Identification of Dynamic Characteristics of a Rocking Foundation Model on Shaking Table Testing

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ABSTRACT
In recent years, rocking foundation mechanism has been suggested to isolate seismic waves for reducing the seismic demand on superstructure in seismic design. To understand the influence of this mechanism, shaking table testing of a rocking-dominant column-footing model has been performed at the National Center for Research on Earthquake Engineering of Taiwan (NCREE). The column-footing model was a column of height 80 cm with a footing of 40cm×40cm. The model was founded on a dense sand layer with a thickness of 1.2m. Rocking may reduce the contact area between the soil and the footing and further changes the footing rotational stiffness. To investigate the influence of this change to dynamic properties of the rocking foundation system (system frequency and equivalent damping ratio) during rocking, this study attempts to apply system identification methods to analyze the acceleration data of the model measured in the shaking table testing. The methods used include Auto regressive moving average (ARMA) model and Short time transfer function (STTF) method. These methods are originally applied to linear time invariant systems. Since the rocking system may be a time varying system, we extend these models by processing the data segmentally. The results show that the ARMA model is effective to identify the system predominant frequency and damping ratio and trace their change during shaking. The dynamic parameters obtained also agreed well with those identified from the STTF method.

Keywords: Rocking, Shaking table testing, Shallow foundations, System identification.

1 INTRODUCTION

In conventional structural analysis, the base of columns in a structural model is commonly assumed to be fixed based on the assumption of the rigid base (AASHTO, 2009). For a structure with footings, the foundation is susceptible to rocking under intensive seismic loading. The assumption of the fixed-base condition no longer holds. The seismic response of structure will be governed by rocking strength of the footings so that originally expected hinging mechanism in the structure may not occur. Many studies have been conducted to investigate the influence of foundation rocking on the response of structures under seismic loading (Gazetas. 1983; Gajan et al., 2005; Shirato et al., 2008; Paolucci et al., 2008; Gajan and Kutter, 2009; Anastasopoulos et al., 2012; Gazetas et al., 2013; Anastasopoulos and Kontoroupi, 2014; Antonellis et al., 2015). From these studies, one of the major characteristics of rocking foundations is the uplift behavior of the footing, which will lead to a reduction in the contact area between the soil and footing. Therefore, the rotational response of the footing could exhibit nonlinear response even though the soil behaves less nonlinearly in stiff soil. As a result, the dynamic properties of foundation systems, including the predominant frequency and equivalent damping, may change during the excitation.

For better understanding of the dynamic response of a rocking governed footing, Chiou et al. (2015) conducted a series of shaking table tests on a rocking-governed column-footing model. In order to further investigate the change in dynamic characteristics of the system, this study applies system identification methods to analyze the experimental data for retrieving the evolution of the dynamic properties of the model.

2 MODEL SETUP AND EXPERIMENTAL PROGRAM

A series of shaking table tests were performed at the National Center for Research on Earthquake Engineering of Taiwan (NCREE). A column-footing model of a column height 80 cm and a footing of 40cm×40cm was designed, as shown in Figure 1.

Figure 1. Arrangement of shaking table testing (from Chiou et al. 2014)

The column was made of an aluminum pile with an outer diameter of 89 mm and a thickness of 3 mm. The weight of the model was 0.165 kN. Mass blocks with weights of 0.905 kN were made to be placed on the top of the column for simulating the weight of a structure. The fixed-base predominant period of the model was about 0.125 sec (8 Hz). The combined weight of the model and the mass blocks was 1.07 kN. The column-footing model was founded on a soil specimen of size 1880 mm × 1880 mm × 1520 mm in a laminar box. The laminar shear box is composed of 15 layers of sliding frames to simulate the behavior of level ground subjected to horizontal seismic excitations. Vietnam sand was used for the soil specimen and compacted to have a relative density of about 75% (dense sand). The mass density of the soil was 1.585 t/m^3; the friction angle of the soil was about 39º, and the shear modulus of the soil was about 14.6 MN/m^2.

The input motions were applied at the bottom of the shear box in the Y-direction via the shaking table to simulate seismic loading. Three types of input motions were adopted, including white noise wave, sinusoid signals, and historical earthquake records (Chiou et al., 2015). The test series adopted for analysis are listed in Table 1.

Table 1 Selected analysis cases in the shaking table experimental program

Test Sequence	Input Motion	Record	PGA (g)
WN-1	White Noise	0-30 Hz	0.03
S-1	Sine Wave	2 Hz	0.02
S-2	Sine Wave	4 Hz	0.02
S-3	Sine Wave	8 Hz	0.02
S-4	Sine Wave	10 Hz	0.02
WN-2	White Noise	0-30 Hz	0.03
921-RS-1	921	TCU079	0.05
921-RS-2	921	TCU079	0.35

Cases WN-1 and Cases S-1~4 represent the test cases using input motions of white noise signals and sinusoidal waves, respectively. Cases 921-RS-1 and 921-RS-2 are the input motions using the strong ground motion records at the seismic station TCU079 in Chi-Chi 1999 Taiwan earthquake. The acceleration spectra of cases 921-RS-1 and 921-RS-2 is shown in Figure 2.

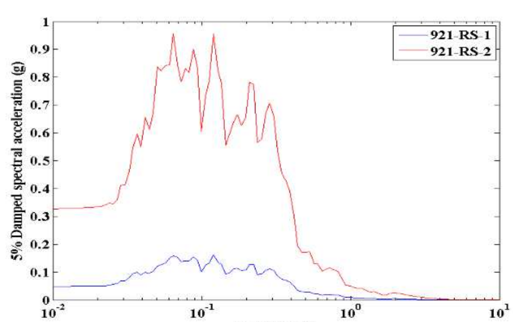

Figure 2. Damped spectral acceleration of input motions

3 SYSTEM IDENTIFICATION METHODS

In this study, system identification methods are applied to analyze the shaking table testing data for identifying the dynamic characteristics of the system. The Transfer Function method (TF method) and the auto-regression moving average model (ARMA model) are adopted. These two models are originally applied to linearly time-invariant (LTI) systems. Dividing the entire time history into several segments, this study extends the LTI system to time-varying systems for tracing the change in system parameters during shaking.

3.1 Short time transfer function method

The short time Fourier transform (STFT) is a modified Fourier transform used to find the frequency content and the corresponding phase in each time segment of a signal. The algorithm of the STFT is to divide the whole signal into shorter time sections in equal length and then to perform the Fourier transform on each time section separately. In practice, the STFT is performed through the convolution of the time window and the Fourier transform of the signal over time. This can be expressed in a two dimensional form as:

$$x(t,f)=\int_{-\infty}^{\infty} w(t-\tau)x(\tau)e^{-i2\pi f\tau}d\tau \qquad [1]$$

where $x(\tau)$ is the signal and $w(t-\tau)$ is the window function.

Based on the Heisenberg uncertainty principle, the resolution of the time domain and frequency domain is fixed, which means that higher frequency resolution is accompanied with poor time resolution and vice versa. In other words, a large window gives better frequency resolution but worse time resolution, and a narrow window gives good time resolution but bad frequency resolution. Moreover, the number of samples by which the window sections overlap will affect the performance of the resolution, which implies that a smaller range of overlapping between every two windows results in larger leakage of the signal. On the other hand, the transfer function is a complex function in frequency domain to describe the relationship between the output and input signals.

Through the transform function, the dynamic properties of a system, such as the predominant frequency and equivalent damping ratio, can be identified. In this study, STFT is applied to build the transfer function of each time window of acceleration records. This way can help observe the time-varying dynamic behavior of the soil-structure system. The Hamming window is adopted as the window function for the STFT. An appropriate window size of 256 data points is set to reach a better trade-off between the time and frequency resolution. The range of overlapping of each time window is 50% window length.

3.2 ARMA (Auto regressive moving average) model

This study also uses the ARMA model to characterize the model parameters. Considering a linear time invariant single input and single output (SISO) system, its dynamic response can be expressed in a linear difference equation:

$$\begin{aligned} & y(t)+a_1 y(t-1)+a_2 y(t-2)+\ldots+a_{n_a} y(t-n_a) \\ & =b_0 x(t)+b_1 x(t-1)+b_2 x(t-2)+\ldots+b_{n_b} x(t-n_b) \end{aligned} \quad [2]$$

where $y(t)$ and $x(t)$ are the system output and input sequences, respectively, at the time t; a_i and b_i (i=1,2…n) are the coefficients of the output and input sequences; n_a and n_b are the orders of the autoregressive functions of output $y(t)$ and input $x(t)$, respectively. Due to the high signal to noise ratio of the signal adopted, the noise term is not considered in this study.

If the z-transform is applied to Equation 2, because of the backward-shift characteristic of the z-transform, the equation can be written as

$$\begin{aligned} & Y(z)+a_1 z^{-1} Y(z)+a_2 z^{-2} Y(z)+\ldots+a_{n_a} z^{-n_a} Y(z) \\ & =b_0 X(z)+b_1 z^{-1} X(z)+b_2 z^{-2} X(z-2)+\ldots+b_{n_b} z^{-n_b} X(z-n_b) \end{aligned} \quad [3]$$

Besides, Equation 3 can be further expressed as

$$G(z)=\frac{Y(z)}{X(z)}=\frac{(b_0+b_1 z^{-1}+b_2 z^{-1}+\ldots+b_{n_b} z^{-n_b})}{(1+a_1 z^{-1}+a_2 z^{-1}+\ldots+a_{n_b} z^{-n_b})} \quad [4]$$

The polynomial ratio $G(z)$ is referred to as the system transfer function. To obtain the poles and zeros of $G(z)$, the transfer function is rewritten as

$$G(z)=\frac{Y(z)}{X(z)}=\frac{b_0(z-z_1)(z-z_2)\ldots(z-z_{n_b})}{(z-p_1)(z-p_2)\ldots(z-p_{n_b})} \quad [5]$$

Note that the p_j's are the poles of $G(z)$ when $X(z)$ is zero, and the z_i's are the zeros of $G(z)$ when $Y(z)$ is equal to zero. Based on the p_j's, the damping ratio ξ_j and the frequency f_j of each vibration mode j of the system are determined as

$$\xi_j=-\frac{\ln(r_j)}{\sqrt{\ln(r_j)^2+\phi_j^2}} \quad [6]$$

$$f_j=\frac{1}{2\pi\Delta t}\sqrt{\ln(r_j)^2+\phi_j^2} \quad [7]$$

where r_j and ϕ_j are the amplitude and the phase of the jth pole (p_j) of the system, and Δt represents the sampling time period. The order of the polynomial of output n_a determines the number of the modes of the system. Since the root of the mode is conjugate, if n_a is even, the number of the modes in the system is assumed as half of n_a.

There are several basic suggestions for selecting the model parameters. First, n_a needs to be lager or equal to n_b. Second, n_a and n_b are appropriate to set as even number. Third, more model parameters can enhance the accuracy of the prediction, but the identified frequency and damping ratio may be wired due to high order polynomial fitting. In other words, because this approach regards the response to be linear in each time segment, it is supposed that it may have few modal frequencies in each section. Therefore, when we apply too many model parameters to describe the system, the model will fit the local fluctuation of test data (i.e., overshooting effect), which may lead to irrational results. Through some tests, 95% fitting percentage is suitable to obtain better model parameters without overshooting effect.

In this study, regarding the model as a single-degree-of-freedom system, the horizontal accelerations at the surface of the soil is considered as the input motion, and the top of the mass is regarded as the output motion. Similarly, if the soil system is assumed as a single soil layer, the shaking table record is used as the input signal while the record on the soil surface is used as the output motion.

As the way adopted in the short time transfer function method, the segmentation scheme mentioned by Glaser and Baise (2000) is applied to the ARMA model. Dividing the original signal into several equal-length time segments, each signal segment is regarded as an independent linear time invariant system and the ARMA model is on each time segment to recognize the nonlinear behavior of the dynamic characteristics of the soil and column-footing model.

4 TEST RESULTS

4.1 White noise sweeping

At the beginning, the white noise case is analyzed to determine the initial condition of the model. The predominant frequencies of the structure model and soil specimen are identified through the transfer functions. Figure 3 shows the transfer function of the structure model of Case WN-1. The fundamental frequency of the model is about 5.9 Hz. On the other hand, another significant amplification is at the frequency of around 24 Hz. According to the previous numerical modal analysis of the model (Hwang, 2016), this frequency is regarded as the predominant frequency of the model in the torsional direction.

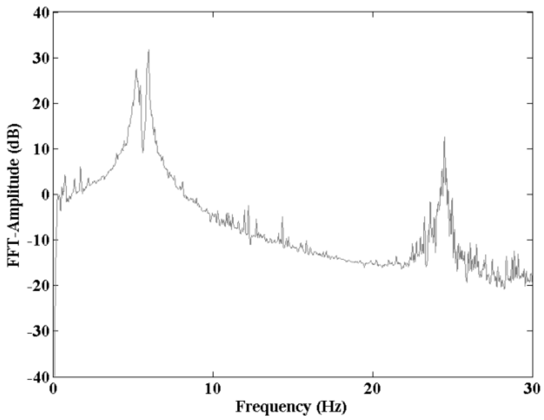

Figure 3. Transfer function for structure in Case WN-1

Figure 4 shows the transfer function of the soil specimen in case WN-1. The fundamental frequency of the soil specimen is around 20 Hz, and the frequency of the second mode is 40 Hz. According to the theoretical transfer function of a uniform soil layer subjected to vertically propagating wave, the equivalent shear wave velocity of the soil is 96 m/s, and the maximum shear modulus is about 14.6 MPa.

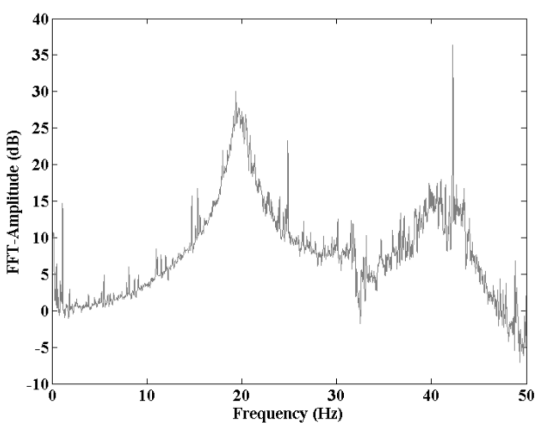

Figure 4. Transfer function for soil in Case WN-1

4.2 Sinusoid signals

The dynamic characteristic of the system response can be roughly captured from the sinusoidal input motions. Table 2 summarizes the peak structure acceleration (PSA) and the peak ground acceleration (PGA) of the sinusoidal cases.

The amplification factor is calculated by dividing the peak acceleration of the system by the peak acceleration of the input motion. The system response will be larger as the input frequency is closer to the predominant frequency of the system. As a result, the comparison shows that Cases S-1 and S-2 have relatively large dynamic amplification in structure for the frequencies of the input motions at 2Hz and 4Hz, respectively. Although the maximum PGA occurs in Case S-4, the PSA of the Case S-4 is the smallest, because the frequency of the input is relatively far from the predominant frequency of the system.

Table 2 Peak acceleration for Case S-1~S-4

Case	S-1	S-2	S-3	S-4
PGA (g)	0.022	0.026	0.025	0.027
Amplification Factor	1.102	1.309	1.256	1.361
PSA (g)	0.027	0.052	0.027	0.017
Amplification Factor	1.210	1.970	1.087	0.620

4.3 Historical earthquake excitation

Before conducting Cases 921-RS-1~2, the white noise case WN-2 is performed to measure the initial condition of the soil-structure system. The predominant frequency of the structure is 5.9 Hz, and the fundamental frequency of the soil specimen is 20 Hz.

4.4 Short time transfer function method

The time segment of 3 seconds is adopted to perform the short time transfer function, i.e., each segment contains 600 data point. Figures 5 and 6 show the short time frequency response functions of Cases 921-RS-1~2.

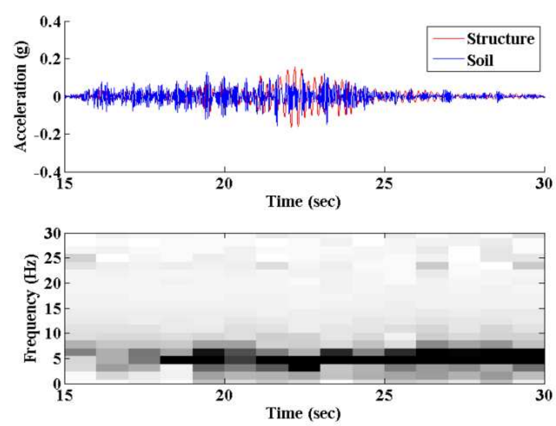

Figure 5. Short time transfer function for structure in Case 921-RS-1

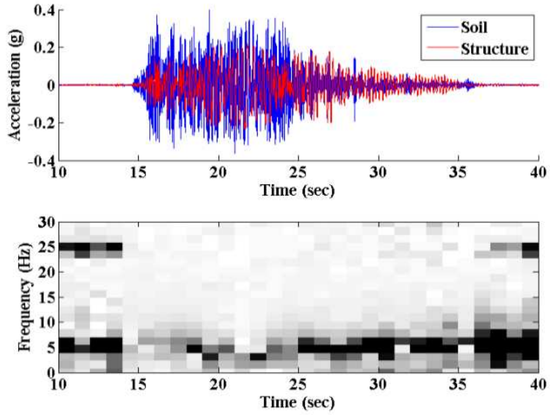

Figure 6. Short time transfer function for structure in Case 921-RS-2

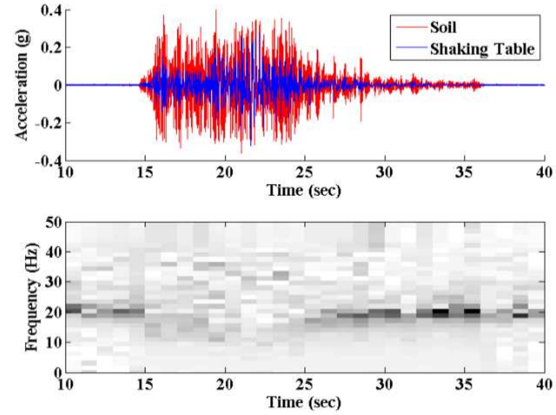

Figure 8. Short time transfer function for soil in Case 921-RS-2

From these figures, two frequencies of significant amplification can be recognized. The first one is the fundamental frequency at around 5.2Hz, and the second one is at around 24Hz. In the short time frequency response function analysis of Case 921-RS-1~2, it can be found that the predominant frequency drops from 5.2Hz to about 3Hz at 15~20sec, after that the frequency rise back to 5Hz. This phenomenon indicates that the reduction of the predominant frequency is mainly due to the rocking behavior of the footing. Comparing Cases 921-RS-1 and 921-RS-2, the intensive rocking behavior causes the significant degradation of the system vibration frequency. The second modal frequency, 24Hz, is considered as the twisting response of the structure model, which exists in the transient state and the free vibration state.

From the spectra, the predominant frequency of the structure model in the free vibration part is lower than in the transient state. As a result, it seems that the soil underneath the footing may have permanent deformation.

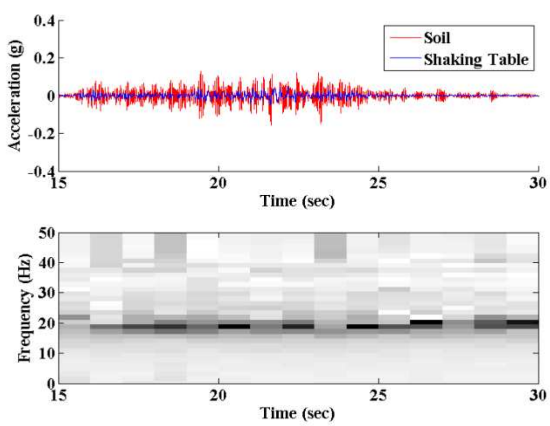

Figure 7. Short time transfer function for soil in Case 921-RS-1

As shown in Figures 7 and 8, the segmentation method is also applied to observe the changes of dynamic soil properties in Cases 921-RS-1~2. In order to give better time resolution, the time segment of 2 seconds for the soil system is set. From these figures, the predominant frequency of the soil is around 19 Hz. Due to the nonlinear soil behavior, the predominant frequency is reduced to 16 Hz and 17.8 Hz for Cases 921-RS-1~2, respectively. The degradation degree of the predominant frequency increases as the earthquake intensity increases, and the frequency rises back after the free vibration state.

4.4.1 Short Time System Identification analysis

The dynamic structure response of Cases 921-RS-1~2 is shown in Figures 9 and 10. The left-hand top of the figure presents the output response of the system, and the evolution of the vibration frequency, rotational stiffness and damping ratio are displayed at the right-hand top, left-hand bottom, and right-hand bottom of the figure, respectively.

From Figure 9, because of small excitation, the structure system is almost in the linear response. The predominant frequency of the structure model is 5.2 Hz for Case 921-RS-1

As shown in Figure 10, the severe degradation of the rotational stiffness and the vibration frequency during the excitation shows that the foundation uplifting behavior and the nonlinear soil response are significant for Case 921-RS-2. The strong excitation causes much larger rocking behavior, which makes the vibration frequency of the structure down to 3.34 Hz.

In general, although the vibration frequency may decrease during excitation, the vibration frequency will rise back as the excitation stops, such as Case 921-RS-1. But the vibration frequencies at the free vibration state of the other cases identified are less than that generated at the transient state. It brings about indirect evidence that the soil underneath the footing might have permanent deformation, which makes the stiffness of the system decrease in the free vibration state.

According to the figures, the estimated damping ratio ranged from 2%~8% for Case 921-RS-1. The evolution of the damping ratio in Case 921-RS-2 shows that the damping ratio grows with the increasing of earthquake intensity, and the maximum damping ratio occurs when the structure system reaches its peak acceleration response. The maximum damping ratio for structure response of Case 921-RS-2 is 40%, which might contain the effect of depressed acceleration due to rocking behavior. As a result, the results show that rocking behavior is involved into the estimated damping from the system identification process.

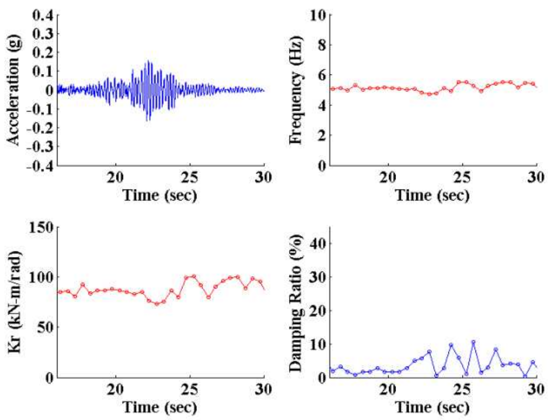

Figure 9. Short time system identification for structure in Case 921-RS-1

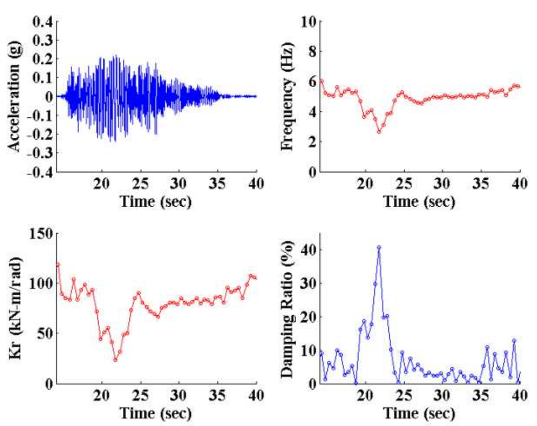

Figure 10. Short time system identification for structure in Case 921-RS-2

Figures 11 and 12 show the results of the applied short time system identification on the soil. The shear stiffness of the soil specimen is displayed at the left-hand bottom of the figure. Case 921-RS-1 shows that the predominant frequency of the soil is about 20 Hz, and the estimated damping ratio is around 5% during the excitation. Based on the observation, the nonlinear soil behavior is not significant in the small excitation condition.

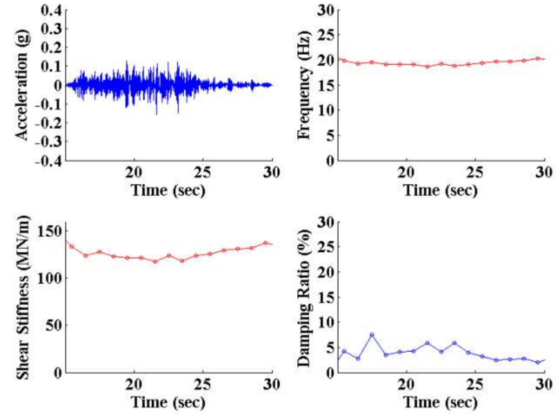

Figure 11. Short time system identification for soil in Case 921-RS-1

On the other hand, as shown in Figure 12, the severe nonlinear soil behavior is remarkable in the Cases 921-RSN-2. As a result, the reduction of the predominant frequency and the enlargement of the damping ratio occur when the input excitation amplified the dynamic response of the soil. The maximum damping ratio for soil response of Case 921-RSN-2 is 19%.

However, the predominant frequency rises again as nearly the same as the beginning state after the main excitation, which indicates that the soil specimen is stiff enough to resist the strong excitation. The trend of the results show that the degradation of the natural frequency is similar to that observed in the structure behavior.

The degradation of the predominant frequency of the soil specimen shows that the strain softening behavior is due to the large dynamic response of sand particle at the soil surface. The evolution of the shear stiffness is consistent with the evolution of the predominant frequency and the damping ratio. The degradation of shear stiffness also implies that the soil is in nonlinear response during the excitation.

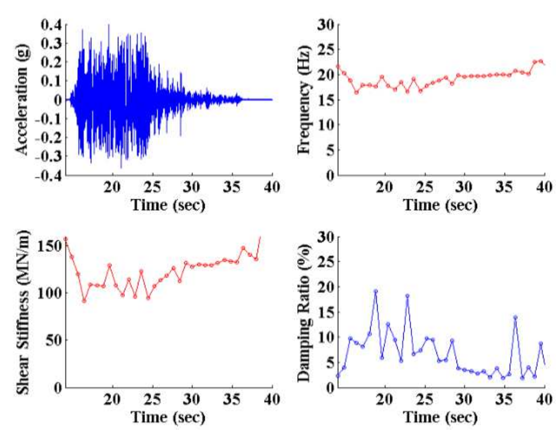

Figure 12. Short time system identification for soil in Case 921-RS-2

5 CONCLUSIONS

In this study, the methods of system identification are applied to investigate the change of the dynamic characteristics of a column-footing model in shaking table testing under different excitation conditions.

With the aid of the segmentation technique, the short-time transfer function and ARMA method can effectively capture the evolution of the dynamic system of the model and soil.

According to system identification results, during the process of shaking, the predominant frequency of the model is not constant. The predominant frequency decreases as the shaking intensity increases, but it may increase as the shaking intensity decreases. The uplifting behavior is significant in the large excitation, which causes the degradation of the rotational stiffness as well as the reduction of the vibration frequency of the system.

The evolution of the identified damping ratio is also compatible with the degradation behavior of the vibration frequency. The damping ratio increases as the vibration frequency decreases, and the maximum damping ratio occurs when the vibration frequency approximately reaches its lowest value. Due to the ultimate moment capacity of the footing, the limited structure acceleration is a major contribution to a large damping ratio identified for shaking at a larger acceleration level. As a result, the foundation rocking can dissipate the seismic energy, which is more significant for larger shaking.

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