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Uncertainty of Site Amplification Derived from Ground Response Analysis

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ABSTRACT

Site-specific geotechnical ground response analyses (GRAs) are typically performed to evaluate stress and strain demands within soil profiles and/or to improve the estimation of site response relative to generic site terms from empirical prediction equations. Implementation of GRA results in probabilistic seismic hazard analysis (PSHA) requires knowledge of the mean and standard deviation of site amplification from GRA. We provide expressions for evaluating within-event standard deviations of ground motion given the parameterization of site amplification for a particular intensity measure into a nonlinear equation. A key component of those expressions is the standard deviation of site amplification, ϕ_{lnY}. We evaluate from the literature estimates of ϕ_{lnY} from ground response simulations using variable inputs and from interpretations of ground motion recordings. We find data-based studies to exhibit relatively consistent values of $\phi_{lnY} \approx 0.3$ over a wide range of oscillator periods, whereas simulation-based studies produce much higher values below the fundamental period of the analyzed soil column and lower values at longer periods. Values derived from the data-based studies are recommended for application.

Introduction

Along with source and path effects, site response analysis is a vital component of earthquake ground motion prediction. Site amplification is quantified by amplification factors (Y), which represent the ratio of a ground motion intensity measure on the ground surface (Z) to the intensity measure (IM) on the reference condition (typically rock), X:

$$Y = \frac{Z}{X} \quad or \quad \ln Y = \ln(Z) - \ln(X) \tag{1}$$

A nonlinear expression that has proven to be effective for representing X-dependent amplification is as follows (e.g., Seyhan and Stewart, 2014):

$$\ln \overline{Y} = f_1 + f_2 \ln\left(\frac{x_{IMref} + f_3}{f_3}\right) \tag{2}$$

where f_1, f_2, and f_3 are model parameters, and x_{IMref} is the amplitude of shaking for the reference site condition which is usually taken as the median peak ground acceleration (PGA) for rock. The mean amplification from Equation (2) can be estimated from generic models or from site-specific analysis. When generic models are used, such as the site terms in ground motion

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prediction equations (GMPEs), the site response estimate is referred to as *ergodic*. Such models take the mean site amplification for a given IM as a function of site descriptors, such as the time-averaged velocity in the upper 30 m (V_{S30}) and various sediment depth parameters. Site-specific estimates of $\ln \overline{Y}$, for example as derived from ground response analyses, are also referred to as at least partially *non-ergodic*.

Implementation of site amplification in probabilistic seismic hazard analyses (PSHA) requires knowledge of the probability distribution of Y, which is typically taken as log-normal. The standard deviation of Y (denoted ϕ_{lnY}) contributes to the within-event dispersion of earthquake ground motions, which is commonly denoted as ϕ. We examine in this paper factors affecting ϕ_{lnY} and the effects of site response variability on the total standard deviation of ground motion σ.

We begin in the following section by defining notation that partitions ground motion variability in the manner required for discussion of ergodic and non-ergodic site response. We then synthesize findings from the literature that can be used to evaluate ϕ_{lnY} based on wave propagation analyses that consider input variability and based on analysis of ground motion data. This paper is a synthesis of a portion of a PEER report presenting guidelines for performing GRA and implementing the results in PSHA (Stewart et al., 2014).

Partitioning of Ground Motion Variability

Earthquake ground motions are affected by source, path, and site effects, each of which has corresponding terms in GMPEs. Each of those terms may be systematically in error for a particular earthquake source, wave path, and site. Provided sufficient data exists, those systematic component errors can be estimated through mixed effects methods of residuals analysis (e.g., Pinheiro et al., 2013). A general expression to help visualize such effects is as follows (adapted from Al Atik et al., 2010 with some modification):

$$\ln z_{ij} = \left(\mu_{\ln Z}\right)_{ij} + \eta_{Ei} + \eta_{Pi,j} + \eta_{Sj} + \varepsilon_{ij} \tag{3}$$

where z_{ij} represents a recorded ground motion for event i and site j, $(\mu_{lnZ})_{ij}$ represents the mean from a GMPE (in natural log units), and η_{Ei}, $\eta_{Pi,j}$, and η_{Sj} represent event, path, and source terms, respectively. The term ε_{ij} represents the remaining residual when all of the above systematic biases are removed. In Al Atik et al. (2010), ε_{ij} is further partitioned into record-to-record variability of site-specific amplification and the remaining residual. The record-to-record variability of site amplification is the difference between site amplification for event i and the median site amplification across all events. This difference is unknowable in the absence of vertical arrays, so we do not further partition ε_{ij} for the present application.

When site-specific analysis of ground motion amplification is available, it is combined with a GMPE applied for reference rock conditions to estimate ground motions. In this case $\left(\mu_{\ln Z}\right)_{ij} + \eta_{Sj}$ is replaced with $\left(\mu_{\ln X}\right)_{ij} + \ln \overline{Y}_{ij}$, and Equation (3) is re-written as:

$$\ln z_{ij} = \left(\mu_{\ln X}\right)_{ij} + \eta_{Ei} + \eta_{Pi,j} + \ln \overline{Y}_{ij} + \varepsilon_{ij} \tag{4}$$

Each of the event, path, and site terms has corresponding standard deviations. Following the notation introduced by Al Atik et al. (2010), the standard deviation of between-event terms, and for repeatable path and site terms are denoted τ, ϕ_{P2P}, and ϕ_{S2S}, respectively. The site-to-site standard deviation (denoted ϕ_{S2S}) is especially important in the present context, where site-specific 1D GRA are being incorporated into PSHA. This dispersion term contributes to the within-event standard deviation provided by GMPEs (ϕ), in which site response effects for the many sites contributing to the dataset are captured only through an ergodic, V_{S30}-based site term.

The remaining aleatory standard deviation (of the ε_{ij} term) is taken as ϕ_{lnY}, although this term will represent sources of within-event aleatory variability beyond the site amplification (e.g., some path effects may be included). The total standard deviation can then be computed as:

$$\sigma = \sqrt{\tau^2 + \phi_{P2P}^2 + \phi_{S2S}^2 + \phi_{\ln Y}^2} \tag{5}$$

The three 'phi-squared terms' in Equation (5) sum to ϕ^2, which is the total within-event variance. If the site response were perfectly represented by the mean amplification function (including the effects of both the soil and rock components of the site), the site-to-site contribution to the variance would be eliminated. Under such conditions, the within-event component of ground motion variability can be shown to be (Stewart et al. 2014 and references therein):

$$\phi = \sqrt{\left(\frac{f_2 x}{x + f_3} + 1\right)^2 \phi_{P2P}^2 + \phi_{\ln Y}^2} = \sqrt{\left(\frac{f_2 x}{x + f_3} + 1\right)^2 \left(\phi_{\ln X}^2 - \phi_{S2S}^2\right) + \phi_{\ln Y}^2} \tag{6}$$

Where f_2 and f_3 are parameters in the nonlinear site amplification function (Equation 2). This form of the within-event standard deviation function would be appropriate when the site amplification is derived from on-site recordings. On the other hand, when the site amplification is computed by models such as GRA, we cannot be sure that the mean amplification function is unbiased and that site-to-site variability is eliminated. Under these conditions, we re-write Equation (6) as:

$$\phi = \sqrt{\left(\frac{f_2 x}{x + f_3} + 1\right)^2 \left(\phi_{\ln X}^2 - F\phi_{S2S}^2\right) + \phi_{\ln Y}^2} \tag{7}$$

where F represents the degree of confidence in the effectiveness of GRA in representing the ergodic site response and ranges from zero to one. $F = 0$ indicates no confidence that site-specific amplification factors account for site effects more reliably than ergodic site terms in GMPEs. $F = 1$ corresponds to the ideal conditions in which site-to-site variability is completed removed. Using either Equations (6) or (7) to estimate ϕ requires estimates of ϕ_{S2S} and ϕ_{lnY}. In the following sections, we summarize available prior work from which estimates of ϕ_{lnY} can be derived and provide our recommendations for estimating this important parameter. We defer to the Stewart et al. (2014) report and the references therein for ϕ_{S2S}.

Uncertainty in Site Amplification from Simulations

One way of quantifying the uncertainty in site amplification (ϕ_{lnY}) is to perform a suite of GRAs that includes random realizations of input parameters. Sources of variability that can be captured in this manner are variable input motions, randomness in V_S profiles, randomness in modulus reduction and damping (MRD) curves, and model-to-model variability (through the use of alternate methodologies). Sources of variability that are not captured comprise epistemic uncertainties associated with limitations of the 1D assumption with respect to geologic structure and wave propagation. These include effects of 3D geological structure and surface waves. We summarize below four studies that have used this approach.

Li and Assimaki (2011) (hereafter LA 2011) investigated the effect of variability in V_S profiles and MRD curves on the results of 1D GRA (as implemented in a research nonlinear code, Assimaki et al., 2008) for three sites. Site-specific V_S profiles were used for the median and uncertainty in V_S was evaluated using the Toro (1995) model for generic site conditions. The median and dispersion of MRD curves were taken from generic relationships by Darendeli (2001). Both weak and strong input ground motion were considered. Figure 1 shows a representative example of site amplification dispersion for a deep-soil site (Los Angeles, La Cienega) subjected to strong input motions (which cause the MRD variability to affect the results, along with the V_S variability). Labelled in Figure 1 as 'LA 2011', the results illustrate several characteristic features of site amplification variability as derived from GRA: (1) the level of variability at short periods is quite high at about 0.5-0.6 (comparable to total within-event variability ϕ in GMPEs); (2) there is an increased variability near the inelastic period of the soil column considered in the analysis (in this case, at approximately 1.2 sec); and (3) beyond the soil column period, the dispersion drops markedly.

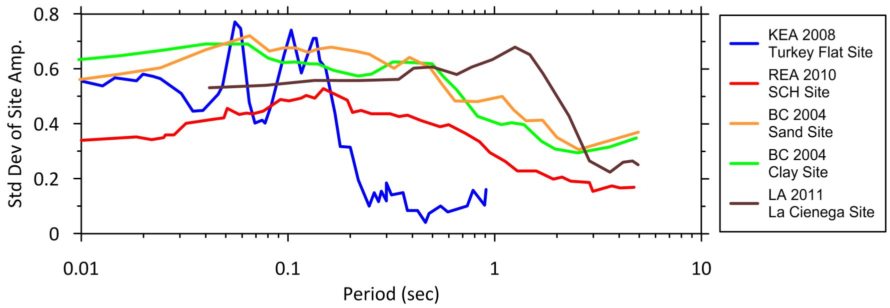

Figure 1. Synthesis of standard deviations of site amplification from GRA-based studies.

Rathje et al. (2010) performed analyses similar in scope to those of LA 2011 for a deep alluvium site (Sylmar County Hospital, SCH); in particular, V_S profile variability and MRD variability were characterized using the same models as LA 2011. The distinctions are the code used for GRA (STRATA, which uses an random-vibration theory based equivalent linear approach; Kottke and Rathje, 2008) and consideration of input motion variability, which was captured using suites of 5, 10, and 20 motions scaled to a target spectrum. As shown in Figure 1, the results are broadly similar to those of LA 2011, but the fall-off at long periods is more gradual. Interestingly, even though an additional source of variability is considered (input motions), the site response variability at short periods is not increased relative to LA 2011.

Kwok et al. (2008) investigated the site response at the Turkey Flat vertical array site during the Parkfield earthquake. They considered effects of V_S profile variability (based on site-specific measurements), MRD curves (from Darendeli, 2001), and model-to-model variability (using six different 1D GRA formulations). As shown in Figure 1, the site response variability results follow identical trends to those described above for LA 2011, except that the site period in this case is relatively short at 0.15 sec.

Bazzurro and Cornell (2004) investigated the effects of variable input motions and soil parameters on the standard deviation of site amplification as computed using the nonlinear model SUMDES (Li et al., 1992). They considered generic sites comprised predominantly of sandy and clayey soils. For randomizing small-strain shear modulus G_{max}, which is related to V_S, they used a method similar to Toro (1995). Figure 1 shows the standard deviation of site amplification reflecting all of the considered soil parameter and input motion variations. The resulting trends are broadly similar to those of the other studies, but with relatively graduate reductions in ϕ_{lnY} at long periods (as with Rathje et al. 2010).

Inference of Site Response Uncertainty from Ground Motion Recordings

For sites having ground motion recordings from multiple earthquakes, it is possible to interpret variations in ground motions in such a way that site response variability (akin to ϕ_{lnY}) can be estimated. There are two general ways that such inferences have been made. One approach is to partition residuals of a predictive model as in Equation (3), which requires only ground surface records. Using this approach, ϕ_{lnY} represents aleatory variability from variable site amplification and other (unknown) sources of ground motion dispersion unrelated to the fixed effects in Eq. (3). As such, it provides an upper bound on the dispersion associated solely with site amplification. The second approach, which requires vertical array data, evaluates site amplification empirically using surface and downhole recordings. Neither approach requires performing GRA, but GRA has been used in some of the predictive models considered in residuals analysis. We summarize relevant results from three studies in Figure 2.

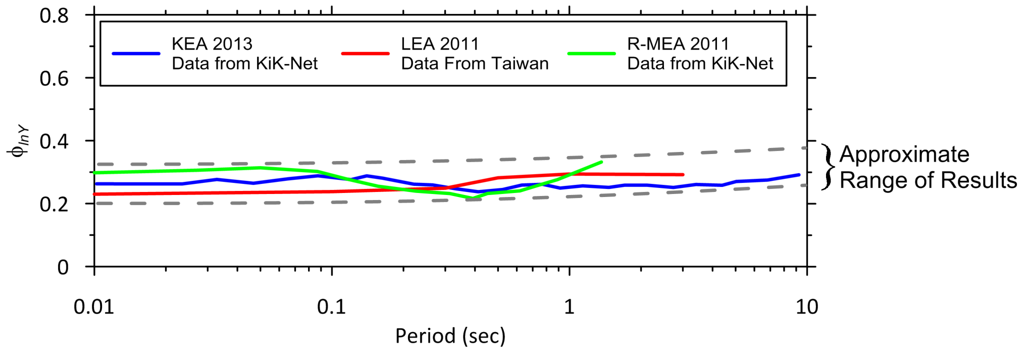

Figure 2. Synthesis of ϕ_{lnY} results obtained from data-based studies.

Kaklamanos et al. (2013) (hereafter KEA 2013) investigated site effects and their dispersion using residuals analysis. They considered vertical array data from the KiK-net arrays in Japan. The predicted motions to which data are compared consist of the downhole recording modified by a 1D ground response analysis. Referring to Equation (4), the downhole record represents the sum (in natural log units) of the mean rock motion (μ_{lnX}), the event term (η_{Ei}), and the path term ($\eta_{Pi,j}$). The site term is assumed to be zero for the downhole motion:

$$\ln x_{ij} = \left(\mu_{\ln X}\right)_{ij} + \eta_{Ei} + \eta_{Pi,j} \tag{8}$$

If the computed mean site response from model k is denoted $\ln \bar{Y}_{ijk}$, then residuals of the surface motion are computed as:

$$R_{ijk} = \ln z_{ij} - \left(\ln x_{ij} + \ln \bar{Y}_{ijk}\right) + c_k + \eta_{Sjk} + \varepsilon_{ijk} \tag{9}$$

where z_{ij} represents the surface recording, c_k is the mean residual when the source, path, and modeled site effects are accounted for, η_{Sjk} is a site term, and ε_{ijk} represents the remaining residual having a mean of zero. The site term η_{Sjk} represents the mean residual for site j relative to the predictions of model k. The standard deviations of η_{Sjk} and ε_{ijk} are taken as ϕ_{S2S} and ϕ_{lnY}, respectively. As shown in Figure 2, the resulting values of ϕ_{lnY} vary from 0.2-0.3 for oscillator periods between 0.01 and 9 sec.

Lin et al. (2011) partitioned residuals into source, path, and site components using a data set of surface recordings from Taiwan and a region-customized GMPE (modified from Chiou and Youngs, 2008). The partitioning of residuals matched Equation (3), except that the event term (η_{Ei}) was separated into two components as:

$$\eta_{Ei} = \eta_{SRi} + \eta_{E0i} \tag{10}$$

where η_{SRi} is the mean event term for the cluster of earthquakes at the location of the i^{th} event, and η_{E0i} is the event term after removing η_{SRi}. Standard deviations were computed from the partitioned residuals including site-to-site and within-site terms for six spectral periods. As shown in Figure 2, ϕ_{lnY} results have a flat trend with oscillator period (similar to KEA 2013).

Rodriguez-Marek et al. (2011) utilized KiK-Net vertical array data from Japan and developed array-specific GMPEs for prediction of borehole or surface ground motions. We denote the means from these GMPEs as $(\mu_{lnX})_{ij}$ and $(\mu_{lnZ})_{ij}$ for downhole and surface locations, respectively. The site amplification is evaluated from recorded data (z_{ij} at surface and x_{ij} downhole) as:

$$\ln y_{ij} = \ln z_{ij} - \ln x_{ij} = \left(\mu_{\ln Z}\right)_{ij} - \left(\mu_{\ln X}\right)_{ij} + R_{ij}^{Y} \tag{11}$$

Equation (11) is used to compute amplification residuals from the data, which are denoted R_{ij}^{Y}. This approach has the advantage of representing a direct analysis of site amplification statistics without the use of an underlying site response model. These residuals can be partitioned by mixed-effects analysis as:

$$R_{ij}^{Y} = \eta_{Sj}^{Y} + \varepsilon_{ij}^{Y} \tag{12}$$

where η_{Sj}^{Y} represents the mean of the amplification residual for site *j,* and ε_{ij}^{Y} represents the remaining residual. The standard deviation of η_{Sj}^{Y} is taken as ϕ_{S2S} and the standard deviation of ε_{ij}^{Y} as ϕ_{lnY}. As shown in Figure 2, ϕ_{lnY} is seen to have a relatively flat trend with respect to period, consistent with the other results from the aforementioned data-based studies.

Synthesis of Findings

The results of data-based studies consistently indicate that ϕ_{lnY} falls within the range of 0.23–0.30. This consistency is found despite significant differences in the manner by which the ϕ_{lnY} values were computed. In particular, Kaklamanos et al. (2013) and Rodriguez-Marek et al. (2011) evaluated site response effects and ϕ_{lnY} relatively directly from vertical array recordings, whereas Lin et al. (2011) used surface records for which other effects (non-site) may affect the residuals from which ϕ_{lnY} was computed.

In Figure 3, we re-plot results from the simulation-based studies along with the approximate band of results from data-based studies (from Figure 2). The trends from these two groups of studies are significantly different. The simulation-based studies overestimate ϕ_{lnY} at short periods. As shown in Stewart et al. (2014), the overestimation of ϕ_{lnY} at short periods occurs for both weak and strong motions, and hence is not likely related to over-randomization of the MRD curves (which do not affect significantly ground motion prediction for weak motions). Moreover, because the overestimation occurs for studies that did not consider input motion variability, it appears that the overestimation of ϕ_{lnY} at short periods is likely due principally to over-randomization of the V_S profiles. In light of these findings, it would be worthwhile to re-visit the randomization scheme provided by Toro (1995), or at the very least, to use the site-specific version of the randomization scheme in lieu of the generic version.

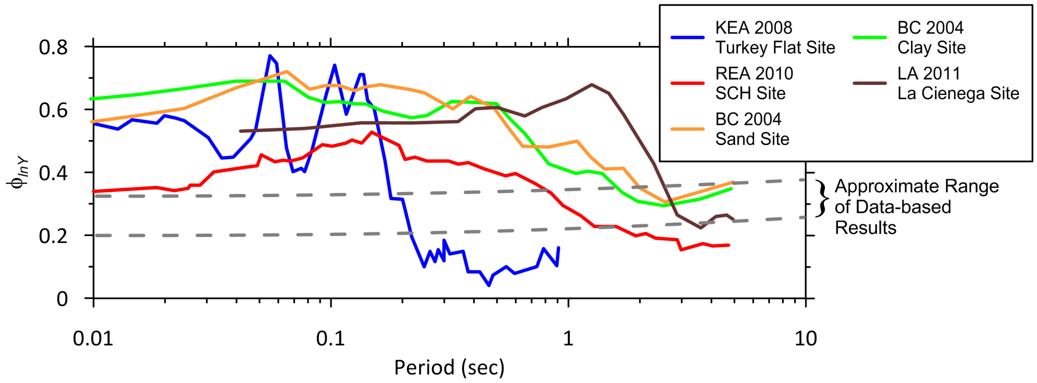

Figure 3. Difference between simulation-based studies and data-based studies on ϕ_{lnY}.

Another difference between the two categories of studies is underestimation of ϕ_{lnY} at long periods (beyond the site period) by the simulations. At these long periods, seismic quarter-wavelengths become long relative to the profile thickness, so there is little site response and hence little variability in site response. In reality site effects at these long periods are associated with physical processes that are not captured by 1D GRA, such as surface waves and various basin effects. Variability in those processes control long-period ϕ_{lnY} from data-based studies.

Conclusions

Standard deviations of site amplification computed directly from GRAs are considered unreliable—the values are generally too high below the fundamental period of the analyzed soil column and too low above that period. The overestimation at short periods may reflect problems in the randomization of shear wave velocity, which may not have been tailored to address true site-specific issues. This requires further investigation to reach a definitive conclusion. The underestimation at long periods occurs because GRA is unable to capture the physics of site response in that period range. For those reasons, we recommend the use of ϕ_{lnY} inferred from ground motion data analysis, which indicates remarkable consistency at $\phi_{lnY} \approx$ 0.25-0.3 over a broad range of oscillator periods. These values of standard deviation are useful for evaluation of within-event standard deviation of ground motion when GRA are used to replace the site term in a GMPE, as shown in Equation (7).

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