Partitioning the displacement components of seismic demand due to soil-structure interaction

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ABSTRACT
In the present study, seismic demand of compliant soil-foundation-structure systems (SFSS) is investigated through numerical time history analyses of the elastic SFSS. Due to kinematic and inertial interaction effects, structure's demand may be substantially different from the traditionally calculated from the fixed-base structure on free-field approach. In the first part of this study, we propose appropriate modification factors of the acceleration demand to account for soil-structure interaction (SSI). Based on the proposed modification factors, the herein presented methodology is helpful in engineering practice for calculating the acceleration demand accounting for both inertial and kinematic interaction effects. The modification factors clearly demonstrate the beneficial SSI effects on the structure, as the maximum average acceleration at the top of the actual SFSS can be of about 55% - 82% of the acceleration response in case we consider the free-field demand. At the second part, we present the components of the displacement demand when considering SSI. Some of the main conclusions are the following: (i) horizontal displacement demand of the structure is more important for squatty structures, (ii) the effect of structural mass is more important for slender structures, (iii) when the structure to soil stiffness ratio $1/\sigma$ is greater than 0.2-0.3 the structure's mass doesn't play an important role on the horizontal movement of foundation (iv) as the aspect ratio of the structure increases, the horizontal foundation movement decreases, whereas the foundation rocking increases and (v) as soil becomes very soft the structure tends to follow the earthquake induced movements like a rigid block. Both acceleration modification factors and displacements are presented in terms of dimensionless parameters such as structure-to-soil stiffness ratio $1/\sigma$ and aspect ratio h/B (where h is the structure's height and B the half width of foundation), and show remarkable trends.

1 INTRODUCTION

Soil – structure - interaction (SSI) affects the seismic response of a structure, even if this structure is founded on shallow foundation. SSI effects can be categorized into inertial and kinematic interaction effects. Inertial interaction effect are attributed to inertia of the mass of both structure and foundation systems during the earthquake shaking. Inertial interaction effects raise base shear and moment and cause additional displacements comparing with the free field. Many studies have proved that inertial interaction effects are concentrated in a narrow frequency range around the first-mode frequency for translational motions (Kim and Stewart 2003). Additionally, inertial interaction effects increase the structural period and the associating damping. Kinematic interaction effect is a result of the embedment and wave averaging effect of a massless rigid body resting upon a soil profile and subjected to seismic waves. The kinematic effect is significant in a higher range of frequency of the motion (Kim and Stewart 2003).

Concerning the SSI effects in seismic design practice, the structure is conventionally assumed fixed at its base and thus it is subjected to the free-field ground motion. Until recently, most codes and studies were silent on the issue of SSI and some addressed the issue but mostly limited to inertial effects that result into change in fundamental period and damping of the system. Afterwards, various studies (Mylonakis and Gazetas 2000, Aviles and Perez-Rocha 2003) highlighted the effects of both inertial and kinematic interaction on dynamic structural response. Most of the currently used approaches (for example NIST 2012, FEMA- 440 2005, FEMA-356 2000) treat the SSI phenomenon using the sub-structure and not the direct method, which is utilized in the present study, mainly due to the high computational cost of the direct analysis. They include the inertial or kinematic interaction separately in the proposed methodologies and thus are unable to consider the combined effect. NIST (2012) and FEMA-440 (2005) are two of the few codes/ guidelines that consider the SSI effect in the design and assessment methodologies they propose. The basis of treatment of the kinematic effect in FEMA-440 (2005) is based on Kim and Stewart (2003) study, which calibrated the model proposed by Veletsos et al. (1974) based on recordings of instrumented buildings.

Pandey et al. (2012) studied the modification of free-field motions by SSI for shallow foundations and Fraino et al. (2012) studied the SSI effects on instrumented bridges. In the first study, the researchers examine the phenomenon in two instrumented buildings, while the second study concerns ten instrumented bridges. In both cases, the SSI effects are studied comparing the acceleration response spectrum at the foundation level with the one at free field conditions.

To conclude, the combined effect of SSI has until now been studied mainly through the analyses of actual records obtained from instrumented buildings or bridges, but these recordings are not sufficient for proposing a full-methodological path. The numerical results of the present study try to fill this research gap.

In the present study, we examine the influence of the SSI effects on the structure's demand through the results of a thorough parametric numerical study. More specifically, appropriate modification factors for the direct evaluation of acceleration demand considering the SSI effects are proposed. Displacement modification due to SSI is also quantified. The aim of this paper is to elucidate the effects of SSI on coupled system's response for the simplest case of linear systems. For linear elastic systems, soil-foundation compliance directly affects seismic demand, even for surface or almost surface foundations. Moreover, as it will be further explained in the ensuing paragraphs, for flexible-base linear elastic structures only one pair of demand spectral coordinates exists, a single point for any soil-foundation-structure system. Spectral values are evaluated herein directly by a finite-element model (FEM) for different configurations, covering a wide range of typical cases for engineering practice.

2 CONFIGURATION AND NUMERICAL MODELLING

In order to investigate the seismic demand of compliant SSI systems, we conducted 2D linear elastic time history analyses of a coupled SSI system using the finite element method. The same configuration was also used in Karatzetzou and Pitilakis (2017) where the effect of SSI on dynamic foundation response, was examined. When using finite elements, to ensure that the numerical model is compatible with the actual problem, some special considerations must be done. In the following paragraphs these considerations are analyzed and discussed.

The examined superstructure is a single-degree-of-freedom structure (SDOF), the degree of freedom being the translational displacement of the structural mass, m_s. The SDOF structure is characterized by its stiffness k_s, its damping c_s and its height h. The structure is founded on a massless rigid surface foundation of width equal to 2B resting on the soil surface. Both structure and the foundation are modelled with the Opensees software (McKenna et al. 2000) by elastic beam column elements of 1m length. Full connection is assumed between the foundation and the soil nodes. The entire superstructure's mass is lumped at the top of the superstructure without any contribution from the assumed massless pier.

The idealized SDOF can be interpreted (i) as an equivalent representation of the fundamental mode of vibration of a multi-storey structure or (ii) as a bridge system.

Two-dimensional plane strain analyses are performed in time domain. Using the four-node plane strain formulation with appropriate bilinear isoparametric quadrilateral elements of two degrees-of-freedom at each node, we modeled the soil profile. The soil remains in the linear elastic range and the use of absorbing elements (Zienkiewicz et al. 1988) provides a relatively simple and practical way, to model the unbounded soil domain. We considered a homogeneous soil profile subjected to an incoming seismic vertically propagating SH wave. By modeling the problem using a 2D dimensional approach we are implicitly assuming a periodicity of 1m following the direction normal to the plane of the model. Elastic bedrock is modelled using Lysmer-Kuhlemeyer (1969) dashpots at the base of the soil, used to account for the finite rigidity of the underlying half- space. Concerning the vertical boundaries, we use the tied lateral boundary approach by declaring equal degree of freedom for every pair of nodes that share the same y-coordinate as described in Zienkiewicz et al. (1988). The soil profile is excited at the base by a horizontal force time history, which is proportional to the known velocity time history of the ground motion (Joyner and Chen 1975, Lysmer 1978).

The size of the soil mesh was selected based on the results of a sensitivity analysis, to assure free field and "quasi transparent" conditions at the boundaries. In this way, the geometry of the soil mesh has a total width of 200m with a depth of 50m. The geometry of the mesh is based on the concept of resolving the propagation of the shear waves at or below a pre-defined frequency allowing an adequate number of elements to fit within the wavelength of the chosen shear wave. This ensures that the mesh is refined enough to capture propagating waves. Following the principle that the element size must be 8 to 10 times smaller than the minimum expected wavelength, the minimum required element size is calculated as λ_{min} = Vs/(10fmax) =Vs/100m. The adopted element size has been set equal to 1m (as the smaller value of the utilized shear wave velocity Vs in the analyses is equal to 100m/s). Thus, for a maximum frequency of 10Hz, the quadratic elements of 1m×1m, suggest a dense discretization. The soil mesh comprises of 10000 four-node quadrilateral elements. The width of the finite element soil mesh (200m) was sufficiently large, to avoid spurious reflections at the boundaries.

3 PARAMETRIC ANALYSES

Two distinct geometries of structures are chosen to demonstrate the effects of soil-foundation-structure interaction on seismic acceleration and displacement demand and propose appropriate modification factors of the free- field demand to evaluate SSI demand.

Both two structures could represent bridge piers or equivalent multi-storey buildings. Material and geometrical properties of the systems vary based on the relative structure-to-soil stiffness ratio $h/T_{fix}V_s$ and the aspect ratio h/B, where h=distance from the base to the centroid of the inertia force, T_{fix} is the resonant period of the fixed-base structure, Vs is the shear wave velocity of soil, and B is the half-width of foundation.

The modulus of elasticity is E=32GPa for the structure, corresponding to concrete type C30/37, while the cross-section diameter d of the circular pier ranges from 0.6m to 3.0m. The structure's height is 3m and 6m, to cover typical constructions that could be founded on shallow footings. For both 3m-and 6m-tall structures the footing width is 6m wide. For the structure, the Rayleigh damping is equal to 5% at frequencies f_{SSI} and f_{fix}. Relative structure-to-soil stiffness varies between 0.01 and 0.98 and aspect ratio h/B varies between 1 and 2.

The mass of the superstructure m_s is 100Mg, 200Mg, 400Mg and 800Mg. For ordinary structures, normalized mass (m_n=m/ρr^{2}h) varies between 0.4 and 0.6 according to Stewart JP et al (2003), which, for the examined foundation geometries and soil properties, means that the total mass for ordinary buildings range between 20Mg and 300Mg. In this range, we considered two masses, 100Mg and 200Mg. For the bridges, the unit weight of the deck is taken equal to 200kN/m, which is a realistic value according to Ciampoli and Pinto (1995). The spans of the selected bridges are equal to 20m and 40m and thus the resulted masses 400Mg and 800Mg, respectively. These masses equal to the concentrated mass of the two adjacent half spans of the bridge deck considering a massless pier. In this study, the 2D mass and stiffness of the pier are considered equal to the actual 3D values.

The soil profile is simplified to a homogeneous half-space, with mass density ρ=2Mg/m^3 and Poisson's ratio v=1/3. The elastic bedrock ($V_{s,bedrock}$ = 1500m/s) lies at 50m beneath the ground surface. For the shear wave velocity Vs of the homogeneous soil profile we assumed values of 400m/s, 300m/s, 200m/s and 100m/s, classifying the soil profile in soil class B, C and D according to Eurocode 8 (CEN 2004). For the soil, we used Rayleigh damping with two target frequencies, equal to 4% for the first and the third mode of the soil profile (Kwok et al. 2007). We kept soil material damping constant in all analyses for meaningful comparison of peak acceleration demand between different systems.

The fundamental period of the soil profiles varies from 0.5s to 2.0s, whereas the resonant period T_{SSI} of the flexible-base system is in the range of 0.12s to 2.7s, spreading over a wide range of civil engineering applications. T_{SSI} is calculated directly from the numerical analyses, from the ratio of the Fourier spectra of the response at the top of the structure and the free-field. Table 1 shows the details on the system properties.

From all possible configurations, we did not retain the ones that gave very low non-realistic values for fixed-base period T_{fix} (lower than 0.1s), as well as the combinations where the safety factor for bearing capacity under earthquake loading, according to Eurocode 8 (CEN 2004), was lower than one. The soil strength values considered for the bearing capacity evaluation stem from Eurocode 8 – Part 1 (CEN 2004) and depend on the soil classification scheme determined by shear wave velocity.

4 EARTHQUAKE MOTIONS

The soil-foundation-structure system is subjected to ten earthquakes records covering significant range in magnitudes (Mw=5.6 to 7.6), epicentral distances (R=3.4 to 43km) and predominant periods (T_p=0.10 to 0.50s).

All selected earthquakes were recorded on rock or very stiff sites with $V_{s,30}$ larger than 600m/s. No scaling was applied, whereas a second order Butterworth band-pass filter with corner frequencies 0.25Hz and 25Hz was used. The selected earthquake records were chosen to have relatively low peak accelerations values varying form 1.03m/s^2 to 4.14m/s^2. Figure 1 shows the time series of the utilized earthquake records. All combinations of the input parameters sum up to more than 2500 free-field and coupled soil-foundation-structure interaction analyses.

Table 1.Details on the utilized system properties.

V_s (m/s)	h(m)	2B (m)	m_s (Mg)	T_{fix} (s)	h/B
100/200/ 300/400	3	6	100/200/ 400/800	0.10- 1.18	1
100/200/ 300/400	6	6	100/200/ 400/800	0.10- 1.88	2

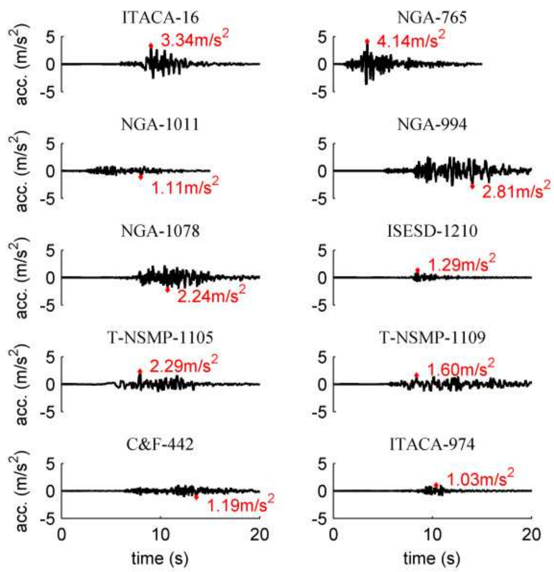


Figure 1. Time series of the utilized earthquake records

5 MODIFICATION FACTORS TO ESTIMATE ACCELERATION DEMAND

The pseudo-spectral acceleration when considering kinematic and inertial interaction effects is traditionally obtained (NIST 2012) by i) evaluating the foundation level spectrum modified due to kinematic interaction effects and ii) entering the spectrum drawn of the previous step for effective damping ratio at the corresponding elongated period T_{SSI} (inertial interaction effects). In the present study, these two interaction phenomena (inertial and kinematic) are treated in a single step. Firstly, we define the spectral SSI acceleration demand üstr (maximum acceleration at the top of the structure- foundation system founded on soil and subjected to earthquake loading) directly from the time history analyses. Secondly, we plot üstr at the same graph with the free- field acceleration demand üstr f (maximum acceleration at the top of a equivalent single-degree-of-freedom structure fixed at its base, with period equal to the period of the flexibly-supported structure, T_{SSI}, subjected to the free-field motion). The proposed dimensionless modification factor is the üstr / üstr f ratio.

The proposed modification factors can be used in engineering practice for the estimation of seismic acceleration demand considering SSI effects in a single step. The user needs only to calculate the T_{SSI} period and the free- field acceleration of the system under investigation. T_{SSI} can be calculated from the proposed in literature closed form solutions (Veletsos and Meek 1974) and the free- field motion (FFM) from a simple one dimensional analysis of the soil profile. Knowing the FFM, the free-field spectral acceleration for the period T_{SSI} can be easily evaluated. Multiplying this free- field spectral acceleration üstr f with the proposed modification factor, the actual SSI acceleration demand üstr is calculated.

Figure 2 presents the radial lines that show the üstr to üstr f ratio for the geometry of the SDOF system being (a) h=3m and 2B=6m (h/B=1) and (b) h=6m and 2B=6m (h/B=2). The black continuous line is the 1:1 radial line and in grey is the range between the average line plus and minus the standard deviation (0.027-0.048).

Figure 2 shows that for h/B =1 (SDOF system of 3m height and 6m foundation width) and $1/\sigma$ < 0.1, the modification factor is 0.76. Similarly, for soil-to-structure stiffness ratio $1/\sigma$ > 0.1, the proposed modification factor result equal to 0.55 for the slenderness ratio h/B=1. When the slenderness ratio is equal to two, the proposed modification factor for $1/\sigma$ < 0.1 is equal to 0.82. For the softer soil profiles of $1/\sigma$ > 0.1, the modification factor is 0.66 for the 6m high column. These results are of great interest, as it seems that the maximum acceleration at the top of the actual SFSS, can be a percentage of about 55% - 82% of the response in case we consider the ‘‘free field demand’’.

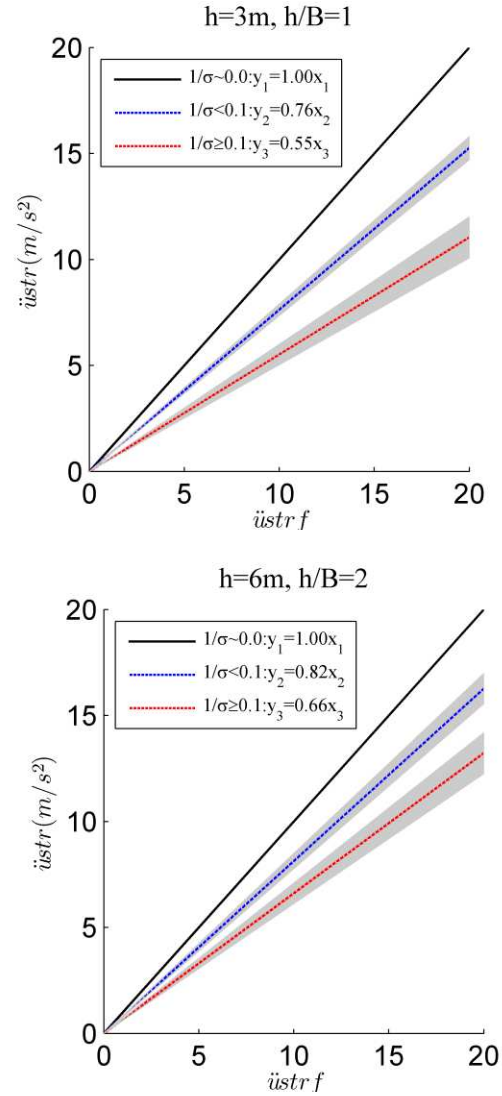
For all examined cases when the soil-to-structure stiffness ratio $1/\sigma$ is greater than or equal to 0.1, shifting of the resulting modification factor from unity is more intense. Irrespectively the SDOF system’s geometry, the modification factor is more or less similar for same slenderness ratios.

The modification factors concern surface foundations and thus they include mainly the inertial interaction but the proposed methodology could be extended also to embedded foundations. When the modification factor is not equal to unity, the structure’s acceleration demand differs from the one at free field conditions. The modification factors are important as they capture the whole interaction phenomenon, namely inertial and kinematic interaction effects.

6 DISPLACEMENT DEMAND

In this section the resulting displacements of the time history visco-elastic parametric analyses will be presented. The displacements are presented in terms of soil- to- structure stiffness ratio $1/\sigma$ and normalized structure to the mobilized mass m_{norm} $(m/B^3/\rho)$,

The total horizontal displacement at the top of the structure is composed by the displacement due to foundation’s horizontal swaying u_f, the horizontal displacement due to foundation’s rocking u_θ and the horizontal displacement at the top due to structural bending u_{sb} (Figure 3).



(a)

(b)

Figure 2. Radial lines that show the üstr to üstr f ratio for the geometry of the SDOF system being (a) h=3m and 2B=6 and (b) h=6m and 2B=6m.The black continuous line is the 1:1 radial line and in grey is the range between the average line plus and minus the standard deviation.

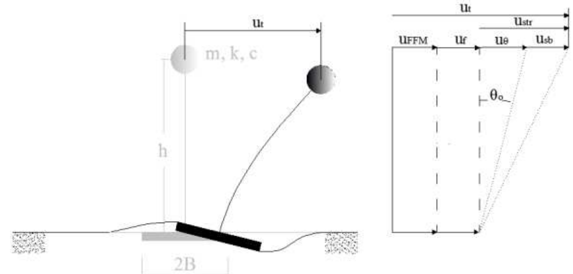


Figure 3. Response at the top of the structure when considering soil-foundation-structure interaction effects in terms of displacements.

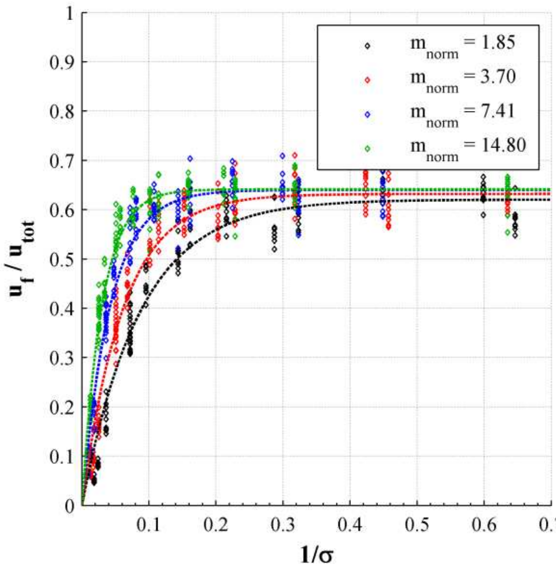
Figure 4 illustrates the displacement ratios of the displacement at the top of the structure due to horizontal foundation's swaying as a fraction of the total displacement u_f/ u_{tot} for (a) h=3m and 2B=6m and (b) h=6m and 2B=6m. The displacement ratios are shown in terms of relative soil to structure stiffness ratio 1/σ and normalized structure to the mobilized m_{norm} ratios. Total displacement u_{tot} is the sum of u_f, u_θ and u_{sb}. Similarly, Figure 5 and Figure 6 show, for the same structure's geometries, the displacement at the top of the structure due to foundation's rocking divided to the total displacement u_θ/ u_{tot} ratio and the displacement at the top of the structure due to structure's slenderness divided to the total displacement u_{sb}/ u_{tot} ratio.

Each point on each figure is one finite element analysis of the coupled soil- foundation- structure system. The displacement ratios are categorized according to the normalized structure to the mobilized mass ratio m_{norm}. The dashed lines resulted using the Matlab nonlinear curve fitting tool (MathWorks2009). It is important to note that even if the values of the following figures stem from the time history analyses, there is a clear trend with relatively low dispersion.

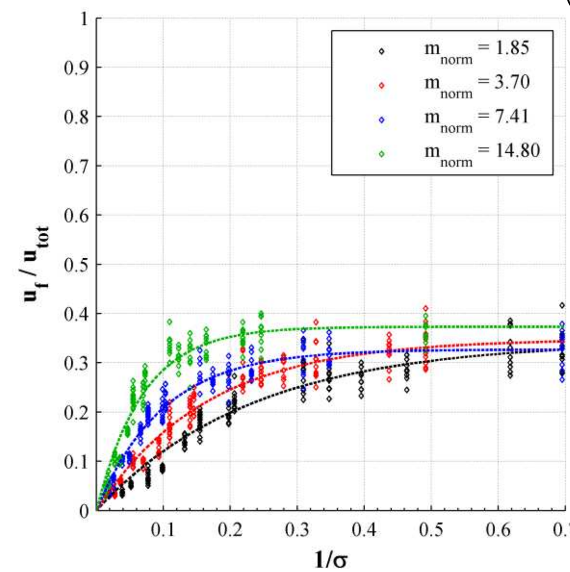
Displacement due to horizontal foundation's movement over total displacement (u_f/u_{tot}) for aspect ratio equal to unity (h/B=1) takes values up to 0.6, whereas for the more slender structures (h/B=2) the horizontal movement ratio takes values up to 0.4 (Figure 4). This in other words shows that horizontal displacement due to horizontal movement of the foundation is more important for less slender structures. Irrespectively the structure's slenderness, for the same value of relative soil to structure's stiffness ratio 1/σ, as the mass increases, u_f/u_{tot} ratio increases too. Finally, when 1/σ is larger than 0.2-0.3, the structure's mass does not play an important role on the horizontal movement of foundation value.

On the other hand, the u_θ/u_{tot} displacement ratios take values up to 0.4 for the less slender and 0.6 for the more slender structures (Figure 5). Thus, as the aspect ratio of a structure increases, the horizontal foundation movement decreases, whereas the foundation rocking increases. The structure's mass affects more the u_θ/u_{tot} ratio of the more slender structures. Finally, as 1/σ increases the effect of structure's mass on u_θ/u_{tot} ratio decreases.

As far as the u_{sb}/u_{tot} ratio is concerned, it seems from Figure 6 that as 1/σ ratio increases, contribution of bending to the total displacement approaches zero. This, in other words means, that as soil becomes very soft (1/σ>0.4 for h/B=1 and 1/σ>0.7 for h/B=2), the structure tends to follow the earthquake induced movements like a rigid block and with almost no structural bending at all. The structure's mass affects the displacement due to structural bending and as the mass increases, the structure's bending increases also. The effect of the mass on the u_{sb} component is more important for the slenderer structures.



(a)



(b)

Figure 4. Displacement at the top of the structure due to foundation's horizontal movement to total displacement u_f/ u_{tot} ratio for the geometry of the SDOF system being (a) h=3m and 2B=6and (b) h=6m and 2B=6 in terms of 1/σ and m_{norm} ratios, where total displacement u_{tot} is the sum of u_f, u_θ and u_{sb}.

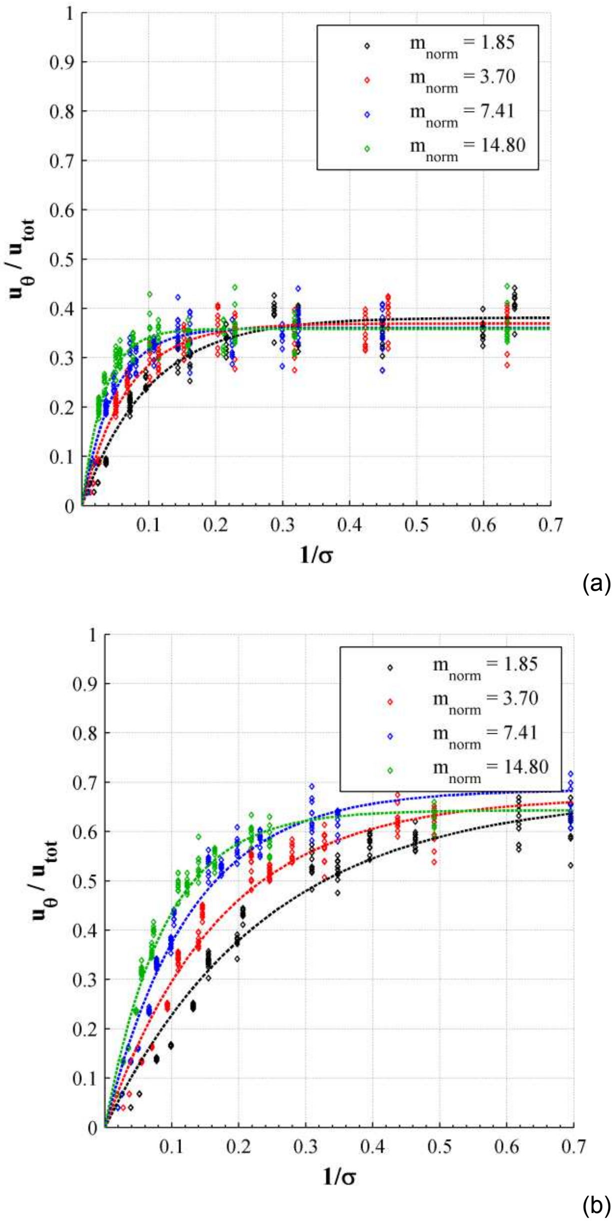


Figure 5. Displacement at the top of the structure due to foundation's rocking to total displacement u_θ/ u_{tot} ratio for the geometry of the SDOF system being (a) h=3m and 2B=6 and (b) h=6m and 2B=6 in terms of 1/σ and m_{norm} ratios, where total displacement u_{tot} is the sum of u_f, u_θ and u_{sb}.

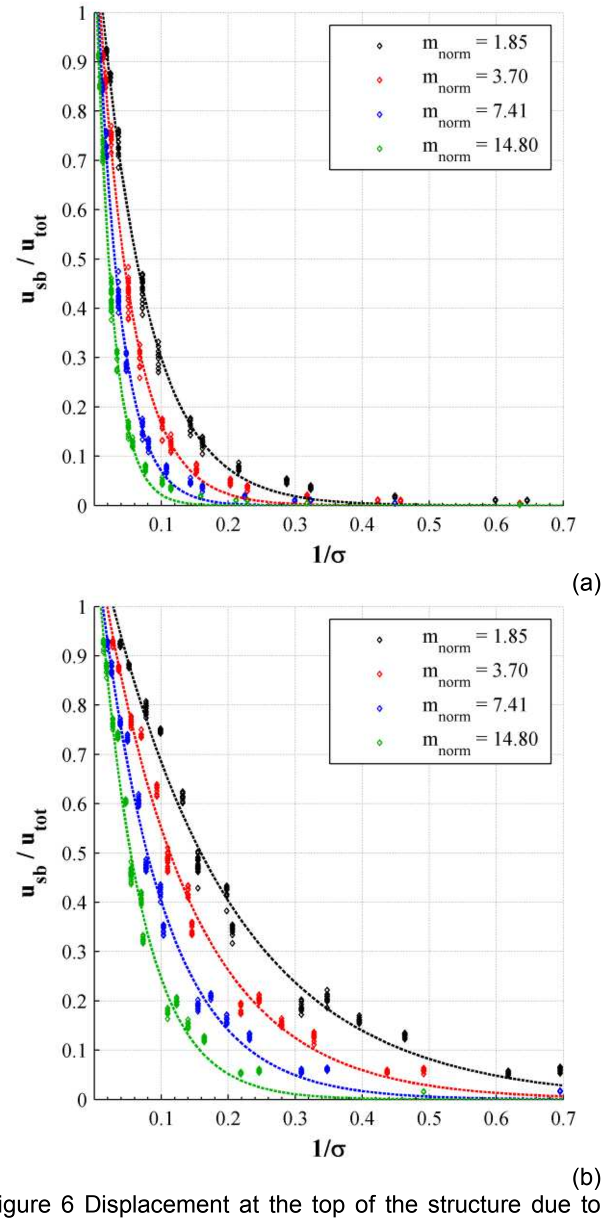


Figure 6 Displacement at the top of the structure due to structure's slenderness to total displacement u_{sb}/ u_{tot} ratio for the geometry of the SDOF system being (a) h=3m and 2B=6 and (b) h=6m and 2B=6 in terms of 1/σ and m_{norm} ratios, where total displacement u_{tot} is the sum of u_f, u_θ and u_{sb}.

7 CONCLUSIONS

The main conclusions the stem from the investigation of the SSI demand for RC structures could be summarized in the following paragraphs.

At the first part, we propose modification factors to evaluate the actual acceleration demand including in a single step both inertial and kinematic interaction effects. By multiplying these modification factors with the free-field demand, engineers in practice can easily evaluate seismic demand including SSI, without having to perform any numerical analysis. When the structure-to-soil stiffness ratio 1/σ is greater than or equal to 0.1, notably for soft soil profiles, modification factor is lower than 1, which means that interaction effects are pronounced. Irrespectively of the SDOF system's geometry, modification factors are similar for similar aspect ratios. The proposed modification factors show that maximum acceleration at the top of the actual SFSS, and thus the acceleration demand, can be a percentage of about 55% -

82% of the demand in case we consider the typical procedure of evaluation demand of an equivalent fixed-base SDOF with T_{ssi} subjected to a free-field motion. Earthquake record characteristics have a minor effect on the acceleration modification factor and the scatter is not important.

At the second part of this study we present the structural displacements when considering soil- structure interaction effects. As relative structure-to-soil stiffness ratio increases, displacement components at the top of the structure due to foundation rocking and swaying increase, whereas displacement due to structural bending decreases. Horizontal displacement due to horizontal swaying of the foundation is more important for lower aspect ratios. For very soft soils, the structure's mass does not play an important role on the horizontal movement of foundation. As the aspect ratio of the structure increases, the horizontal foundation movement decreases, whereas foundation rocking increases. As soil becomes very soft ($1/\sigma>0.4$ for $h/B=1$ and $1/\sigma>0.7$ for $h/B=2$), the structure tends to follow the earthquake induced movements like a rigid block and with almost no structural bending at all.

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