On the Importance of Coupled Stiffness of Fixed-Head Piles

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ABSTRACT

Oftentimes the design of foundations and structures is governed by serviceability requirements such as deformations and vibration amplitudes. These requirements require accurate modeling of structural stiffness with due consideration of foundation flexibility. For structures supported by pile foundations, the foundation flexibility is affected by the individual pile stiffness as well as pile-soil-foundation interaction. Fixed-head piles are frequently used to increase the lateral stiffness of foundations, which would result in limited lateral displacements and improved performance. Most common modeling techniques simulate fixed-head pile condition in the form of boundary conditions at the pile head, which tend to ignore the coupled stiffness components. Consequently, the lateral stiffness could be largely over-estimated, which could lead to inaccurate representation of the actual behavior and may result in a foundation design that provides unsatisfactory performance. This paper investigates the common methods for modelling fixed-head piles and their limitations, and the effect of coupled stiffness on the lateral stiffness of the foundation system. In addition, two approaches are provided to better represent the boundary conditions of fixed-head piles.

1. INTRODUCTION

Fixed-head piles are defined as piles that do not exhibit any rotation at the pile head as a result of induced lateral displacement at the pile head. On the other hand, pinned-head piles are free to rotate at the pile head, as shown in Figure 1. Imposing a fixed-head condition on the pile can provide a lateral stiffness being more than twice that of pinned-head piles (e.g. Novak, 1974).

The pile head fixity requires a connection between the pile head and the pile cap (or other foundation elements), which is capable of resisting rotational forces (i.e. a rigid moment connection). Therefore, it is true that all fixed-head piles must be rigidly connected to their foundations. However, not all rigidly connected pile heads can provide the same lateral stiffness of a fixed-head pile. This is due to the relative stiffness of the pile cap (or foundation element) and associated effect of the coupled stiffness. The rotational force (i.e. moment) that is required to prevent a fixed-head pile from rotating at its head due to a unit lateral displacement along the i th direction, Ui = 1, is termed the coupled stiffness, Kij, where i is the translational degree of freedom (DoF) along the direction of displacement and j refers to the rotational degree of freedom (DoF) in the plane of displacement.

In addition to the coupled stiffness, the flexural stiffness of the foundation that connects the pile heads plays a significant role in determining the lateral stiffness of fixed-head piles. The lateral stiffness of a rigidly connected pile head approaches that of a fixed-head pile with increasing foundation stiffness, and approaches that of a pinned-head pile with decreasing foundation stiffness as shown in Figure 2.

The lateral performance of a pile can be characterized by its load-displacement curve. Typically, the load—displacement curve is developed without prior knowledge of the rigidity (or lack of) of pile cap (or the foundation element); that is the load displacement curve is developed considering the group effect in terms of pile-soil-pile interaction only. Therefore, the provided load-displacement curves are developed under the assumption that the foundation is infinitely rigid; that is without due consideration of pile-foundation interaction. However, the pile-foundation interaction should be evaluated accounting for the effect of foundation flexibility on the stiffness of the system as a whole.

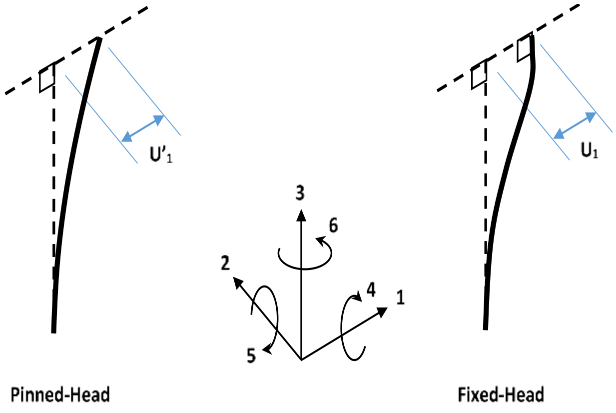

Figure 1. Deformed shape of a fixed-head pile versus a pinned-head pile (pile-soil interaction only)

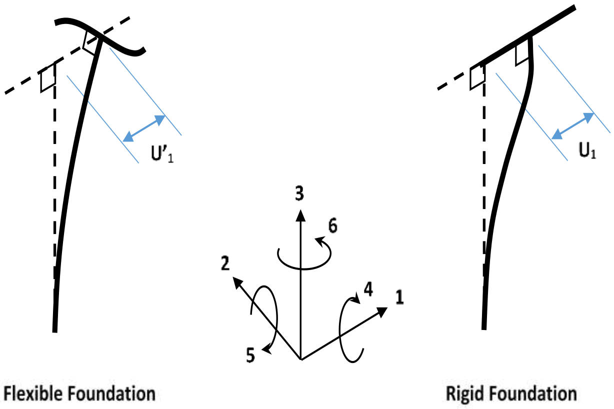

Figure 2. Deformed shape of a pile that is rigidly connected to a flexible foundation versus a rigid foundation (pile-soil-foundation interaction)

2. BOUNDARY CONDITIONS

2.1. Background

The lateral performance of a pile can be characterized by its load-displacement curve, similar to that shown in Figure 3. The load-displacement curve for a pile describes the lateral response of a pile due to a lateral applied load at its head. It can be obtained experimentally via full-scale testing as outlined in ASTM D3966 (ASTM, 2007) procedure for lateral load testing.

Alternatively, the load-displacement curves can be estimated analytically. Perhaps one of the most common analytical approaches for estimating the lateral response of vertical piles is the p-y curves approach proposed by Reese (1984). In this method, the pile is simulated as a beam and the soil resistance is simulated as a series of springs attached to the pile along its shaft as shown in Figure 4. The force-deformation relationship of these springs is represented by p, defined as the soil reaction per unit length of the pile at a given depth and y is the corresponding lateral deflection of the pile at that depth. The p-y curves could represent linear or non-linear force-deformation behaviour and are a function of the soil parameters.

Moreover, the pile can be modeled as linear elastic or non-linear material. The p-y curves method is incorporated in several computer codes that are used for pile analysis such as LPile (Ensoft, 2011).

Similar to the p-y curve approach, El Naggar and Novak (1995,1996) developed a computationally efficient model for evaluating the lateral response of piles and pile groups based on the Winkler hypothesis, accounting for nonlinearity using a hyperbolic stress–strain relationship. In addition, they accounted for slippage and gapping at the pile–soil interface. El Naggar and Bentley (2000) further developed this model by employing dynamic p-y curves that account for the hysteretic behavior of the soil and energy dissipation during dynamic loading. Badoni and Makris (1996) utilized nonlinear soil springs in conjunction with distributed dashpot placed in parallel to investigate the dynamic response of piles. Gerolymos and Gazetas (2005) extended the p-y curve approach to simulate both the lateral soil reaction and pile inelasticity and to compute the nonlinear response of pile under monotonic and cyclic lateral load with due consideration to effects of pile and soil nonlinearity as radiation damping. Allotey and El Naggar (2008) developed a dynamic beam on nonlinear Winkler Foundation (BNWF) model with different rules for loading, reloading, and unloading capable of accounting for cyclic degradation, opening and closing of gap and reduced radiation damping. The model comprises a four segment multi-linear backbone curve. The backbone curves are fitted to American Petroleum Institute (API) static p–y curves (1993). Heidari et al. (2014) extended this model further to incorporate backbone curves based on the strain wedge model proposed by Ashour et al. (1998).

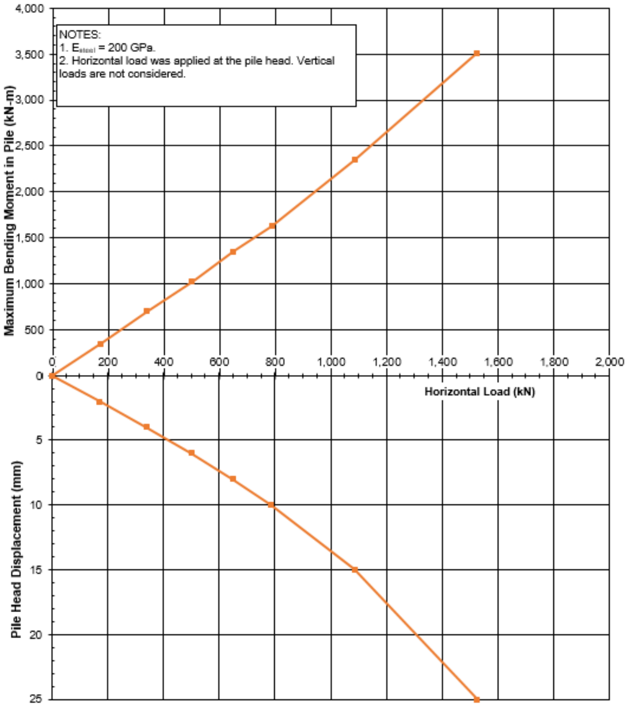

Figure 3. A typical load-displacement curve for a fixed-head pile using the p-y method. The curve also shows the associated moment at the pile head for each load increment

Another commonly used analytical method for estimating the pile head displacement and rotation is the elastic continuum approach as presented by Fleming et al. (2009). This approach provides closed form solutions, which are derived from extensive finite element and boundary element modeling of the pile and soil continuum. However, this method is limited to linear elastic pile and soil medium.

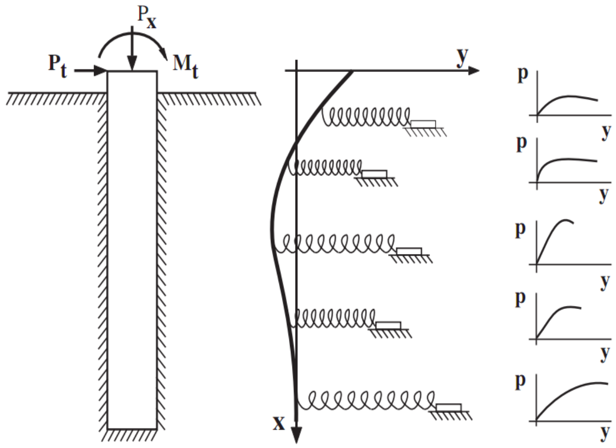

Figure 4. Discretization of a laterally loaded pile using p-y nonlinear springs (from Ensoft, Inc.)

From the resulting load-displacement curves, as shown in Figure 3, the stiffness of the pile at the pile head and at a specific load level, Pii, can be estimated as follows:

$$k_{ii} = \frac{P_{ii}}{U_{ii}} \quad [1]$$

$$k_{ij} = \frac{M_{ij}}{U_{ii}} \quad [2]$$

where U_{ii} is the displacement response due to applied load P_{ii} along the i th direction, and M_{ij} is the associated in-plane moment resulting from a displacement equal to U_{ii}.

There are other analytical methods in the literature that can be used to evaluate the stiffness of the pile head directly as presented by Novak (1974) among others. Novak (1974) provided a solution based on the continuum approach, and considering plane strain conditions, to evaluate the static and dynamic stiffness of piles at the pile head. The solution is limited to linear elastic pile and soil materials. This approximation is representative of cases where the applied loads on the pile result in shear strains in the soil less than 10-5, which is typical for foundations supporting vibration machinery.

2.2. Common Practices in Evaluating Pile-Foundation Stiffness

The load-displacement curves are typically developed with little to no knowledge of how stiff the foundation is; thus, considering only pile-soil interaction. Therefore, it is essential to account for the effect of foundation rotational stiffness on the lateral stiffness of fixed-head piles, i.e., the pile-foundation interaction, when evaluating the response to lateral loads.

The common practice by structural engineers is to represent the pile as a joint spring located at the pile head, in lieu of modeling the entire pile-soil domain. The joint spring has six degrees of freedom, three translational and three rotational, with spring constants along these degrees of freedom equal to the stiffness values obtained from the aforementioned methods (i.e. neglecting foundation flexibility).

Alternatively, some design engineers model the pile along with the foundation and structure, and simulate the soil resistance with discretized springs along the pile shaft, whose constants are derived based on the BNWF or the modulus of subgrade reaction approach (Matlock, 1970). Even though the latter modeling approach provides more accurate representation of the pile connectivity as it accounts for soil-pile-foundation interaction all at once, it is computationally expensive and exposes structural engineers to additional liabilities related to geotechnical input. On the other hand, the former modeling approach can provide excellent results provided that all the stiffness components of the piles are addressed properly.

All commercially available structural analysis software packages, such as SAP2000 (2015) and STAAD Pro (2014), allow for modelling piles employing spring elements with a formulation that accounts for 6 DoF as mentioned previously, including three translational and three rotational. The stiffness matrix of such springs is as described by Eq. 3.

$$[K] = \begin{bmatrix} k11 & 0 & 0 & 0 & 0 & 0 \\ 0 & k22 & 0 & 0 & 0 & 0 \\ 0 & 0 & k33 & 0 & 0 & 0 \\ 0 & 0 & 0 & k44 & 0 & 0 \\ 0 & 0 & 0 & 0 & k55 & 0 \\ 0 & 0 & 0 & 0 & 0 & k66 \end{bmatrix} \quad [3]$$

where k11 to k33 are the translational stiffness constants while k44 to k66 are the rotational stiffness constants (spring constants) at the pile head, as indicated in Figure 1.

For a single pile or a symmetric pile group, k11 is equal to k22, and k44 is equal to k55. In addition to asymmetric pile groups, there are some obvious cases where k11 does not equal k22 such as the two-pile group case that is shown in Figure 5. Since the torsional stiffness of the beam is much lower than its flexural stiffness, structural engineers tend to assume that k11 and k55 will be that of a fixed-head pile while k22 will be that of a pinned-head pile with k44 = 0; k33 and k66 are not affected though. This approximation accounts indirectly for pile-foundation interaction in the transvers direction (i.e. along direction 2 as shown in Fig. 5).

It is important to note that the approach described by Eq. 3 assumes that the relative rotational stiffness of the foundation is much greater than that of the pile (i.e. rigid foundation). If that is the case, this approach will yield accurate results. However, if the rotational stiffness of the foundation is low, the lateral stiffness of the system will be largely over-estimated. In other words, the approach described by Eq. 3 will yield the same lateral stiffness for any foundation system regardless of how rigid or flexible the foundation is.

For example, consider a pile foundation that comprises 4 fixed-head piles connected with a 4 m x 4 m x 0.5 m concrete pile cap that supports a pipe anchor. The piles are equally spaced with center-to-center spacing equal to 3.2 m. The average stiffness of each pile at the pile head is obtained using the p-y approach and is shown in Table 1. The concrete pile cap is discretized at its mid-thickness using shell elements and the piles are represented by spring elements in SAP2000. The spring element is modeled in the same manner as described in Eq. 3 with spring constants as shown in Table 1.

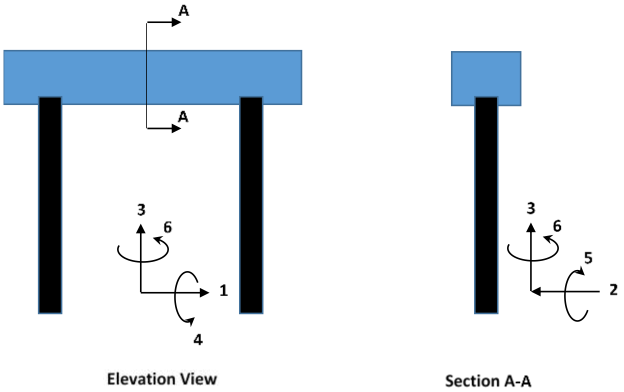

Figure 5. A two-pile group rigidly connected to a beam/pile cap

The pile cap is subjected to a horizontal pipe anchor load, P1, equal to 1000 kN. The displacement at the load point of application, U1, can be estimated, assuming a rigid cap, to be:

$$\mathrm{U1} = \frac{1000\ \mathrm{kN} \times 1000\ \mathrm{mm/m}}{4 \times 70{,}000\ \mathrm{kN/m}} = 3.6\ \mathrm{mm}$$

The displacement obtained from SAP2000 is 3.7 mm which is slightly higher than the estimated value since the cap's in-plane diaphragm is not infinitely rigid as assumed.

By increasing the flexural stiffness (i.e. out-of-plane flexural stiffness) of the cap by a factor equal to 1000, the resulting displacement from SAP2000 is found to be the same as the displacement obtained assuming a flexible cap. Therefore, this modeling technique yields the exact same lateral stiffness (and consequently same lateral displacement) regardless of how stiff or flexible the cap is.

Table 1. Pile head stiffness values

Direction	Stiffness	Unit
K11, Horizontal	70,000	kN/m
K22, Horizontal	70,000	kN/m
K33, Vertical	550,000	kN/m
K44, Rotational	440,000	kN.m/rad
K55, Rotational	440,000	kN.m/rad
K66, Torsional	100,000	kN.m/rad
K15, Coupled	160,000	kN/rad
K24, Coupled	160,000	kN/rad

2.3. Effect of Coupled Stiffness

Some structural analysis software packages, such SAP2000, offer spring elements with formulation which facilitates accounting for off-diagonal stiffness components (i.e. coupled stiffness components) as shown below:

$$[\mathrm{K}] = \begin{bmatrix} k11 & k12 & k13 & k14 & k15 & k16 \\ k21 & k22 & k23 & k24 & k25 & k26 \\ k31 & k32 & k33 & k34 & k35 & k36 \\ k41 & k42 & k43 & k44 & k45 & k46 \\ k51 & k52 & k53 & k54 & k55 & k56 \\ k61 & k62 & k63 & k64 & k65 & k66 \end{bmatrix} \quad [4]$$

Solving the same example, while considering the coupled stiffness terms k15 and k24, the resulting displacement from SAP2000 would be 6.9 mm, which is almost twice the displacement obtained from the analysis ignoring the coupled stiffness terms. Consequently, the lateral stiffness of the foundation drops by almost 50% when the coupled stiffness terms are accounted for. In addition, the reduction in stiffness would reduce the natural frequency of the foundation by almost 25%. Even if the cap is constrained to behave as a rigid body, the displacement would be 4.5 mm which is still 20% greater than the case without coupled stiffness.

This difference in response associated with consideration of coupled stiffness in the spring formulation is attributed to the development of bending moment in the spring corresponding to the applied lateral load. The resulting moment causes local rotations at each connection between the springs and the cap, as well as global rotation in the foundation as the foundation is resisting the applied moments via tension and compression at each spring.

Therefore, considering coupled stiffness of fixed-head piles will always lead to reduced foundation lateral and torsional stiffness and their corresponding natural frequencies. Therefore, caution should be exercised when modeling fixed-head piles for structures and foundations that are sensitive to lateral and torsional stiffness.

2.4. Equivalent Pile Approach

The spring element formulations of most commercially available software packages are not capable of modeling the off-diagonal (coupled) stiffness terms. In such case, piles that support a relatively flexible pile cap or grade beams should be modeled as a beam on a Winkler foundation (i.e. with springs along its shaft). This procedure is tedious and the constants of soil springs that should be used along the pile shaft might not be readily available.

Alternatively, since the formulation in Eq. 4 is essentially the same as the formulation of a frame element, a fixed-head pile may be represented by an equivalent frame element that is restrained at its base as shown in Figure 6. The length, cross-sectional area, and inertia of the equivalent pile are selected such that the resulting stiffness values at the free end are equal to the stiffness values of the pile at its head. The general stiffness relations of the equivalent pile at the free end are as follows:

$$\begin{aligned} \mathrm{k11} &= \frac{12\ \mathrm{E}\ \mathrm{I}_{22}}{\mathrm{L_e}^3} & \mathrm{k55} &= \frac{4\ \mathrm{E}\ \mathrm{I}_{22}}{\mathrm{L_e}} \\ \mathrm{k22} &= \frac{12\ \mathrm{E}\ \mathrm{I}_{11}}{\mathrm{L_e}^3} & \mathrm{k66} &= \frac{\mathrm{G}\ \mathrm{I}_{33}}{\mathrm{L_e}} \\ \mathrm{k33} &= \frac{\mathrm{EA}}{\mathrm{L_e}} & \mathrm{k15} &= \frac{-6\ \mathrm{E}\ \mathrm{I}_{22}}{\mathrm{L_e}^2} \\ \mathrm{k44} &= \frac{4\ \mathrm{E}\ \mathrm{I}_{11}}{\mathrm{L_e}} & \mathrm{k24} &= \frac{6\ \mathrm{E}\ \mathrm{I}_{11}}{\mathrm{L_e}^2} \end{aligned} \qquad [5]$$

where:
I_{ii} is the section inertia about the i th axis,
Le is the equivalent pile length,
E is the modulus of elasticity of the equivalent pile,
G is the shear modulus of the equivalent pile, and
A is the cross-sectional area of the equivalent pile

Since K11 to K66, K15 and K24 are known, the equivalent pile length, Le, can be easily calculated by substituting any of those values into the relations described by Eq. 5. Each relation could yield a different value of Le. However, to reduce the error in the approximation it is recommended to use Le that is obtained from K11 and K22 since both values are a function of Le^3.

For example, considering the pile whose stiffness values are presented in Table 1 (k11 = 70,000 kN/m, E = 200 GPa and I22 = 0.002594 m^4), Le can be estimated as follows:

$$\frac{12\ \mathrm{E}\ \mathrm{I}_{22}}{\mathrm{L_e}^3} = 70{,}000\ \mathrm{kN/m}$$

$$\therefore \mathrm{L_e} = \sqrt[3]{\frac{12\ \mathrm{E}\ \mathrm{I}_{22}}{70{,}000}} = 4.464\ \mathrm{m}$$

Similarly, the area of the equivalent pile can be calculated as follows:

$$\frac{\mathrm{E\ A}}{\mathrm{L_e}} = 550{,}000\ \mathrm{kN/m}$$

$$\therefore \mathrm{A} = \frac{550{,}000 \times 4.464}{\mathrm{E}} = 0.0123\ \mathrm{m}^2$$

Finally, the polar inertia of the equivalent pile section can be calculated as follows:

$$\frac{\mathrm{G\ I}_{33}}{\mathrm{L_e}} = 100{,}000\ \mathrm{kN.m/rad}$$

$$\therefore \mathrm{I}_{33} = \frac{100{,}000 \times 4.464}{\mathrm{E}/2.6} = 0.0058\ \mathrm{m}^4$$

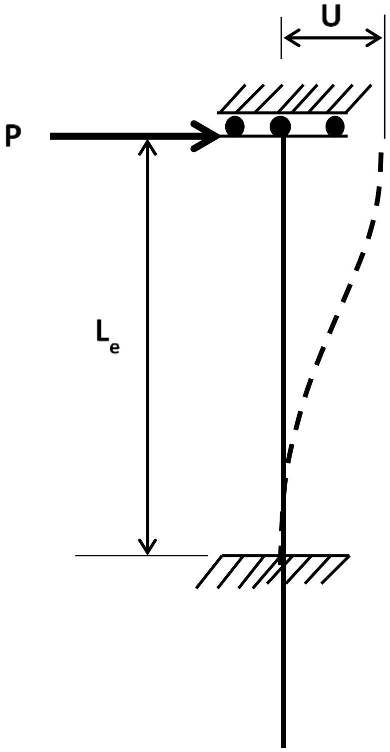

Figure 6. Equivalent Pile

Once I33, A and Le are determined, the rest of the stiffness values can be calculated using Eq. 5. A comparison between the stiffness values of the equivalent pile and the actual stiffness values is shown in Table 2. It is important to note that the sign convention of the coupled stiffness terms is very important and that it follows the right-hand rule.

Table 2 shows that the stiffness values obtained using the equivalent pile method agree well with the actual stiffness values, with a maximum error of 6% in the rotational stiffness and exact values for the translational stiffness for the example presented. Similar results are obtained for other cases. Therefore, all springs in this example can be replaced by frame elements with length (Le) = 4.464 m, I11 = I22 = 0.002594 m^4, I33 = 0.0058 m^4, A = 0.0123 m^2, and E = 200 GPa. Employing these springs in the SAP2000 model to calculate the lateral response yields lateral displacement of 6.5 mm, and for the case of constrained pile cap (i.e. rigid body) the displacement is 4.4 mm.

Table 2. Comparison between actual pile stiffness and equivalent pile stiffness

Direction	Actual Stiffness	Equivalent Stiffness	% Error
K11, Horizontal	70,000	70,000	0
K22, Horizontal	70,000	70,000	0
K33, Vertical	550,000	550,000	0
K44, Rotational	440,000	464,960	6
K55, Rotational	440,000	464,960	6
K66, Torsional	100,000	100,000	0
K15, Coupled	-160,000	-156,238	2
K24, Coupled	160,000	156,238	2

For comparison purposes, Table 3 provides a summary of the displacement obtained considering the different methods and boundary conditions. It can be shown that the equivalent pile method yields excellent results compared to the method described in Eq. 4 (i.e. stiffness matrix considering coupled stiffness).

Table 3. Comparison between displacement value for different boundary conditions

	No Coupled Stiffness	With Coupled Stiffness	Equivalent Pile
Displacement (mm)	3.7	6.9	6.5

3. CASE STUDY

To illustrate the importance of considering the effect of coupled stiffness on the calculate response in a practical setup, an example that involves a pile foundation supporting a 3000 HP centrifugal slurry pump is examined. The pump operating speed is 725 RPM, which means an operating frequency of 76 rad/sec. The pump weighs approximately 280 kN while the steel skid (i.e. foundation) supporting the pump weighs around 110 kN. The foundation also supports the enclosure structure including a 15 tonne over-head crane, as well as a pipe anchor support for the discharge nozzle of the pump as shown in Figure 7. The overall weight of the pump house is approximately 1550 kN. The pump nozzles' axial forces are approximately 1500 kN and the unbalanced dynamic force of the pump, along the 2-3 plane, is 4 kN.

The foundation is composed of rigidly connected steel beams and is supported on 18 steel piles with 0.5 m projection above the ground surface. The structural analysis was performed with the aid of SAP2000.

The foundation was designed to satisfy the ultimate limit states, serviceability limit states, and fatigue limit states. One of the ultimate limit states is to limit the absolute lateral displacement along the pipe anchor support to 8 mm, beyond which the pump discharge nozzle would fail. In addition, the natural frequency of the system must be 20% less than or greater than the operating frequency to avoid resonance. The serviceability limits are to maintain vibration amplitudes less than 0.02 mm for smooth operation of the pump.

The vertical, horizontal, coupled, and rotational stiffness components of the piles at their heads are provided for both fixed and pinned-head piles. An initial trial with pinned-head piles showed unacceptable levels of vibrations and high displacements at the pipe anchors. Because of space limitations, no additional piles could be used. The lateral stiffness of fixed-head piles was found to be approximately 2.5 times the lateral stiffness of pinned-head piles. Therefore, fixed head piles are considered for supporting the pump and associated foundation. The connections between the pile heads and the foundation are designed as rigid moment connections.

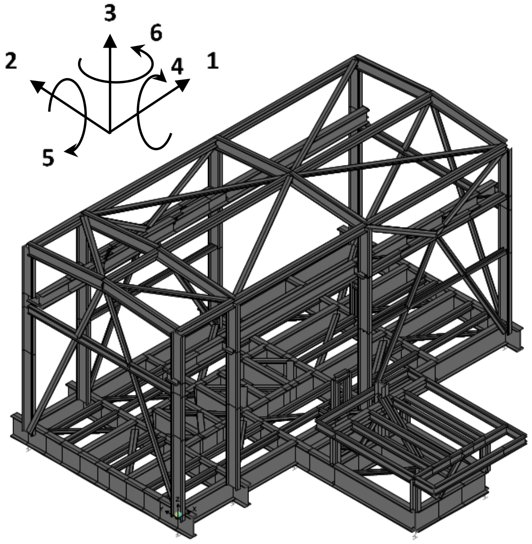

Figure 7. Pump-House isometric view

The piles were represented in SAP2000 as ground springs, and their constants were inputted in the model as suggested by Eq. 3 (i.e. without coupled stiffness) and Eq. 4 (i.e. with coupled stiffness). The steady state vibration response, the anchor displacement, and the natural frequencies for both cases are shown in Table 4. In addition, the vibration response in the frequency domain is shown in Figure 8. It should be noted that Figure 8 is produced utilizing only 10% of the available system damping to emphasize the locations of peaks and should not be used to extract response amplitudes.

Table 4 shows that the analysis performed without considering the coupled stiffness yielded satisfactory results since the vibration amplitude and the anchor displacement are lower than the specified limits. In addition, there is no resonance condition observed, i.e., the operating frequency is away from the natural frequency of the system. However, when the springs are modeled considering the coupled stiffness, the natural frequencies decreased due to the reduction in the system stiffness. Thus, the vibration and displacement responses increased and the torsional natural frequency became coincident with the operating frequency as shown in Figure 8.

Table 4. Response comparison between a case considering coupled stiffness and a case without coupled stiffness

	Direction	No Coupled Stiffness	With Coupled Stiffness
Vibration Response (mm)	2	0.012	0.017
Anchor Displacement (mm)	2	6.7	9.4
Lateral Natural Frequency (RPM)	2	661	545
Torsional Natural Frequency (RPM)	6	901	716

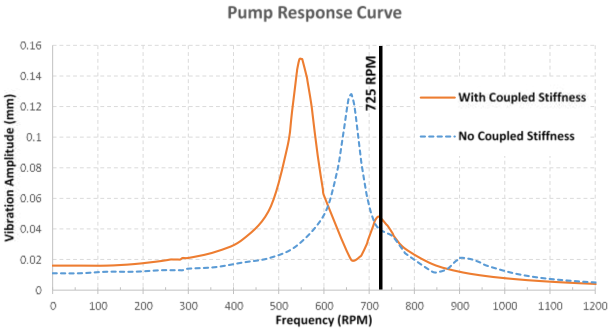

Figure 8. Vibration response of the pump in the frequency domain

4. CONCLUSION

The lateral stiffness and response of pile foundations can be affected by the pile head connectivity to the pile cap and the relative rigidity of the pile cap (or other foundation elements) that connects the piles. The boundary conditions representing fixed-head piles should be selected carefully for structures and foundations that are sensitive to lateral and torsional stiffness. It is shown that the effect of the coupled stiffness terms of fixed-head piles is to reduce the lateral and torsional stiffness of the foundation system. Therefore, the coupled stiffness terms must be accounted for in the boundary conditions through any of the following approaches: by inputting the coupled stiffness terms into the spring element formulation; or by finding an equivalent pile that can produce the same stiffness of the actual pile at the its head. Finally, attention must be paid to the sign convention of the coupled stiffness terms. It is recommended to follow the right-hand rule for sign convention.

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