathrm{p}z}z\right),$$

$$T_{\mathrm{S}32}^{\mathrm{R}} = \mathrm{i}k_{\mathrm{s}x} \exp(\mathrm{i}k_{\mathrm{s}z}z),\ \ T_{\mathrm{S}31}^{\mathrm{R}} = -\mathrm{i}k_{\mathrm{p}z} \exp\left(-\mathrm{i}k_{\mathrm{p}z}z\right),\ \ T_{\mathrm{S}34}^{\mathrm{R}} = \mathrm{i}k_{\mathrm{s}x} \exp(-\mathrm{i}k_{\mathrm{s}z}z).$$