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Macroelement Modeling for Single Vertical Piles

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ABSTRACT

In geotechnical engineering, macro-elements became recently very popular for their simplicity, efficiency and accuracy. This paper presents a new macroelement for single piles in sand (with pile-head on the ground surface) developed within the framework of hypoplasticity. The incremental nonlinear constitutive equations are as usual defined in terms of generalized forces, displacements and rotations. The new macroelement is based on the general approach proposed by Salciarini and Tamagnini (2009) and the concept of "internal displacement" introduced after Niemunis and Herle (1997), necessary for cyclic loadings. After a presentation of the 3D failure envelope of a single pile in sand and the calibration of the macroelement parameters, comparisons with available centrifuge experimental results under static monotonic and cyclic loadings show the performance of this new numerical tool. The proposed macroelement remains simple and computational fast and therefore is suitable for numerical parametric studies and engineering design.

Introduction

In the field of geotechnical modeling of shallow foundations, different macroelement models can be found in the literature. Nova and Montrasio (1991) first introduced the macroelement to simulate the behavior of a perfectly rigid strip footing resting on a frictional soil. Paolucci (1997) proposed a perfect plastic model with a non-associated flow rule suitable for seismic calculations. Grange et al (2009), inspired from the macroelement of Crémer et al (2001), used the plasticity theory of multi-mechanisms to efficiently couple plasticity with an adequate overturning mechanism (uplift). Salciarini and Tamagnini (2009) introduced a hypoplastic macroelement for shallow foundations suitable for monotonic and cyclic loading. Recently, Correia (2011) introduced a plasticity theory-based macroelement for a single concrete pile in cohesive soil. Coupling is considered in the horizontal force overturning moment space but not with the vertical force.

This paper presents a new macroelement for single piles, with pile-head on the ground surface, developed within the framework of hypoplasticity. The macroelement adopts the 3D failure envelope for a single pile in Fontainebleau sand (NE34) proposed by Li et al (2014) and a simple isotropic hardening law to define the coupled plastic responses in the three directions. The incremental nonlinear constitutive equations are defined in terms of generalized forces, displacements and rotations. The macroelement is inspired from the macroelement for shallow foundations of Salciarini and Tamagnini (2009), adopts the "internal displacement" concept mutuated from Niemunis and Herle (1997) to reproduce the behavior under cyclic loading and it

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is implemented into the finite element Matlab toolbox FedeasLab (Filippou and Constandines, 2004). The calibration of the model parameters is presented. Comparisons with experimental centrifuge test show the good performance of the new macroelement.

Hypoplastic Macroelement Formulation

Constitutive Equations of Hypoplasticity

The mechanical response of the pile foundation system is described by means of generalized load vector t (Equation 1) and a generalized displacement vector u:

$$t := \{V, H, M\}^T \tag{1}$$

$$u := \{w, u, \theta\}^T \tag{2}$$

where H and V are the resultant forces (horizontal and vertical) and M the moment acting on the pile head; w, u and θ are the respective displacements and rotations. The generalized velocity vector d is then introduced:

$$d := \dot{u} \tag{3}$$

The basic structure of the hypoplastic macroelement in rate-form reads (Niemunis, 2002):

$$\dot{t} = \mathcal{K}(t, q, \eta) \tag{4a}$$

$$\mathcal{K} = \mathcal{L}(t, q) + N(t, q)\eta^T \tag{4b}$$

$$\eta = d/\|d\| \tag{4c}$$

where q is a pseudo-vector of internal variables accounting for the effects of the previous loading history. In the basic form of hypoplasticity presented by Equation 4, the model is suitable only for monotonic loading. If, following the work of Niemunis and Herle (1997), the "internal displacement" (ID) vector δ is introduced as an internal variable to extend the model to cyclic loading, the tangent stiffness matrix $\mathcal{K}$ in equation (4b) is modified as follows:

$$\mathcal{K}(t, q, \delta) = [\rho^\chi m_T + (1 - \rho^\chi)m_R]\mathcal{L} + \tilde{\mathcal{K}}(t, q, \delta) \tag{5}$$

$$\tilde{\mathcal{K}}(t, q, \delta) = \begin{cases} \rho^\chi(1 - m_T)(\mathcal{L}\eta_\delta)\eta_\delta^T + \rho^\chi N\eta_\delta^T & \text{if: } \eta_\delta^T\eta > 0 \\ \rho^\chi(m_R - m_T)(\mathcal{L}\eta_\delta)\eta_\delta^T & \text{otherwise} \end{cases} \tag{6}$$

where χ, m_T, m_R are model constants, and:

$$\eta_\delta = \begin{cases} \delta/\|\delta\|, & \delta \neq 0 \\ 0, & \delta = 0 \end{cases} \tag{7}$$

The evolution rate of the internal displacement (ID) is defined as:

$$\dot{\delta} = \begin{cases} \left(I - \rho^{\beta r}\boldsymbol{\eta}_\delta\boldsymbol{\eta}_\delta^T\right)\boldsymbol{d} & \text{if: } \boldsymbol{\eta}_\delta^T\boldsymbol{\eta} > 0 \\ \boldsymbol{d} & \text{otherwise} \end{cases} \tag{8}$$

where the scalar $0 \leq \rho \leq 1$ is the normalized magnitude of $\boldsymbol{\eta}_\delta$: $\rho = \|\boldsymbol{\eta}_\delta\|/R$, β_r and R are model constants and $\boldsymbol{I}$ is the identity matrix.

Elastic stiffness matrix $\boldsymbol{\mathcal{L}}$

The matrix $\boldsymbol{\mathcal{L}}$ that accounts for the stiffness at a load reversal point is defined as:

$$\boldsymbol{\mathcal{L}} = \frac{1}{m_R}\boldsymbol{\mathcal{K}}^e \tag{9}$$

$$\boldsymbol{\mathcal{K}}^e := \begin{bmatrix} k_v & 0 & 0 \\ 0 & k_{hh} & k_{hm} \\ 0 & k_{hm} & k_{mm} \end{bmatrix} \tag{10}$$

where $\boldsymbol{\mathcal{K}}^e$ is the elastic stiffness matrix and k_v, k_{hh}, k_{mm} and k_{hm} the vertical, horizontal, rotational and coupled horizontal-rotational stiffness of the foundation system. Different from the macroelement for shallow foundations of Grange et al (2009), coupling between horizontal forces and moments has to be considered, as the off-diagonal coupling terms are no more negligible in the case of single piles.

Constitutive vector N

The constitutive vector $\boldsymbol{N}$ can be expressed as:

$$\boldsymbol{N}(\boldsymbol{t}) = -Y(\boldsymbol{t})\,\boldsymbol{\mathcal{L}}\,\boldsymbol{m}(\boldsymbol{t}) \tag{11}$$

The scalar function $Y(\boldsymbol{t}) \in [0,1]$ accounts for the degree of nonlinearity, as it depends on the normalized distance $\xi \in [0,1]$ of the current stress state to the failure envelope, and is given by:

$$Y(\boldsymbol{t}) = \xi^\kappa \tag{12}$$

where κ is a model constant that controls the hardening behavior of the macroelement. The unit vector $\boldsymbol{m}(\boldsymbol{t})$ defines the direction of plastic flow (the normal direction of the loading surface at the current stress state point). An associated plastic flow rule is adopted and finally, the direction of the plastic flow $\boldsymbol{m}(\boldsymbol{t})$ is provided by:

$$\boldsymbol{m}(\boldsymbol{t}) = (\partial f/\partial t)/\|\partial f/\partial t\| \tag{13}$$

where f is the function defining the failure locus (or bounding surface) for the single pile in sand, as proposed by Li et al (2014):

$$f = 1.0\left(\frac{H}{H_0}\right)^2 + 1.0\left(\frac{M}{M_0}\right)^2 - 1.5\left(\frac{H}{H_0}\right)\left(\frac{M}{M_0}\right) - \left[1.0 - \left(\frac{V}{V_0}\right)^2\right] \tag{14}$$

In eq. (14), the scalar quantities H_0, M_0 and V_0 (to be interpreted as V_{c0} when the pile is loaded in compression and V_{t0} when loaded in tension) are the horizontal, bending and vertical bearing capacities of the pile, respectively.
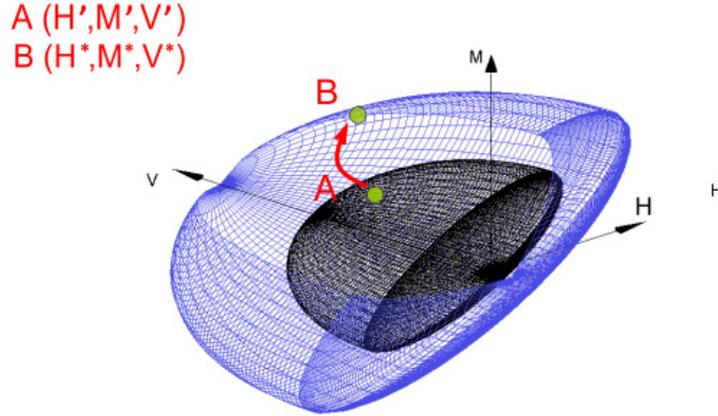

Figure 1. Evolution of the loading surfaces in the H − M −V space

A representation of the evolution of the loading surfaces is shown in Figure 1. Suppose that the current stress point A (H', M', V') lays on the loading surface described by Equation 15. This surface is homothetic to the failure surface, on which an image point B, of coordinates (H^*, M^*, V^*), can be found. For the current and image loading points we have:

$$Point\ A: f = 1.0 \left(\frac{H'}{\xi H_0}\right)^2 + 1.0 \left(\frac{M'}{M_0}\right)^2 - 1.5 \left(\frac{H'}{H_0}\right)\left(\frac{M'}{M_0}\right) - \left[1.0 - \left(\frac{V'}{V_0}\right)^2\right] = 0 \tag{15}$$

$$Point\ B: f = 1.0 \left(\frac{H^*}{\xi H_0}\right)^2 + 1.0 \left(\frac{M^*}{M_0}\right)^2 - 1.5 \left(\frac{H^*}{H_0}\right)\left(\frac{M^*}{M_0}\right) - \left[1.0 - \left(\frac{V^*}{V_0}\right)^2\right] = 0 \tag{16}$$

From the assumption of homothety, we can set $(H', M', V') = \xi(H^*, M^*, V^*)$, with $\xi \in [0,1]$. Then eq. (15) provides the current value of the normalized distance ξ for any loading state. When the current state is on the failure surface, $\xi = 1$ (i.e. $H' = H^*, M' = M^*, V' = V^*$).

Calibration of the Macroelement Constants

The first three parameters of the macroelement, H_0, M_0 and V_0 are linked with the bearing capacity of the pile, i.e., they control the size of the failure surface. For a pile of diameter $D = 0.72$ m, length $L = 13$ m, in homogeneous dry Fontainebleau sand (Rosquöet, 2004), the following values have been proposed by Li et al (2013) and Li et al (2014), after extensive experimental and FEM numerical studies: $H_0 = 0.5\times10^4$ kN, $M_0 = 0.45\times10^5$ kN·m and $V_0 = 2.5\times10^4$ kN (0.5×10^4 kN in tension).

The stiffness constants k_v, k_{hh}, k_{mm} and k_{hm}, can be determined using either a linear elastic FEM model that takes into account the evolution of the soil's elastic properties with depth, or with a more advanced constitutive law (both methods have been used in this paper), by imposing

adequate displacements on the pile-head and evaluating the corresponding reactions. Figure 2 illustrates the general procedure:

a) The horizontal k_{hh} and the coupling stiffness k_{hm} are obtained by applying a small horizontal displacement on the pile head while keeping its rotation fixed.

b) In a similar way, k_{mm} and the coupling stiffness k_{mh} are obtained by applying a small rotation on the pile head while keeping the horizontal displacement fixed (the coupling terms k_{hm} and k_{mh} should be equal).

c) k_v is finally obtained by applying a small vertical displacement on the pile-head.
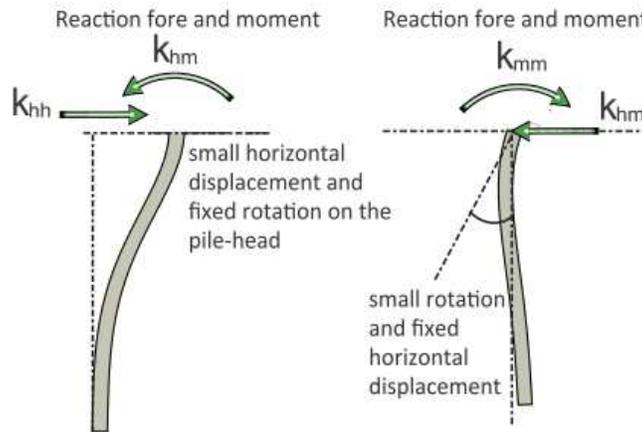

Figure 2. Determining the initial stiffness parameters of the hypoplastic macroelement

For the case at hand, the following values have been adopted: k_{hh} = 2.39×10^5 kN/m, k_{m} = 1.92×10^6 kN·m/rads, k_{hm} = 5.78×10^5 kN/m and k_v = 1.45×10^5 kN/m.

The hardening parameter κ controls the degree of nonlinearity, i.e. the way the current loading state approaches the final failure state. A monotonic deformation path of sufficient length is therefore needed to calibrate it. This kind of test data being unavailable, the results of the FEM model with the advanced hypoplastic constitutive law have been adopted for the calibration (red line in Figure 3). Parametrical studies with different κ are performed with the hypoplastic macroelement (black lines in Fig. 3). In performing the simulations with the macroelement, the constants controlling the cyclic response of the pile have been assigned the values proposed by Salciarini and Tamagnini (2009) for shallow foundations as a first approximation: m_R = 5.0, m_T = 2.0, R = 1.0×10^{-4}, β_r = 0.5 and χ = 1.0. A more accurate calibration of these constants has been performed subsequently, using cyclic test data. The results shown in Figure 3 provide an optimal value κ = 1.2.

Finally, cyclic test data are needed to calibrate the internal displacement constants of the macroelement. A good calibration of these constants is essential to get an accurate stiffness at the loading reversal points and to restrain accumulation of excessive deformations. For this purpose, we have used again the cyclic tests performed by Rosquöet (2004). Adopting a trial-error method it is found that the cyclic response of the macroelement is not very sensitive to the choice of internal displacement constants. The final adopted values are: m_R = 5.0, m_T = 2.0, R = 6.0×10^{-3},

$\beta_r = 0.5$, $\chi = 0.5$. The complete set of model constants for the hypoplastic macroelement for a single vertical pile in Fontainebleau sand are summarized in Table 1.
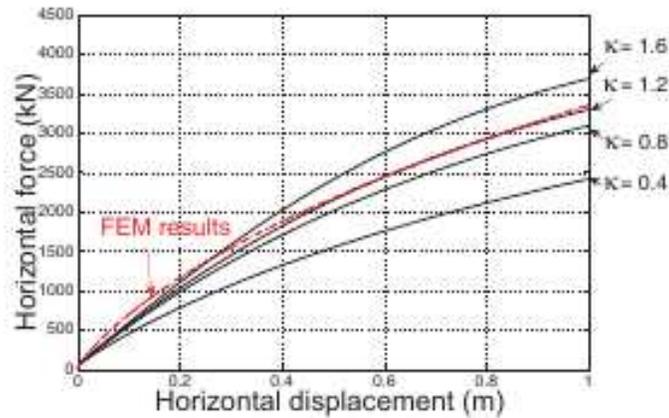

Figure 3. Calibration of macroelement's hardening parameter κ. Dashed curve: monotonic loading path from the FEM model using the advanced hypoplastic constitutive law, solid curves: monotonic loading paths calculated with the hypoplastic macroelement

Table 1. Calibrated parameters of the hypoplastic macroelement for a single vertical pile in Fontainebleau sand using a FEM model

Parameter	Value
H_0	0.5×10^4 kN
M_0	0.45×10^5 kN·m
V_0	2.5×10^4 kN
k_v	1.45×10^5 kN/m
k_{hh}	2.39×10^5 kN/m
k_{mm}	1.92×10^6 kN·m/rads
κ	1.2
m_R	5.0
m_T	2.0
R	6.0×10^{-3}
β_r	0.5
χ	0.5

The macroelement parameters were calibrated using a finite element model, Li et al (2015). However and for the practical applications, empirical equations can be used for some of them. The constants H_0, M_0 and V_0 of the failure locus can be estimated using the formulas introduced

by Meyerhof and his co-workers (Meyerhof el al., 1983, 1988, 1989). The stiffness parameters k_{hh}, k_{mm} and k_{hm} can be evaluated using the formulas proposed by Gazetas (1991) or those reported in Eurocode 8–part5 (Eurocode 8, 2003). As for the parameters κ, m_R, m_T, R, β_r and χ, due to the lack of information in the literature, the values listed in Table 1 could be used as a first approximation.

Validation of the Macroelement

In order to validate the performance of the calibrated hypoplastic macroelement, new cyclic test data have been simulated, considering the same material and same geometrical characteristics for the pile. The pile is now laterally loaded using one-way or two-way cyclic forces with a different amplitude (Rosquöet, 2004). Rosquoët (2004) applied monotonic and cyclic horizontal loadings on a single vertical pile embedded in dense Fontainebleau sand submitted to a centrifuge gravity level of 40 g (g being the gravity acceleration). The diameter of the pile is 0.72 m (prototype) and its slenderness (length/diameter) ratio is 15. The pile has a section stiffness of E_pI_p=2638 $\times 10^6$ N•m^2. The comparison of the experimentally observed behavior with the numerical predictions obtained with the macroelement is provided in Figures 4 and 5. It is apparent that the macroelement captures quite well the behavior of the pile under the different loading conditions considered. Although the calibration of the parameters was done using only the horizontal force-displacement curve (section 3), the results concerning the rotations are also relatively good (see column (c) in Figures 4 and 5). The coupling in the observed response in the horizontal and rotational degrees of freedom is therefore well captured by the macroelement.
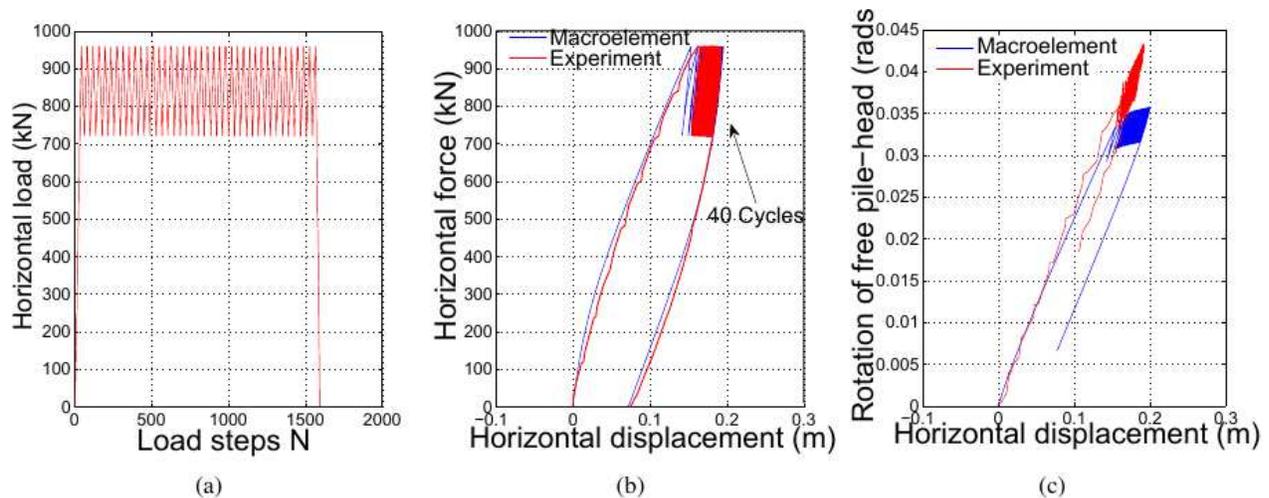

Figure 4. Validation of the hypoplastic macroelement for a single vertical pile (calibrated with a FEM model and cyclic tests): one-way cyclic loading, 12 cycles, amplitude = 240 kN, experimental data from Rosquöet (2004): (a) loading; (b) pile-head horizontal displacement-force curve; (c) pile-head horizontal displacement-rotation curve.
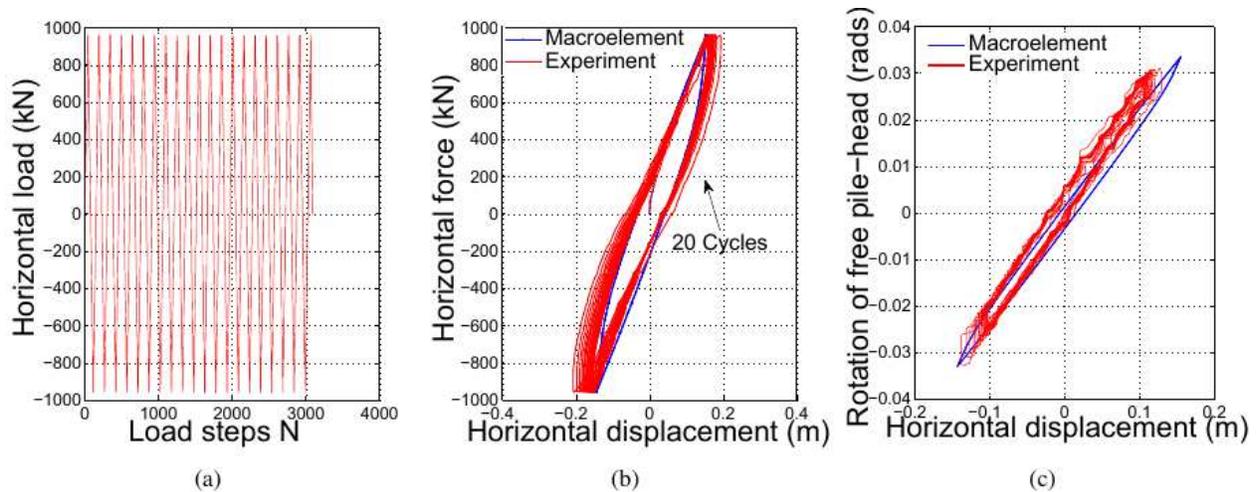
Figure 5. Validation of the hypoplastic macroelement for a single vertical pile (calibrated with a FEM model and cyclic tests): two-way cyclic loading, 20 cycles, amplitude = 1920 kN (experimental data from Rosquöet (2004)): (a) loading; (b) pile-head horizontal displacement-force curve; (c) pile-head horizontal displacement-rotation curve.

Conclusions

This paper presents a novel hypoplastic macroelement for single piles in sand with pile head at the ground surface. The mathematical formulation of the model is simple and the different model constants can be calibrated in a (relatively) simple way, as shown for the present case. Comparisons with experimental results demonstrate the good performance of the macroelement for the different loading cases considered. Compared to continuum-based approaches, the proposed macroelement has the advantage of being simple and computationally much more efficient. In addition, it captures well the coupling between the different degrees of freedom, which cannot be simulated using the classical "Beam on Non–linear Winkler Foundation" (BNWF) or p-y method. The proposed macroelement is therefore suitable for finite element calculations and engineering design purposes. A more detailed presentation is given in Li et al (2015).

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