*Earthquake Geotechnical Engineering for Protection and Development of Environment and Constructions – Silvestri & Moraci (Eds)*
 *ISBN 978-0-367-14328-2*

# Granular mechanics of the seismic stability of soil slopes

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ABSTRACT: The analysis of the stability of slopes made of grains submitted to their own weight and to the maximum seismic force of inertia is presented in this paper. The basic hypothesis is that the stresses are transmitted along linear, conjugated and symmetrical chains of contact forces. The obtained stresses fulfill the differential equations of pseudo-static equilibrium and the boundary conditions, and serve to find the potential slip line. Then, the shear stress as well as the shear strength at a constant angle with the major principal stress are calculated. As the soil is in elastic state, the strength parameters must be reduced to be compatible with the failure of the soil. In granular mechanics, the reduction factor leads to different slip lines whether the initial or the reduced shear parameters are used to define the chains of forces. The results are compared with the experimental and numerical slip lines and the factor of safety as well, reported by several authors, finding a good concordance.

## 1 INTRODUCTION

The evaluation of slope response to seismic stresses can be done using pseudo-static or dynamic methods. In the first case, the stability of the slope is defined by the concepts of safety factor and critical acceleration and, in the second case, by introducing the simplified displacement concepts (e.g. Newmark 1965, Sarma 1981, Chlimintzas 2003, Tan 2006), and the numerically calculated displacements (e.g. Clough et al. 1967, Duncan 1996). For the evaluation of the stability, it is necessary to previously know the geometry of the slip line. In the limit equilibrium method, the straight line, the circumference, the logarithmic spiral, and other curves have been used (e.g. Fellenius 1936, Bishop 1955, and Morgenstern & Price 1965). Then, the parameters defining the critical slip line are attained by minimizing the factor of safety. In the isostatic theory of plasticity, the soil obeys the Mohr-Coulomb law of failure at every point. So that, the Cartesian stresses are calculated directly by solving the system of the three equations: the Mohr-Coulomb law and the two differential equations of equilibrium. From these, the stress field and the slip lines field are obtained (Sokolovski 1965). Granular mechanics presents a similar but much simpler theory, based on the particulate nature of coarse and fine soils.

## 2 CHAINS OF FORCES IN DENSE GRANULAR SOILS

Soils are materials made of discrete solid particles in contact with each other, which, in the first instance, can be described as a packing of spheres. From the mechanical point of view, the most important property of this material is the transmission of stresses following discrete paths, involving some grains and excluding others, forming a network of force trajectories with certain tendencies, which change according to the nature of the stress. This phenomenon has been observed in the experiments on two-dimensional arrays of discs made of photo elastic material, (e.g. Majmudar & Berhinger 2005), (Figure 1a), and, also, in the numerical simulations of the granular matter, such as the Distinct Element Method (e.g. Cundall & Strack 1979).

In order to establish a mathematical model for these materials, several authors have used the methods of Statistical Mechanics in the description of the geometry of random granular

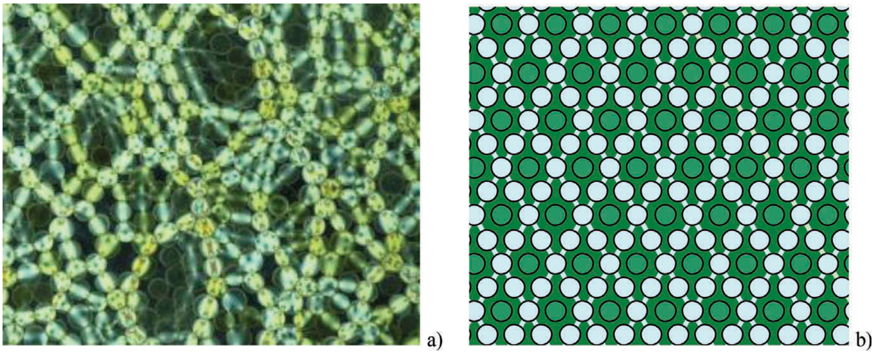


Figure 1. a) Photograph of chains of contact forces in a medium consisting of bi-refringent disks (Majmudar & Behringer 2005). b) Two-dimensional packing of spheres and chains of contact forces in an idealized granular medium.

assemblies and the network of contact forces (e.g. Snoeijer et al. 2004), but, in general, the solutions attained are not immediately applicable to the practice of Geotechnical Engineering. In this paper, a model based on the simplifying hypothesis that the granular material is constituted by ordered equivalent spheres, and that the stresses are transmitted following discrete, linear, conjugated and symmetrical chains of forces inclined the angle $\theta_0$ to the vertical is described (Figure 1b). For the sake of simplicity, each chain of forces is associated to an inclined band, so that the concepts of the continuum mechanics can be directly applied.

## 3 ZONING AND CLASSIFICATION OF BILINEAR SOIL SLOPES

When considering slopes with simple geometry, the surface of a soil in active state is described by two straight lines: the crest, inclined an angle $j$, and the slope, inclined an angle $i$, both to the horizontal, (Figure 2a). In terms of the polar coordinates, $(r, \theta)$, the intersection of these boundary lines with the network of forces, in a gentle slope, defines three zones: the slope zone, BOD, the core zone, DOE, and the crest zone, AOE, defined by the conditions: $i \leq \theta \leq 90° - \theta_0$; $90° - \theta_0 \leq \theta \leq 90° + \theta_0$; and $90° + \theta_0 \leq \theta \leq 180° + j$; respectively. But in steep slopes, there are only two zones: an incomplete core zone, and the crest zone, AOE, where $i \leq \theta \leq 90° + \theta_0$, and $90° + \theta_0 \leq \theta \leq 180° + j$, respectively. In this case, a special zone appears, limited by the slope line and the chain of forces coming up from the toe, in which the chains of forces pointing to the slope line are in unbalanced state. Tension phenomena, debris and earth falls and toppling are originated by this zone, named "hanging zone". Because its analysis is rather different, steep soil slopes are excluded from consideration in this paper.

## 4 STRESSES IN BILINEAR SLOPES

The weight of the rhomboidal element formed by the intersection of two bands of conjugate and symmetric contact forces can be expressed as $W_0 = \gamma S h$, where $\gamma$ is the unit weight of the soil, $S$, the area of the base and $h$, the half-diagonal of the element. Each elementary contact force is given by: $f = W_0/(2\cos\theta_0)$ and the total force of a chain made of $N$ grains is equal to: $F = Nf$. Thus, for the two chains that intersect at the grain $(x, z)$, the total forces are: $F_1 = N_1 f$ and $F_2 = N_2 f$. This means that the forces caused by the chains of forces 1 and 2 can be expressed as the components of the weight of the inclined columns of heights $z\text{-}z_1 = N_1 h$ and $z\text{-}z_2 = N_2 h$, respectively. By dividing each Cartesian component of these forces by the horizontal width, $S$, and by the vertical width, $S\cot\theta_0$, of the band of forces, partial stresses are obtained. The sum and the difference of these partial stresses yield the components of the average stress tensor:

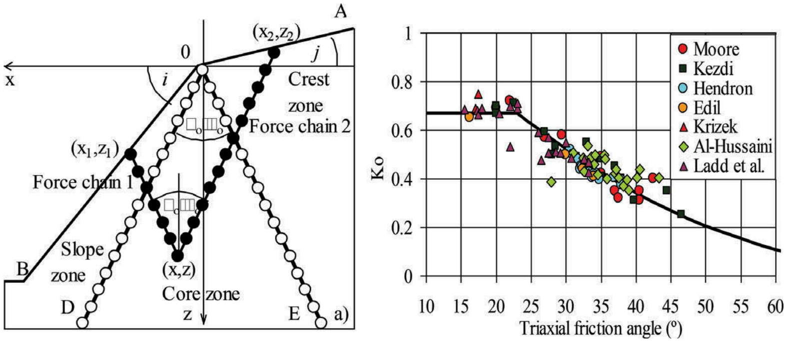


Figure 2. a) Geometry, chains of forces, and zones in a moderate bilinear slope. b) Comparison of the experimental data with the theoretical relationship between the triaxial internal friction angle and the lateral pressure coefficient at rest (Yanqui, 1982).

$$\sigma_{xx} = \frac{\gamma}{2\eta^2}(2z - z_1 - z_2) \quad \sigma_{zz} = \eta^2\sigma_{xx} \quad \tau_{xz} = \frac{\gamma}{2\eta}(z_2 - z_1) \tag{1}$$

where $z_1$ and $z_2$ are the topographic elevations of the intersections of the chains of forces with the slope profile. When these stresses are associated to conjugated inclined slices, the stability analysis of a slope of general geometry with stratified soil and phreatic surface may be performed. But the development of this theme is beyond of this paper.

Particularly, in the core zone of a bilinear gentle slope, the intersections $z_1$ and $z_2$ are obtained in terms of the coefficients $m = \tan i$, $k = \tan j$, and $\eta = \cot\theta_0$. Hence, the components of the two-dimensional stress state are written as:

$$\sigma_{xx} = \frac{\gamma}{2\eta}\frac{(2\eta + m - k)z - \eta(m+k)x}{(\eta + m)(\eta - k)} \tag{2}$$

$$\tau_{xz} = \frac{\gamma}{2}\frac{(m+k)z + [\eta(m-k) - 2mk]x}{(\eta + m)(\eta - k)} \tag{3}$$

$$\sigma_{zz} = \eta^2\sigma_{xx} \tag{4}$$

These equations, that satisfy the differential equations of equilibrium, may be used to calculate the Cartesian stresses in the slope zone, just by doing $k = m$:

$$\sigma_{xx} = \gamma\frac{z - mx}{\eta^2 - m^2} \qquad \tau_{xz} = m\sigma_{xx} \qquad \sigma_{zz} = \eta^2\sigma_{xx} \tag{5}$$

This set of equations describes also the stresses in the crest zone by doing $m = k$, and, in general, the stress state of an infinite slope as well; which gives rise to the following conclusions: (a) the condition of zero stresses at the boundary is satisfied; (b) all of the points located at the same depth exhibit the same stress state, so that the failure plane is parallel to the soil surface, and (c) in regarding the inexistence of tension stresses, an infinite slope cannot be steeper than the gradient of the chains of forces $\eta$.

## 5 VALIDATION OF THE MODEL

Stresses in an infinite slope are elastic if $m < \mu$, and become critical when $m = \mu$, where $\mu = \tan\varphi$, and $\varphi$ is the internal friction angle of the soil in plane strain. Making $m = \mu$ in equations (5) and substituting in the Mohr-Coulomb failure law, the relation $\eta^2 = 1 + 2\mu^2$ is attained.

Likewise, the geostatic stress state, characterized by the coefficient of lateral pressure at rest, $K_0$, is obtained from the condition $m = k = 0$ in equations (5), giving the relationship $K_0 = 1/\eta^2$. Eliminating the parameter η of these expressions yields:

$$\tan^2\theta_0 = \frac{1}{\eta^2} = K_0 = \frac{1}{1+2\mu^2} \tag{6}$$

This equation allows the validity of the present theory to be weighed, since experimentally both the internal friction angle and the coefficient of lateral pressure at rest are determined independently from each other. To do that, φ must be expressed in terms of the triaxial internal friction angle, $\varphi_t$. From Figure 2b it is concluded that the present granular model describes properly the behavior of coarse and fine soils.

## 6 ISOGONAL LINE IN A BILINEAR SLOPE

Once the inclination of the major principal axis, $\alpha_1$, is obtained from the stress transformation equations, and the oblique angle of a line, $\theta_d$, is defined, the angle of this line, β, with respect to the abscissa, is obtained from the equation:

$$\beta = \frac{\pi}{2} + \frac{1}{2}\arctan\left(\frac{2\tau_{xz}}{\sigma_{xx}-\sigma_{zz}}\right) - \theta_d \tag{7}$$

In a soil mass, the oblique line becomes an isogonal trajectory to the major principal stress, since it changes in direction at each point. Known the angle β, the differential equation of the isogonal trajectories of the oblique lines is easily deduced in the polar coordinate system ($r$, θ) using the usual procedures of the differential calculus: $r' + r\cot(\theta - \beta) = 0$. Since β is a function of the polar angle θ, the solution of this differential equation is a non-canonical logarithmic spiral:

$$r = C\exp\left[-\int\cot(\theta-\beta)d\theta\right] \tag{8}$$

in which the integration constant, $C$, is determined by knowing some condition, such as, for example, the condition that the isogonal line passes through the toe of the slope. Then, $C=r_0 = H/\sin i$. At the surface of the slope, where $\theta = i$, and all of the components of the stress are zero, the angle β is attained by substituting the stresses given by equations (5) in equation (7). Then:

$$\beta = \frac{\pi}{2} + \frac{1}{2}\arctan\left(\frac{2m}{1-\eta^2}\right) - \theta_d \tag{9}$$

The integral of the exponent of equation (8) cannot be expressed as a close-form function, and, therefore, some method of numerical integration may be used. For that purpose, suppose that at point $n$, belonging to the isogonal trajectory and defined by the polar coordinates ($r_n$, $\theta_n$), the Cartesian stresses and, therefore, the angle of the isogonal line, $\beta_n$, are known. At point $n$+1, belonging to the same trajectory, the polar angle is given by $\theta_{n+1} = \theta_n + \Delta\theta_n$, where $\Delta\theta_n$ is the chosen increment of the polar angle. Applying the law of sines to the differential triangle gives the radius vector of this point, $r_{n+1}$, as:

$$r_{n+1} = r_n\frac{\sin(\theta_n-\beta_n)}{\sin(\theta_n-\beta_n+\Delta\theta_n)} \tag{10}$$

Known the values of the polar coordinates ($r_{n+1}$, $\theta_{n+1}$) of the point $n + 1$, the Cartesian stresses given by the formulas (2), (3) and (4) are calculated, and with them, the direction of the oblique line, β, given by equation (7). With this data, the previous procedure is repeated and the polar coordinates ($r_{n+2}$, $\theta_{n+2}$), corresponding to point $n + 2$, are determined. By applying successively this procedure the isogonal curve is obtained, starting from the initial polar coordinates of the toe at the base ($r_0$, $i$).

## 7 FACTOR OF SAFETY AGAINST SLIDING

The normal and tangential components of the stress, acting on an isogonal line, can be obtained from the Cartesian stresses by means of the equations of the stress transformation:

$$\sigma = \frac{\sigma_{zz} + \sigma_{xx}}{2} + \frac{\sigma_{zz} - \sigma_{xx}}{2}\cos 2\beta - \tau_{xz}\sin 2\beta \qquad \tau = \frac{\sigma_{zz} - \sigma_{xx}}{2}\sin 2\beta + \tau_{xz}\cos 2\beta \tag{11}$$

In a uniform stress field, the failure occurs when the shear stress $\tau$ at the oblique line equals the shear strength, $\tau_f = \mu\sigma + c$. If the specimen is in elastic state, the inequality $\tau < \tau_f$ is valid. This means that for the soil to reach the ultimate state, the strength parameters, $\mu$ and $c$, must be reduced, dividing them by a quantity, *FS*, called factor of safety. But, according to equation (6), the packing of the grains and the structure of the chains of forces depend on the angle of internal friction of the soil. Therefore, there are two ways to determine *FS*:

a. The chains of forces depend on the reduced coefficient of friction, so that the following condition must be fulfilled at the ultimate state: $(\mu_R\sigma + c_R)/\tau = 1$, where, $\mu_R=\mu/FS$ and $c_R=c/FS$. In this case, since the shear stress is equal to the shear strength of the soil, the oblique angle is equal to the failure angle: $\theta_d = \theta_f = \pi/4 - \varphi_R/2$.
b. The chains of forces depend on the original coefficient of friction, so that the factor of safety is calculated as: $FS = \tau_f/\tau$.

When the stress field is not uniform, the previous reasoning is still applicable if the shear resultant forces are found by integration of the shear stresses along the entire isogonal line, *L*. Since the stress functions (11) cannot be integrated in close-form functions, it is more useful to express the integrals as summations. Then:

$$\frac{\Sigma(\mu_R\sigma_n + c_R)\Delta L_n}{\Sigma\tau_n L_n} = 1 \qquad FS = \frac{\Sigma(\mu\sigma_n + c)\Delta L_n}{\Sigma\tau_n L_n} \tag{12}$$

for the first criterion and the second criterion, respectively. In the first case, *FS* is defined as the number by which the strength parameters must be divided so that the resulting shear force is equal to the shear strength force. This procedure is similar to that of the limit equilibrium method proposed by Bishop (1955). In the second case, as the strength remains constant, *FS* can be withdrawn from the summation to get an explicit form. This criterion is related to the development of the shear band in dense granular soils, where the shear strain is maximum or, reciprocally, where the safety factor is minimum: $d(FS)/d\theta_d = 0$. The angle that meets this condition is called the critical oblique angle and is a characteristic of the slope (Tan 2006). This procedure is similar to that adopted in the Finite Element Method.

By applying the law of sines to the triangle drawn by the radius vector $r_n$, the radius vector $r_{n+1}$ and the increment of the arc length $\Delta L_n$, this quantity is found to be:

$$\Delta L_n = r_n \frac{\sin\Delta\theta_n}{\sin(\theta_n - \beta_n + \Delta\theta_n)} \tag{13}$$

## 8 PSEUDO-STATIC STRESSES

One way to introduce the effect of an earthquake on a soil slope is to consider the force of inertia caused by the earthquake as static force acting on the rhomboidal element of the soil mass. For this purpose, the maximum inertial force is considered. The Cartesian components $E_h$ and $E_v$ of this force can be written in terms of the Cartesian components $a_h$ and $a_v$ of the maximum acceleration caused by the earthquake, as follows: $E_h = W_0\alpha_h$ and $E_v = W_0\alpha_v$; where $\alpha_h = a_h/g$ and $\alpha_v = a_v/g$ are called seismic coefficients. These forces are added algebraically to the weight of the grain to obtain the components of the resultant. If the seismic force is supposed to come from a harmonic movement, it changes its sign in each half-period of the displacement. It is usually considered that the most critical state is given when the horizontal component points in the direction of the slope, and the vertical component points upwards. This means that the resultant force has the inclination $s$ with respect to the vertical given by: $\tan s = \alpha_h/(1-\alpha_v)$.

If the inclinations of the chains of forces do not change due to the seismic stress, the unitary contact forces are obtained by decomposing the resultant force in the two directions of the chains of forces, $\theta_0$: $f_1 = W_0(1-\alpha_v-\eta\alpha_h)/(2\cos\theta_0)$ and $f_2 = W_0(1-\alpha_v + \eta\alpha_h)/(2\cos\theta_0)$. The total contact forces, $F_1$ and $F_2$ acting on the grain $(x, z)$ are obtained by adding the unit forces as in the static problem. Then, the Cartesian components of these forces are divided between the area of the horizontal and vertical sections of each band to obtain the pseudo-static Cartesian stresses, $s_{xx}$, $s_{xz}$ and $s_{zz}$, which can be expressed as a linear combination of self-weight stresses:

$$s_{xx} = (1-\alpha_v)\sigma_{zz} + \alpha_h\tau_{xz} \qquad s_{xz} = (1-\alpha_v)\tau_{xz} + \alpha_h\sigma_{zz} \qquad s_{zz} = \eta^2 s_{xx} \tag{14}$$

## 9 FACTOR OF SAFETY AND CRITICAL ACCELERATION

The oblique line obtained directly from the pseudo-static seismic stresses (14) does not coincide with the failure line observed in landslides. This is because this method does not take into account the vibratory nature of earthquakes, whose acceleration changes in sign and magnitude quickly. In fact, if dynamic condition is taking into account, the real slip surface cannot be readily predicted using simple method of analysis. For this reason, most of the authors agree that the oblique line derived from the static stresses should be considered. First, the critical oblique angle of the isogonal line in static condition is determined; then, maintaining this angle, the step-by-step calculation described above is repeated in terms of the pseudo-static stresses (14) to find the factor of safety. For the pseudo-static analysis, the two criteria of defining the factor of safety are applicable. Another parameter used in the evaluation of the pseudo-static stability of a slope is the critical acceleration defined by Newmark (1965) as the acceleration to be applied to soil mass to produce the failure of the slope. To get this scope, the oblique angle of the critical line in static condition is determined, and then, with this angle, the seismic coefficient is calculated, so that $FS = 1.0$. For instance, in the simplest case of cohesionless infinite slope, whose stresses at a line parallel to the surface are the same as the method of limiting equilibrium, the critical coefficient of acceleration is given by: $\alpha_h=\tan(\varphi-i)$.

## 10 EVALUATION OF RESULTS

One of the purposes of soil slope analysis is determining the true slip line. A source of this information is the reports presented by the authors after earthquakes. For instance, Shannon and Wilson (1964) may be cited, who have studied the grabens developed behind the L-street slide area in Anchorage during the Alaskan earthquake. They found a bilinear slip line before the sliding mass sunk into the vacant space, made of a horizontal line and a line inclined to 45°, approximately, just as the granular mechanics predicts for a soil with zero internal friction angle, as corresponds to a liquefied soil (Figure 3). This mechanism has also been observed in saturated clay soils (e.g. Wright and Duncan 1972; Bjerrum 1968).

In recent years, several experimental methods have been proposed to find the true slip line. For instance, Pinyol et al. (2017) reported a methodology to find out the failure line of dry granular slopes by using a non-invasive technique named Particle Image Velocimetry. The analyzed slope had a height of 25.0 cm, and an inclination of 60°. The granular soil was calcareous-siliceous sand characterized by an angle of internal friction in plane strain of 43° and unit weight of 15.4 kN/m$^3$. Figure 4a shows the excellent correspondence between the theoretical shear band derived from the granular mechanics and the experimental line.

Likewise, several numerical methods, such as FEM, DEM and FDM, have been used in order to determine the slip line by means of the point displacements or the shear band localization. For

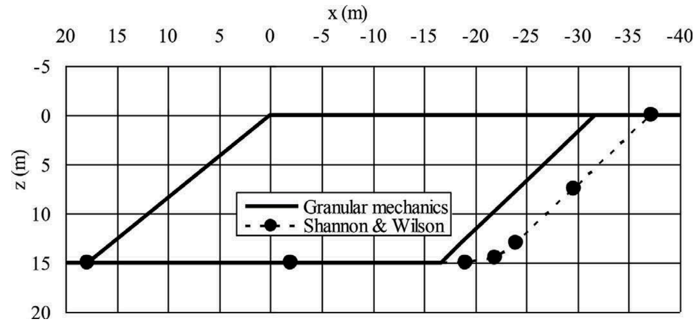


Figure 3. Comparison of the critical slip line given by the Granular Mechanics and that occurred in liquefied soil, at Anchorage during the Alaskan Earthquake (Shannon & Wilson, 1964).

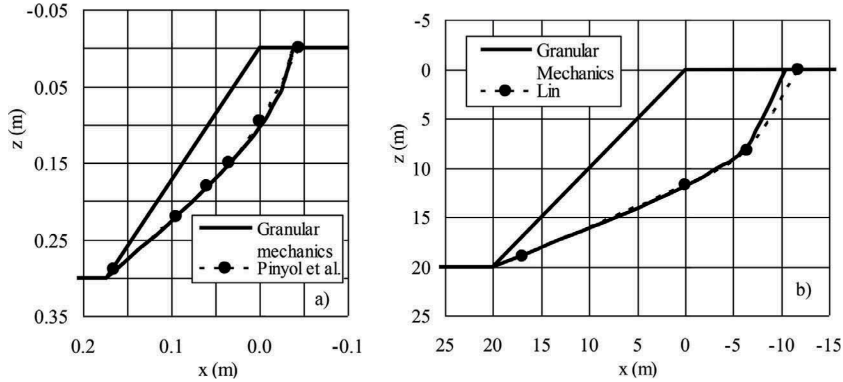


Figure 4. Comparison of the critical slip line given by the Granular Mechanics and that obtained by means of: a) the Particle Image Velocimetry technique, and b) the FLAC3D numerical method.

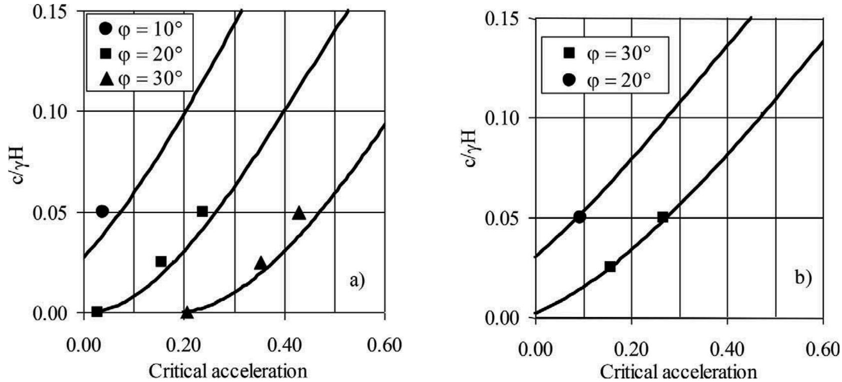


Figure 5. Relationship between the critical acceleration and the factor of stability. Comparison of the curves given by the Granular Mechanics and the data obtained by Tan (2006) by using the Finite Element Method: a) for a 1V:2H slope, and b) for a 1V:1.5H slope.

example, by using the FLAC3D, Lin et al. (2009) analyzed a soil slope in plane strain state of 20.0 m height and an inclination of 45°, made of a homogeneous soil having a unit weight of 25 kN/m$^3$, an internal friction angle of 17°, and an effective cohesion of 42 kPa. The good agreement of the theoretical curve with the numerical results is shown in Figure 4b.

The seismic response of the slopes can be verified in terms of the critical acceleration and the stability factor $c/\gamma H$, for different angles of internal friction and inclination angles of the slope. In Figure 5, the data obtained by Tan (2006) by using the Finite Element Method and the curves given by the granular mechanics are presented. There, it is observed that the bigger the gradient of the slope the better the fitting between them.

## 11 CONCLUSIONS

The simplifying hypothesis that granular matter is made of linear and conjugate chains of contact forces acting in an ordered packing of grains allows analyzing the soil slope stability problem in a very simple way. Firstly, the intersection of the bilinear slope boundary with the network of contact forces defines the internal zones and the classification of the slopes. Secondly, the Cartesian stresses originated by the self-weight of the grains are determined readily because of the linearity of the chains of forces. Thirdly, the seismic pseudo-static Cartesian stresses are expressed as the linear combination of the static stresses. For moderate soil slopes, the critical slip line, which coincides with the shear band, is found as the isogonal trajectory to the principal stress field that minimizes the factor of safety. When compared the theoretical results yielded by the granular mechanics with the observational, experimental and numerical data reported by several authors, a good agreement is found in both the geometry of the critical slip line, and the factor of safety or the critical acceleration.

## REFERENCES


Bishop, A.W. 1955. The use of the slip circle in the stability of earth slopes. *Geotechnique* 5 (4), pp. 7–17.

Bjerrum, L. 1968. Progressive failure in slopes of overconsolidated plastic clay and clay shales. *N.G.I.* 77.

Chlimintzas, G.O. 2003. *Seismic displacements of slopes using multi-block*. Ph.D. Thesis. Imperial College of London.

Clough, R.W. & Woodward, R.J. 1967. Analysis of embankments stresses and deformations. *Journal of Soil Mech. and Found. ASCE* 93 (4), pp. 529–550.

Cundall, A.P. & Strack, O.D.L. 1979. A discrete numerical model for granular assemblies. *Geotechnique* 29: 47–65.

Duncan, J.M. 1996. State of the art: Limit equilibrium and finite element analysis of slopes. *Journal of Geotechnical Eng*. 122 (7), pp. 577–596.

Fellenius, W. 1936. Calculation of the stability of earth dams. *Transactions, 2nd Congress on Large Dams*, Washington, pp. 445–462.

Lin, H., Cao, P., Gong, F., Li, J. & Gui Y. 2009. Directly searching method for slip plane and its influential factors based on critical state of slope. *J. Cent. South Univ. Technol*. 16, pp. 131–136. Springer.

Majmudar, T.S. & Behringer, R.P. 2005. Contact force measurements and stress – induced anisotropy in granular materials. *Nature* 435: 1079.

Morgenstern, N.R. & Price, V.E. 1965. Analysis of stability of general slip surfaces. *Geotechnique*, 15 (1), pp. 79–93.

Newmark, N.M. 1965. Effects of earthquakes on dams and embankments. *Geotechnique*, 15 (2) pp 139–160.

Pinyol, N., Alvarado, M., Parera F. & Yerro A. 2017. Novel procedure to validate MPM results by means of PIV measurements. *Procedia Engineering* 175: 332–340. Elsevier.

Sarma, S.K. 1981. Seismic displacement analysis of earth dams. *Journal of Soil Mech. and Found. ASCE* 107 (12), pp. 1735–1739.

Shannon & Wilson Inc. 1964. *Report on Anchorage area soil studies, Alaska, to U.S. Army Engineer District, Anchorage, Alaska*.

Snoeijer, J.H., Vlugt, T.J.H. van Hecke, M. & van Saarloos, W. 2004. Force network ensemble: a new approach to the static granular matter. *Phys. Rev. Lett*., 92: 054302.

Sokolovski, V.V. 1965. *Statics of granular media*. London. Butterworths Scientific.

Tan, D. 2006. *Seismic slope safety-determination of critical slip surface using acceptability criteria*. Ph.D. Thesis. University of London.

Wright S.G. & Duncan, J.M. 1972. Analysis of Waco dam slide. *J. Soil Mech., ASCE* 98: 869–877.

Yanqui, C. 1982. *Statics of gravitating discontinua*. M.Sc. Thesis. University of South Carolina.

Yanqui, C. 2013. Granular mechanics of the critical state of coarse soils. *Proc. 7th. Int. Conf. on Micromechanics of granular media. AIP* 1542, 197–200.