Earthquake Geotechnical Engineering for Protection and Development of Environment and Constructions – Silvestri & Moraci (Eds)
© 2019 Associazione Geotecnica Italiana, Rome, Italy, ISBN 978-0-367-14328-2

Effect of numerical modeling protocols on the seismic response of a liquefiable slope

K. Ziotopoulou
University of California, Davis, CA, USA

J. Montgomery
Auburn University, Alabama, USA

D. Tsiaousi, P. Tasiopoulou*, J. Ugalde & T. Travasarou
Fugro USA Land, Inc., CA, USA
**currently GR8 GEO, Athens, Greece*

ABSTRACT: The effects of constitutive model calibration and system-level modeling procedures on the dynamic response of a submerged medium-dense sand slope are examined. The numerical simulations for this study are independently performed by two different teams using the same numerical tools and the same input data. The modeling choices made by the different teams during the various stages of the analysis are documented and compared. Comparisons of simulation results are performed at both the element and system level to illustrate the effects of different protocols and practices on the results. The magnitude of uncertainty in the simulation results due to these factors is compared to experimental results from centrifuge tests on a similar submerged slope. Discussions of the importance of thorough documentation, which allows for comparisons such as this one to be made, is included.

1 INTRODUCTION

Nonlinear deformation analyses (NDAs) are widely used in research and practice to evaluate the seismic performance of geotechnical structures affected by liquefaction-related phenomena. Previous studies have shown that the reliability of results from an NDA study depends on several technical and nontechnical factors, including the selection and calibration of the constitutive models, the capabilities and limitations of the numerical modeling procedures ideally validated against case histories and/or model tests, the quality of the site characterization work, the availability and quality of laboratory data, the selection of the input ground motions, the expertise of the analyst, the allotment of sufficient time and resources for completing the work, and the quality of the documentation and review processes (e.g., Boulanger & Ziotopoulou 2018). Increasing stakeholders' confidence in NDA results requires understanding each of these sources of uncertainty.

Previous comparison studies have shown that different numerical modelers working with the same input motions and soil properties can produce very different predictions of dynamic soil response (e.g., Arulanandan & Scott 1993, Stewart et al. 2008, Kutter et al. 2019). The sources of these differences appear to stem largely from differences in calibration protocols, limitations in selected constitutive models, and variations in numerical procedures. Unlike other important factors (e.g., selection of input motions or boundary conditions), there is currently no established guidance on how to account for these sources of uncertainty when applying the results to a project. In light of this, it is important to identify possible sources of such differences and propose verification and validation protocols that will help reducing unnecessary variations in NDA results.

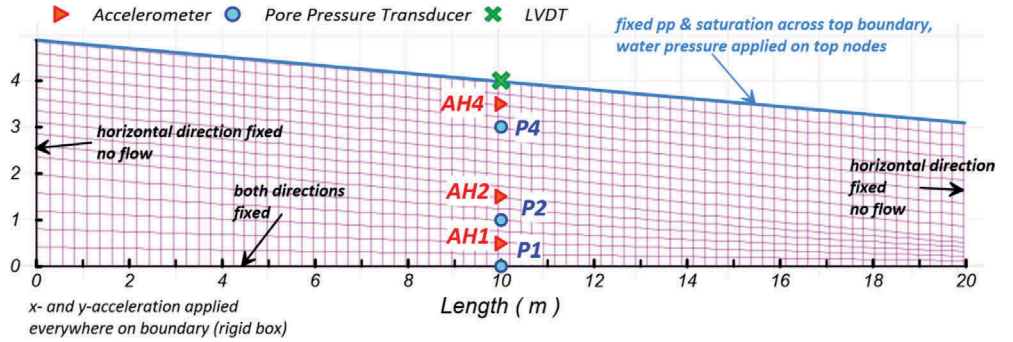

Figure 1. Schematic representation of the centrifuge test layout with FLAC mesh annotated with boundary conditions (rigid box not shown for clarity) and selected instrumentation locations where results are presented in this paper.

This paper focuses on the response of the submerged liquefiable slope (Figure 1) studied within the Liquefaction Experiments and Analyses Project (LEAP, Kutter et al. 2019). Numerical simulations were performed at the single-element level (constitutive model calibration) and the system level, by two different teams composed of researchers from University of California, Davis and Auburn University (denoted as team UA), and Fugro (denoted as team FUG) using the same set of tools (constitutive model PM4Sand Version 3.1 – Boulanger & Ziotopoulou 2017 – and numerical platform FLAC 8 – Itasca 2016) and the same input data (geometry, soil properties, and input motion). The numerical simulations were created as part of a Type B (Lambe 1973) simulation exercise meaning the results of the experiments were unknown to the analysts at the time of their simulations, and only the test configuration and the achieved base excitations and densities were made available to them. The simulation results are compared against a set of experimental data from the Rensselaer Polytechnic Institute (RPI) centrifuge facility. The objectives of this paper are to examine the calibration results at the element level and the dynamic responses at the system level in terms of spectral accelerations, excess pore pressures, and displacements. These comparisons are used to understand how differences in numerical protocols selected by experienced users affect the results and how these effects compare to experimental uncertainty. The overall goal of this work is to foster improvements in validation procedures for numerical modeling of liquefaction effects in research and engineering practice.

2 CALIBRATION PROTOCOLS

Each of the teams independently calibrated PM4Sand to the selected soil (Ottawa F-65) using undrained stress-controlled cyclic triaxial compression results at three different relative densities (62%, 80%, and 90%) from El Ghoraiby et al. (2018). Calibration of PM4Sand requires, at a minimum, three primary input parameters: (1) the apparent relative density (D_R) which controls the dilatancy and stress-strain response characteristics of the soil; (2) the contraction rate parameter (h_{po}) which controls the cyclic strength of the soil; and (3) the shear modulus coefficient (G_o) which controls the small strain stiffness of the model (G_{max}). Depending on the availability of lab data the user may choose to calibrate some of the secondary parameters as well.

The UA and FUG teams performed single element stress-controlled Plane Strain Compression (PSC) simulations to capture the triaxial compression tests. The parameter h_{po} was iteratively adjusted for each D_R until the liquefaction strength curves produced from the simulation reasonably matched the experimental data to the extent possible. Differences in the calibration process between the teams were primarily related to: (a) the D_R and cyclic strength applicable to the targeted RPI centrifuge test, (b) selection of G_o which was poorly constrained by the available lab and centrifuge data (no experimental data for small-strain stiffness or shear wave velocity provided), and (c) activation of secondary parameters. The following

Table 1. PM4Sand parameters calibrated by the two different teams for the RPI experiment based on the data by El Ghoraiby et al. (2018)

Parameter	Description	Cal 1B (UA)	Cal 2B (UA)	Cal FUG
D_R	Apparent relative density	64.7%	64.7%	65%
G_o	Shear modulus coefficient	334.0	779.0	625.0
h_{po}	Contraction rate parameter	0.11	0.07	0.07
e_{max}	Maximum void ratio	0.739	0.739	0.739
e_{min}	Minimum void ratio	0.492	0.492	0.492
n_b	Bounding ratio constant	0.7	0.7	0.5 (default)
φ_{cv}	Critical state friction angle	30°	30°	33°(default)

subsections describe the different approaches and Table 1 summarizes final parameters. More detailed information can be found in the corresponding papers by Montgomery and Ziotopoulou (2019) and Tsiaousi et al. (2019).

2.1 *Calibrations by UC Davis & Auburn (UA – Cal 1B and Cal 2B)*

Montgomery and Ziotopoulou (2019) calibrated PM4Sand against the El Ghoraiby et al. (2018) data during Phase I of LEAP-2017 (Calibrations 1A and 2A – Set A). These calibrations were revisited during Phases II and III (resulting to Calibrations 1B and 2B – Set B) when centrifuge data was available in order to account for the actual achieved soil properties rather than the design values. The differentiating factor between Set A and B was the maximum and minimum void ratios considered which were found to significantly affect results (Kutter et al. 2019) with Set B being closer to the correct values (Carey et al. 2019). Thus, only Set B is presented herein.

In LEAP, the achieved densities in the centrifuge tests varied between the different tests and none exactly matched the target value (D_R=62%). Thus, simulation results from the individual Cyclic Stress Ratio (*CSR*) levels of the single-element simulations were fitted with a power function ($CSR = aN^{-b}$) and the *CSR* to reach 2.5% single amplitude axial strain in 15 cycles for all three D_R values was approximated. The obtained liquefaction triggering curve was used to calibrate h_{po} for untested density levels like the ones obtained at the various facilities.

Similar to h_{po}, G_o is also dependent on D_R and is thus expected to vary between the different experiments. To address this variation, UA performed two calibrations with each one taking a slightly different approach in its selection. For the first calibration (1B), values for G_o were obtained by comparing single-element simulation results to the stress-strain response from the cyclic triaxial tests during the first few cycles of loading. G_o was iteratively changed until a reasonable approximation was achieved. The selected G_o values using this approach were plotted against the corresponding D_R's and by fitting a relationship, applicable G_o values were selected for D_R's different from the tested ones. The second calibration (2B) was performed in an identical manner to 1B, except for G_o which was calculated using the default PM4Sand relationship.

Two of the secondary model parameters were also adjusted to improve the comparison of the simulations results to the stress-strain and stress path results from the cyclic triaxial tests. The critical state friction angle φ_{cv} was reduced to 30° from the default value of 33°, to better match the slope of the frictional envelopes (bounding line) in the stress path plots. The second default parameter that was modified was n^b which controls the bounding ratio and therefore dilatancy and peak effective friction angles. The default value of 0.5 was increased to 0.7 to reduce the rate of strain accumulation in the cyclic mobility regime following the triggering of liquefaction.

2.2 *Calibration by Fugro (Cal FUG)*

Fugro calibrated the primary parameters of PM4Sand and activated e_{min} and e_{max} from the secondary parameters since information on those was available. h_{po} was calibrated to the

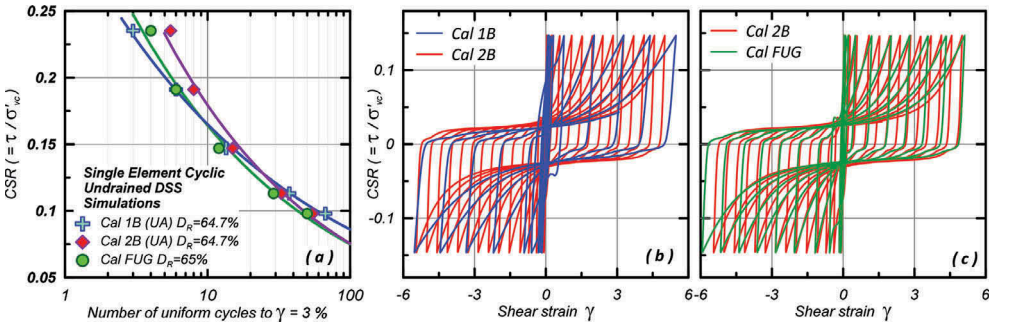

Figure 2. Comparison of three calibrations in single element undrained cyclic Direct Simple Shear (DSS) simulations for D_R=64.7% and 65% under 100 kPa in terms of Cyclic Stress Ratios versus Number of Cycles to γ=3% and stress-strain loops.

liquefaction triggering resistance and G_o was calibrated to the axial strain accumulation rate (SAR). SAR was defined in units of percent of strain per cycle and indicated the half double-amplitude strain accumulation per cycle after liquefaction triggering. For the PSC simulations, SAR increased consistently with larger CSRs. However, this trend was not consistent in the actual triaxial tests. Given this discrepancy, the goal of the calibration was to approximate the average rate of SAR for CSR values between 0.14 and 0.2, which was deemed more relevant for the planned centrifuge tests. The default relationship of G_o as a function of D_R (or N_{160}) was used with a multiplier of 0.80 for the relative density under consideration. This relationship for G_o falls between the two selected by the UA team, and leads to a SAR between those of Cal 1B and 2B (Figure 2). Table 1 lists all model parameters for PM4Sand that were given values other than their default ones. Default values for the remaining parameters are provided by Boulanger and Ziotopoulou (2017).

3 NUMERICAL SIMULATION PROTOCOLS

The two teams independently performed analyses in FLAC 8 (Itasca 2016) using prototype dimensions. For the current study, six analyses were performed yielding different combinations of calibration and numerical modeling-protocols. Both teams simulated the prototype dimensions (Figure 1) using a numerical grid with one uniform layer of sand, 80 zones wide and 16 zones high, yielding 81 gridpoints in the x- and 17 gridpoints in the y-direction. This geometry was used to meet LEAP specifications and to place gridpoints at the prescribed instrument locations. The soil density reported by the facility was used (D_R=65%), with the calibrations already performed for the rest of the constitutive model parameters as described in the previous section and reported in Table 1. Mechanical boundary conditions in the simulations replicated the boundary conditions imposed by the rigid container used in the centrifuge model test, without explicitly simulating the rigid box that surrounded the soil. All bottom nodes were fixed in the x and y directions and all side nodes were fixed in the x direction. The side walls of the rigid container were assumed to have no friction and thus the soil was able to freely slide vertically during all stages of the simulation. Flow of water was allowed across the top surface of the model and restricted across the container boundaries. The experiment was submerged, so pore pressures and saturation were fixed at the top nodes and a pressure was applied to simulate the weight of the fluid. The input motion (component NS1 from the RPI test with PGA = 0.15g) was baseline corrected and applied as a horizontal acceleration time history to the base and sides of the model, and as a vertical acceleration time history to the bottom of the model. The following two subsections discuss modeling choices made by the teams. More details can be found in the corresponding papers (Montgomery & Ziotopoulou 2019, Tsiaousi et al. 2019).

3.1 *UC Davis – Auburn system level analyses (UA 2D-Sim)*

UA simulations were performed using the reported centrifugal acceleration from the facility to establish the pre-shaking stress conditions. The sequence of centrifuge model construction was simulated to establish a reasonable initial stress state for the soil. The soil was sequentially placed in layers under a gravity of $1/N\ g$, where N is the centrifugal acceleration from the experiment, which simulated the stresses in the model before centrifugal spinning. During construction, the sand was assigned a Mohr-Coulomb material model with appropriate stress-dependent stiffness. A K_o value close to 0.45 was expected for normally consolidated sand, so the Poisson's ratio for the Mohr-Coulomb model was chosen to produce this value under one-dimensional conditions. The sand was placed layer-by-layer to avoid any stress arching at the container walls. Saturation was performed by setting the saturation value to 100% everywhere and allowing pore pressures to reach equilibrium. Next, gravity was increased from *1/N g* to *1 g* in 10 increments to simulate the spin-up of the centrifuge. Following each increase in gravity, the stress-dependent stiffness moduli were updated, and the coefficient of earth pressure at rest (K_o) was evaluated. The slight slope of the model produced K_o values that differed only slightly from 0.45. In addition, the values of the surface pressure from the water (both the external pressure and boundary pore pressure) were updated every 0.2 seconds during the simulation to account for any settlement of the soil surface. The elevation of the free water surface was assumed to be at 5 m based on the hydrostatic values from the pore pressure recordings. Water bulk modulus was set to 2 GPa. Rayleigh damping was set to 0.5% at a center frequency of 1 Hz (frequency of input motion) in order to reduce noise in the simulations and to account for small strain damping.

3.2 *Fugro system level analyses (FUG 2D-Sim)*

Fugro followed very similar modeling procedures to those of UA. Mechanical boundary conditions were the same with a triangular pressure applied at the soil surface simulating the load due to the water mass. This pressure was not updated during the dynamic loading. Mohr-Coulomb properties were assigned to the soil during gravity loading, although the spin-up procedure from *1/N g* to *1 g* was not modeled. Instead, a gravitational field with a vertical acceleration equal to $9.81 m/s^2$ was applied and once equilibrium was reached, PM4Sand was assigned to soil elements and equilibrium was reached again. This team set the water table at the elevation of the upper left corner of the grid (4.875m). This choice however is not expected to affect the results in terms of excess pore pressures, as these are computed as differences from the initial condition. Water bulk modulus was set to 0.5 GPa and Rayleigh damping was set to 0.5% at 3.3Hz (average frequency between input motion and overall system accounting for the rigid box as well).

4 RESULTS AND DISCUSSION

Simulation results were extracted in terms of accelerations and spectral accelerations, excess pore water pressures, and displacements. The RPI NS1 results available on the DesignSafe Data Depot (Kutter et al. 2018) are used for comparison, but it should be noted that simulations have not been adjusted to match experimental results (Type-B simulations).

Spectral accelerations obtained from numerical simulations and recorded during the centrifuge model test at instruments AH1, AH2, and AH4 (locations in Figure 1) are illustrated in Figure 3 for the two different numerical simulations and the three different calibrations. There is an overall good agreement both between the different calibrations and between the calibrations and the recordings. Discrepancies are observed at the high frequency range, but these could be attributed to noise in the recording. As expected, Cal 2B and FUG agree well with each other in both 2D simulations since they were similar in their approach to estimating G_o. It also can be seen that the FUG simulation accentuates the difference between Cal 1B and the other two a bit more compared to the UA simulation. Part of the reason for this may be that more of the model liquefied in Cal 1B for both the UA and FUG simulations, compared with the other two calibrations. This suggests that the effects of the system-level modeling choices may have a different degree of importance for liquefied soils and non-liquefied ones.

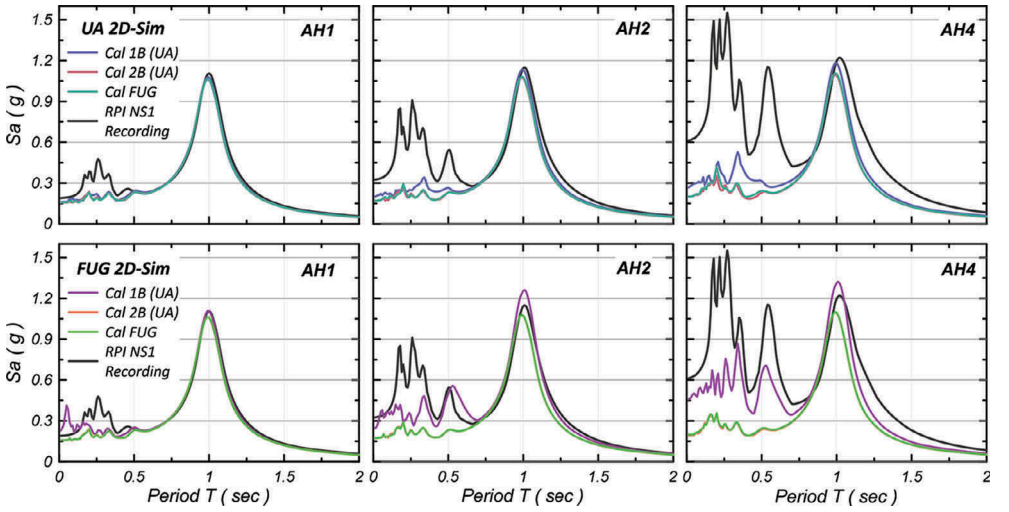

Figure 3. Comparison between simulation and experimental results for acceleration response spectra at accelerometers AH1, AH2, and AH4 using three calibrations (Cal 1B, 2B, and FUG) in two system level simulations (UA 2D-Sim and FUG 2D-Sim).

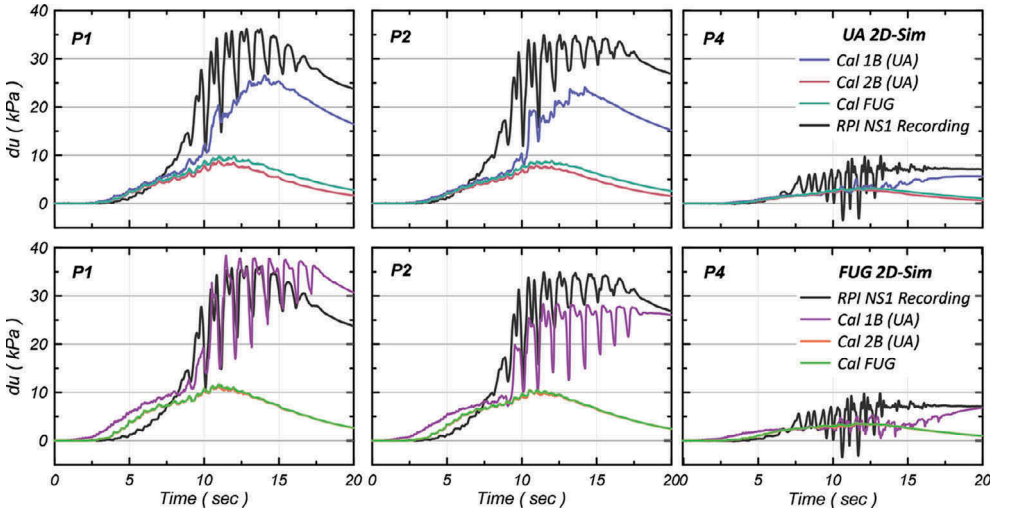

Figure 4. Comparison between simulation and experimental results for excess pore pressures at locations P1, P2, and P4 using three calibrations (Cal 1B, 2B, and FUG) in two system level simulations.

In addition, the choice of Rayleigh damping centered at different frequencies (1Hz for UA versus 3.3 Hz for FUG) needs also to be acknowledged since it is expected to be leading to different patterns of damping especially in the high frequency range.

Figure 4 illustrates excess pore pressure time histories obtained from numerical simulations and recorded during the centrifuge model test at instruments P1, P2, and P4 (locations in Figure 1) for the two different 2D simulations and the three different calibrations. Again, the differences between Cal 2B and FUG are minor, but Cal1B stands out more than it did before in terms of spectral accelerations (Figure 3). Cal 1B captured the observed excess pore pressure generation better than the other two calibrations. There is a chance that instruments in the tests were placed at slightly different locations than planned (Montgomery & Ziotopoulou 2019), which would lead

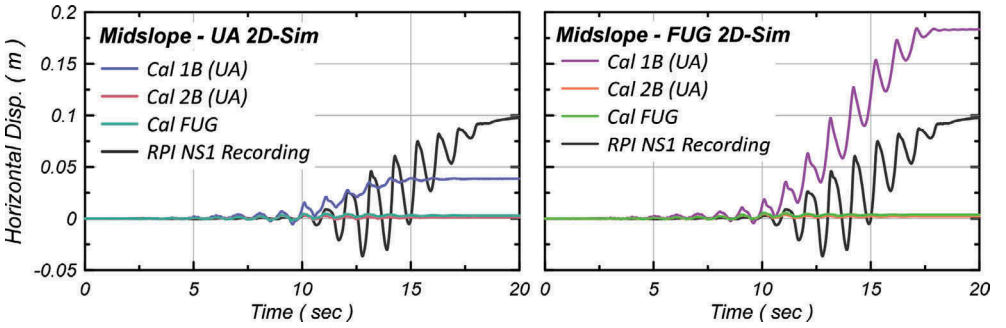

Figure 5. Comparison between simulation and experimental results for horizontal displacements at the midslope using the three calibrations (Cal 1B, 2B, and FUG) in two system level simulations.

to different maximum excess pore pressures. Similar to the spectral accelerations, there is an effect from the simulation choices on Cal 1B indicating that for a liquefied material, numerical simulation choices may be more influential compared to a non-liquefied material.

Figure 5 illustrates horizontal displacement time histories at the midslope obtained from numerical simulations and recorded during the centrifuge model test for the two different 2D simulations and the three different calibrations. Cal 1B does a reasonable job of representing the displacements, while Cal 2B and FUG predict very small displacement. This is consistent with the results seen in terms of pore pressures and can be attributed to the large excess pore pressures generated in the experiment and simulations using Cal 1B, which are not seen in the other two (Figure 4). These results emphasize that different calibrations captured different aspects of the experimental behavior more accurately, suggesting that in lieu of limited and/or uncertain experimental data, investing effort in more than one calibration can envelope the response and increase confidence in the results of system-level simulations.

Studying the results of numerical simulations across the different responses obtained (spectral accelerations, excess pore pressures, and displacements) leads to some interesting observations: the different calibrations and 2D simulations led to differences in all three response measures with discrepancies being more pronounced in excess pore pressures and displacements compared to response spectra. This suggests that spectral accelerations in this problem are less sensitive to the calibration and more dependent on the configuration of the problem, numerical choices (particularly Rayleigh damping and center frequency of its application), as well as the overall ability of the numerical platform to predict a dynamic response. Furthermore, comparing Cal 2B and Cal FUG across all of the results demonstrates that the activation of secondary parameters is likely less influential than the primary parameters. This will require further sensitivity analyses.

It is also instructive to compare the magnitude of uncertainty between the numerical results to those observed experimentally. Kutter et al. (2019) presented a comparison of displacements between numerical sensitivity analyses and experimental results from submerged slopes of varying densities. The experimental results at a D_R of approximately 65% gave displacements at mid-slope that ranged from 0.11m to -0.023m. The numerical results generally fall within this range with the exception of the combination of Cal 1B and the FUG simulation, which gave the highest displacement of any of the simulations. The difference between Calibration 1B and 2B is the selection of G_o which could have been constrained if shear wave velocity data was available.

5 SUMMARY AND CONCLUSIONS

Recent research efforts have focused on the sensitivity and uncertainty of experimental and numerical modeling results across different researchers, tools, and facilities. Particularly for NDAs, uncertainties are commonly examined using both different platforms and different

constitutive models making it challenging to separate individual contributions to the overall uncertainty. As more advanced tools become available, confidence in NDA results depends both on the ability of selected tools to capture key features of the problem at hand, but also on the analyst's expertise and choices made in lieu of experimental data to calibrate against. Thus, it is important to examine results' sensitivity and robustness to diverging calibration and numerical modeling protocols.

This paper investigated the sensitivity of responses to the calibration and numerical modeling protocols followed by two independent and equally experienced teams working with the same set of tools: the constitutive model PM4Sand and the numerical platform FLAC. PM4Sand was calibrated to results from cyclic triaxial tests on Ottawa F-65 sand. The two teams followed generally similar calibration protocols where sufficient data was available to constrain the selection of input parameters. Choices deviated when available data was not sufficient to determine an input parameter (shear modulus coefficient G_o herein) and the analysts had to exercise their own judgement on how to overcome this limitation. It was shown that all calibrations fell within a narrow band (Figure 2a) in terms of liquefaction triggering, guided by the CSRs indicated by the experimental data, but deviated in terms of cyclic mobility behavior (Figures 2b-c). By comparing Cal 2B and Cal FUG, it was also shown that the activation of secondary parameters of PM4Sand was not as influential as the primary parameters (especially G_o) on the obtained response. This demonstrates the value of obtaining data to at least constrain the primary input parameters of the model.

System-level numerical modeling protocols followed by the two teams were generally consistent to each other, closely honoring the geometry and boundary conditions of the specified problem. The overall conclusion herein is that the combination of an experienced analyst investing a reasonable amount of engineering effort and utilizing well-calibrated and validated advanced tools can lead to a good agreement between numerical and experimental data. In the system-level analyses of the two teams choices deviated in the modeling of the spin up process in the centrifuge, the location of the water table (which was not provided), the bulk modulus of water, the center frequency for Rayleigh damping, and accounting for the slightly changed water pressure due to the deformation of the slope. Other than those, the resulting simulations were almost identical to each other. It was shown that the simulation choices played a smaller role in the obtained responses compared to the calibration, with its effect being amplified for softer or liquefied material. This suggests that system-level modeling choices may have different degrees of influence for liquefied and non-liquefied soils and their effect will likely be different for different problem configurations and boundary conditions (e.g., thicker deposits undergoing larger deformations, systems with structural elements and interfaces, or systems with more complicated geometries where the discretization may vary). In terms of dynamic responses, all cases resulted in good agreement of spectral accelerations with the recordings indicating that the response in terms of accelerations is more robust to the calibration compared to excess pore pressures and displacements. It was however shown that numerical choices in the system level simulation can have an effect that is accentuated in spectral accelerations (e.g., Rayleigh damping). The excess pore pressure and displacement time histories demonstrated a higher sensitivity to the calibration and a variable degree of agreement to experimental results, suggesting that enveloping the response via different calibrations is a reasonable approach in lieu of extensive experimental data.

Last but not least, the utility of an NDA study depends on the clear and thorough documentation, since it provides the basis for effective internal and external review processes (Boulanger & Ziotopoulou 2018) and by extension can serve as a means for promoting consistent and/or best practices. As a result, review processes can enable multiple parties to contribute to finding errors and/or extracting insights and benefits. This paper demonstrated that even when using the same set of tools, the ability to resolve or understand any systematic or unusual difference in results depends on the documentation of the practices by the two teams.

This work focused on only one specific and relatively simple problem. Future investigations could examine the effects of input motion characteristics, different geometries and boundary conditions, or inclusion of structural elements, such that more parameters will affect the problem and the associated numerical choices. It is generally expected that the variation of

numerical choices is proportional to the complexity of the problem at hand, but with adequate documentation and established protocols, the uncertainty in results can be reduced.

ACKNOWLEDGEMENTS

The authors are grateful to the organizers of LEAP and all of the participating experimental facilities for providing the data used to create this study.

REFERENCES

Arulanandan K., & Scott R.F. 1993. Verification of numerical procedures for the analysis of soil liquefaction problems, Vol. 1 and 2. Rotterdam: Balkema.

Boulanger, R. W., & Ziotopoulou, K. 2017. PM4Sand (Version 3.1): A sand plasticity model for earthquake engineering applications. Technical Report No. UCD/CGM-17/01, *Center for Geotechnical Modeling, Dept of Civil and Environ Eng*, University of California, Davis, CA.

Boulanger, R. W. & Ziotopoulou, K. 2018. On NDA practices for evaluating liquefaction effects. *Proc. Geotechnical Earthquake Eng & Soil Dynamics V (GEESDV 2018)*, June 10 – 13, 2018, Austin, TX.

Carey T.J., Stone N., & Kutter B.L. 2019. Grain Size Analysis and Maximum and Minimum Dry Density of Ottawa F-65 Sand for LEAP-UCD-2017. In *LEAP-UCD-2017: Reproducibility & sensitivity of centrifuge tests & comparisons to simulations of lateral spreading*.

El Ghoraiby, M.A., Park, H., & Manzari, M.T. 2018. LEAP-2017 GWU Laboratory Tests. DesignSafe-CI, Dataset, doi:10.17603/DS2210X.

Itasca. 2016. FLAC – Version 8, Itasca Consulting Group, Inc., Minneapolis, Minnesota; 2011.

Kutter, B.L., Zeghal, M., & Manzari, M.T. 2018. LEAP-UCD-2017 Experiments (Liquefaction Experiments and Analysis Projects), DesignSafe-CI, Dataset, doi:10.17603/DS2N10S

Kutter, B.L., Manzari, M.T., Zeghal, M., Arduino, P., Barrero, A.R., Carey, T.J., Chen, L., Elgamal, A., Ghofrani, A., Montgomery, J., Ozutsumi, O., Qiu, Z., Taiebat, M., Tobita, T., Travasarou, T., Tsiaousi, D., Ueda, K., Ugalde, J., Wada, T., Yang, M., Zheng, B.-L., & Ziotopoulou, K. 2019. Numerical sensitivity study compared to trend of experiments for LEAP-UCD-2017. *LEAP-UCD-2017: Reproducibility & sensitivity of centrifuge tests & comparisons to simulations of lateral spreading*.

Lambe, T.W. 1973. Predictions in Soil Engineering. *Géotechnique*, Vol. 23, No. 2, pp. 149–202.

Montgomery, J., & Ziotopoulou, K. 2019. Numerical simulations of selected LEAP centrifuge experiments with PM4Sand in FLAC. In *LEAP-UCD-2017: Reproducibility & sensitivity of centrifuge tests & comparisons to simulations of lateral spreading*.

Stewart, J.P., Kwok, A.O., Hashash, Y.M.A., Matasovic, N., Pyke, R., Wang, Z., & Yang, Z. (2008). "Benchmarking of nonlinear geotechnical ground response analysis procedures," *Report PEER 2008/04*, Pacific Earthquake Engineering Research Center, University of California, Berkeley.

Tsiaousi, D., Ugalde, J., & Travasarou, T. 2019. LEAP-UCD-2017 Simulation Team Fugro. In *LEAP-UCD-2017: Reproducibility & sensitivity of centrifuge tests & comparisons to simulations of lateral spreading*.