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DESIGN AND SIMULATION OF A CONTROL SYSTEM OF A SHAKING TABLE FOR SOIL MODELS IN CENTRIFUGAL FORCES

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ABSTRACT

This article presents the non-linear mathematical model of a one degree of freedom electro-hydraulic shaking table to simulate earthquakes in reduced scale models in centrifuge. Concerning the control system the problem presents two challenges: The first one is that when a body is exposed to a certain centrifugal force to simulate gravity on it, the time and the magnitude of the displacement should be divided by the number of gravity (N) that is being applying to get a model N times bigger; and the second one is that the friction in the servo valves will increased. Therefore, the hydraulic system will require a high bandwidth. The model is used to design two techniques of feedback control: Linear Quadratic Gaussian with loop transfer recovery (LQG-LTR) and Model Predictive Control (implemented via filters RST). Finally, Simulations with the non-linear model are presented for the earthquakes of Umbria Italia in 29-04-1984, Mammoth Lakes - Aftershock U.SA in 26-05-1980 and Mexico in 19-09-1985, These simulations are used to compare the feedback techniques using a performance indicator, which is in this case, the sum of quadratic error between the magnitude of the spectrum of the earthquake and the one obtained by the System. The results will help to find the bandwidth limits, possible changes in the model using different gravities, and the robustness of the controllers

Keywords: shaking table, earthquake, Scale Models, electrohydraulic , LQG-LTR, GPC-RST

INTRODUCTION

The behavior of the soils under dynamic conditions like earthquakes is mathematically complex to predict because it requires partial differential equations and the knowing of several non linear parameters. Furthermore trying to study its behavior in real scale is usually not possible. For this reason using reduced scale models in centrifuge is a useful tool to study the behavior of geotechnical structures during earthquakes.

The mechanical behavior of soils is controlled mainly by stresses acting on it, particularly the geostatic stress. Therefore to simulate soil behavior is a reduced scale model it is necessary to reproduce it geostatic stress. One of the techniques to do so is growing artificially the gravity field in the same proportion of the scale reduction (using centrifugal acceleration). Then it is necessary to generate in the reduced scale model an acceleration field using a centrifugal machine that generates a force to simulate gravity (1)

$$a = \omega^2 R = Ng \tag{1}$$

Where, a is the centrifugal acceleration, ω is the angular velocity in rad/s, R is the radius between the center of the body and the rotation axis, N is a scale factor and g is the gravity in m/s^2.

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When the acceleration is applied on a reduced model, N should affect some physical variables to get a prototype [1], these similitude relationships for different variables are shown in the Table 1. Concerning earthquakes, it is necessary that the reduced model has a centrifugal acceleration Ng with a displacement signal divided by N in magnitude and N times faster to simulate a model N times bigger.

Table 1 Scale factor necessary to get a scale model. [1]

Variable	Reduced Model	Prototype
Acceleration	N	1
Density	1	1
Velocity	1	1
Length	1	N
Area	1	N^2
Volume	1	N^3
Force	1	N^2
Energy	1	N^3
Time (dynamic)	1/N	1

This paper is divided into four sections: the first one focuses on the development of the mathematical model of a shaking table that the laboratory of Geothecnical model of the Andes University is building, section two presents the control techniques linear quadratic Gaussian and model predictive control, the next section discusses the result of the simulation comparing it with a performance indicator and last section presents the conclusions of this work.

1 DOF SHAKING TABLE MATHEMATIC MODEL

The mathematic model that will be described is based on the models presented in [2][3][4][5][6][7][8] and the machine that the laboratory of Geothecnical model of the Andes University is building.

As a first approach, it is assumed that the centrifugal acceleration is not affecting the model neither in the hydraulic part nor in the electrical; also it is considered that the delay between the signals generated in the computer and the movement is negligible; finally it is considered that the pipes among the servo-valve are components of the hydraulic actuator. Consequently the model can be described by means of the block diagram shown in Figure 1.

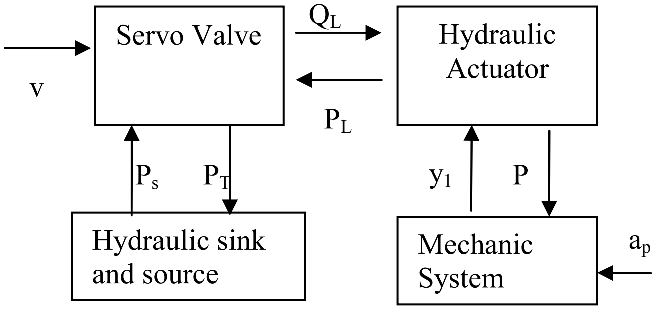


Figure 1 Block diagram for the mathematical model

Servo-Valve

There are three dynamics equations that govern the servo-valve: the behavior of the torque of the motor (inside the servo-valve), the movement equation of the piston and the turbulent fluid equation through an orifice. However, as it is a closed system, it is limited to the specifications given by the manufacturer. Some servo-valve manufacturers as indicated by [4] and [3] provide bode charts between input voltage and position of the piston that could be approached to a second order system. However the manufacturer of the Servo-valve used in this project (Vickers) does not provide these charts, but instead they provide others that are defined as the relationship, without load, between the output flow and a current signal. This chart is given for a specific flow showing the linear features of this relationship. Hence, a second order prototype was used, for which a non linear function was added. This function represents the friction of the piston, which is expected to increase as the acceleration, in addition the friction in the piston increases as well. The system is represented in state variables as follows:

$$\dot{x} = \begin{pmatrix} x_2 \\ -w_n^2 x_1 - 2\xi w_n x_2 - f_f(x_2) \end{pmatrix} + \begin{pmatrix} 0 \\ w_n^2 \end{pmatrix} u_1 \qquad (2)$$

$$y = x_v = x_1$$

Where x_1 is the position of the piston of the valve, ξ the damping factor, w_n the natural frequency and $f_f(x_2) \approx \beta w_n^2\left(1 + 0.05e^{-200|x_2|}\right)\mathrm{sign}(x_2)$, being $\beta = 0.0005u_2$ where u_2 represents the centrifugal acceleration.

Finally, the output flow in the servo valve (Q_l) assuming that the flow through it is turbulent is:

$$Q_l = c_v x_v \sqrt{2}\sqrt{p_s - p_t - sign(x_v)p_l} \qquad (3)$$

Where p_s is the input pressure, p_t the pressure of the sink, p_l the output differential pressure and c_v represents the valve constant that depends on its geometry and the properties of the flow such as the Reynolds number and density.

Hydraulic power source and reservoir

The supply source and recirculation of the hydraulic fluid is composed of an air controlled hydraulic pump which is supported by an accumulator that was pre-charged. Therefore, an ideal source is assumed, running on a (3000 psi) constant pressure coming from the accumulator. It is also assumed that the pipe that connects this part of the system with the servo-valve does not affect the system **[4]**.

Hydraulic Actuator

Basically, the hydraulic actuator is a piston that has an input and an output cavity where a hydraulic flow circulates in and out. This flow is controlled by a servo-valve. Therefore, if it is assumed that there are no leaks, the actuator is symmetrical and the elasticity module of the flow is constant, then the continuity equation is the following:

$$Q_i = 2A\dot{y}_1 + \frac{V}{E}\dot{P}_l \qquad (4)$$

Where y_1 is the displacement of the piston inside the actuator , A is the transversal area of the cylinder, E is the elasticity module of the fluid and V represents the inner volume of the actuator that is assumed constant. Replacing (3) in (4) :

$$\frac{V_T}{E}\dot{p}_l = \left(c_v x_v \sqrt{2}\sqrt{p_s - p_t - sign(x_v)p_l} - 2A\dot{y}_1\right) \tag{5}$$

Mechanical System

The mechanical System (Payload) is a laminar box made by 20 aluminium plates confining the soil (ie. Sand) . The plates can move one over the others. As a first approach the shear stress developed in the sand pile is modeled as an elastic spring [9] [10], then the representation of the mechanical system in state variables uses the following state variables: $x_1 \equiv y_1$, $x_2 \equiv \dot{y_1}$, $\mathrm{x_{2n-1}} \equiv \mathrm{y_n}$, $\mathrm{x_{2n}} \equiv \dot{\mathrm{y}}_\mathrm{n}$, $u_1 \equiv \mathrm{P}$ and $u_2 \equiv \omega^2 r$. With the following state equations (6):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{2n-1} \\ \dot{x}_{2n} \\ \vdots \\ \dot{x}_{2N+1} \end{bmatrix} = \begin{bmatrix} x_2 \\ -\left(\omega^2 + \frac{k}{m}\right)x_1 + \frac{k}{m}x_3 + u_2 + \frac{A}{m}u_1 - \frac{C_v}{M}x_2 \\ \vdots \\ x_{2n} \\ -\left(\omega^2 + \frac{2k}{m}\right)x_{2n-1} + \frac{k}{m}x_{2n-3} + \frac{k}{m}x_{2n+1} + u_2 \\ \vdots \\ -\left(\omega^2 + \frac{k}{m}\right)x_{2N-1} + \frac{k}{m}x_{2N-3} + u_2 \end{bmatrix} \tag{6}$$

Where $\mathrm{y_n}$, represents the position of each soil plate, k is the spring constant that emulates the elastic deformation of the soil pile before cutting loads, m represents the mass of each soil plate y N is the quantity of soil plates.

The constant k that is used to model the elastic deformation under shear stresses, is not an accurate approximation since soil behavior is by far more complicated than an ideal elastic behavior, as a consequence there could appear non linear phenomena that are difficult to model. The principle of this analog behavior to the deformation of solid materials is caused principally by the Coulomb friction that presents the sand particles between them [11]. These particles have different sizes and shapes, this condition makes difficult to simulate this behavior, as shown in the studies presented by [9] [10].

Though the simulation model is the one described until this moment, given the amount of state numbers for the design of the controller, all the mass that moves the actuator is assumed as a rigid body, therefore we have the following:

$$P(t)A + M\omega r^2 = M(\Delta\ddot{y}_1(t) - \omega^2(\Delta y_1(t))) + C_v\,\Delta\dot{y}_1(t) \tag{7}$$

Where,C_v is the viscous friction coefficient of the system caused by the speed of the mass taking account that the system displacement is made under a rail guiding system.

Therefore, the following model from (2), (5) and (7) is used in the controller:

$$\dot{\mathbb{x}} = \begin{pmatrix} x_2 \\ \omega^2 x_1 - \frac{C_v}{M} x_2 - \frac{A}{M} x_3 + u_2 \\ \frac{E}{V}\left(c_v x_4 x_{vmax}\ \sqrt{2}\sqrt{p_s - p_t - sign(x_4) x_3} - 2A x_2\right) \\ x_5 \\ -w_n^2 x_4 - 2\xi w_n x_5 + \frac{w_n^2}{i_{max}} i + f_f(x_4) \end{pmatrix} \tag{8}$$

Where, $x_1 \equiv y_1$, $x_2 \equiv \dot{y_1}$, $\mathrm{x}_3 \equiv \mathrm{P}_l$, $\mathrm{x}_4 = \mathrm{x}_\mathrm{v}$, $\mathrm{x}_5 = \dot{\mathrm{x}_\mathrm{v}}$, $u_1 \equiv \mathrm{v}$ y $u_2 \equiv \omega^2 r$.

Although the first model has more than forty states many of them are no controllable from u_1 which make very difficult to analyze because the parameter k is unknown. Figure 2 presents a comparison between the velocity of the two models from a step at the input showing that the response time and the DC value of the model are very similar.

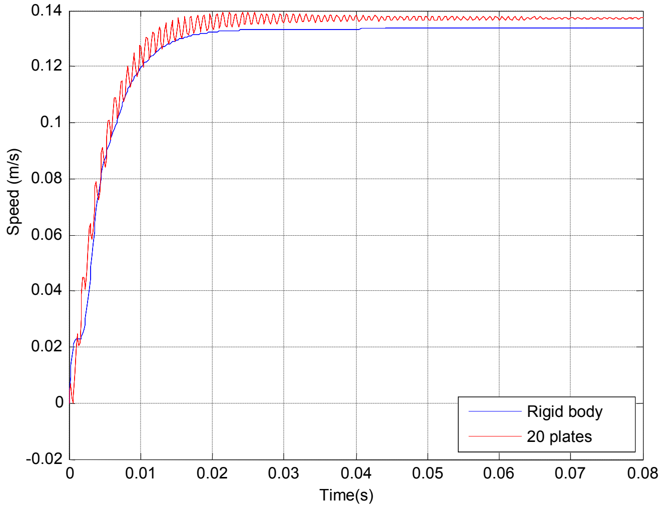


Figure 2. Comparison between the speed of the actuator displacement in the open loop system of the rigid body model and the 20 plate of soil model to step input of 100%.

Linearizing (8) regarding an operation point:

$$\dot{\mathbb{x}} = A_p \mathbb{x} + B_p \mathbb{u} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ \omega^2 & -\frac{C_v}{M} & \frac{A}{M} & 0 & 0 \\ 0 & -\frac{2AE}{V} & \frac{K_c E}{V} & \frac{E}{V} K_x x_{vmax} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -w_n^2 & -2\xi w_n \end{pmatrix} \mathbb{x} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ w_n^2/i\_max & 0 \end{pmatrix} \mathbb{u} \tag{9}$$

Given that the output of the system is y_1

$$\mathbb{y} = C_p \mathbb{x} = (1 \quad 0 \quad 0 \quad 0 \quad 0)\mathbb{x} \tag{10}$$

CONTROLLERS DESIGN

The main parameter to design the controllers is the bandwidth. The reason for that is that the system should be able to reproduce an earthquake forty times faster (40G). This gives a bandwidth of more than 150 Hz, which is not possible because the open loop system has a bandwidth between 60 and 100 Hz.

Another technique that was applied for the problem was feedback linearization [12][13] which is proposed in *[4]*, but this presents many problems in simulation with ODE45 or ODE15s[14] when the friction function is used, hence, this result is not presented.

LQG-LTR

LQG method consists of a state estimator plus a matrix of feedback state gains calculated through the principles of minimization of the rule two of an objective function. As it can be found in [15], it is possible to apply the separation principle in such a way to design first a state estimator sort of Kalman filter. A matrix of state noise covariance is going to be applied on this filter in order to tune the controller to a fixed bandwidth range and with a minimum over peak.

After having obtained the desired performance, the next step is to design the state feedback (LQ) using the Loop Transfer Recovery Technique. With this technique, it is guaranteed that the design of the shaking table in the state estimator keeps the same behaviour. The procedure is the following:

1. Model with Integrator.
   An integrator is added to the equations system (8) regarding the error, hence, the new equation system is (11).

$$\begin{gathered}\dot{\mathbb{x}} = A_a\mathbb{x} + B_a\mathbb{u} = \begin{pmatrix} A_p & 0 \\ -C_p & -0.001 \end{pmatrix}\mathbb{x} + \begin{pmatrix} B_p \\ 0 \end{pmatrix}\mathbb{u} \\ \mathbb{y} = C_a\mathbb{x} = (C\_p \quad 0)\mathbb{x}\end{gathered} \tag{11}$$

Where the integrator is a pole in –0.001, to assure the convergence of the design algorithm of the controller

2. *Design of the Observer by means of a Kalman filter.*
   The Kalman observer is used since there is a Gaussian noise in both the states and measured signal that in our case is the load displacement.

$$\begin{gathered}\dot{\mathbb{x}} = A_a\mathbb{x} + B_a\mathbb{u} + G\mathbb{w} \\ \mathbb{y} = C_a\mathbb{x} + \mathbb{v}\end{gathered} \tag{12}$$

Where $\mathbb{w}$ is the noise of the state variables and $\mathbb{v}$ is the noise of measure outputs. Assuming a covariance matrix:

$$E\{ww'\} = Q, \quad E\{vv'\} = R \; and \; E\{wv'\} = 0$$

It is useful to obtain an estimated value of the state vector, knowing the output and input of the system. In order to achieve that, it is necessary to minimize the quadratic rule of the error between the real value and the estimated. In this case we are going to use the Matlab LQE algorithm that generates a state feedback matrix (K_f) obtaining the following system (13).

$$\dot{\mathbb{x}}_e = A_a\mathbb{x}_e + B_a\mathbb{u} + K_f(y - C_a\mathbb{x}_e) \tag{13}$$

Choosing:

$$G = \begin{bmatrix} C_p \\ 1 \end{bmatrix} \qquad R = 1$$

Q value is tuned to obtain an approximated system bandwidth of 100rad/s, generating the sensibility and complementary sensibility shown in Figure 3.

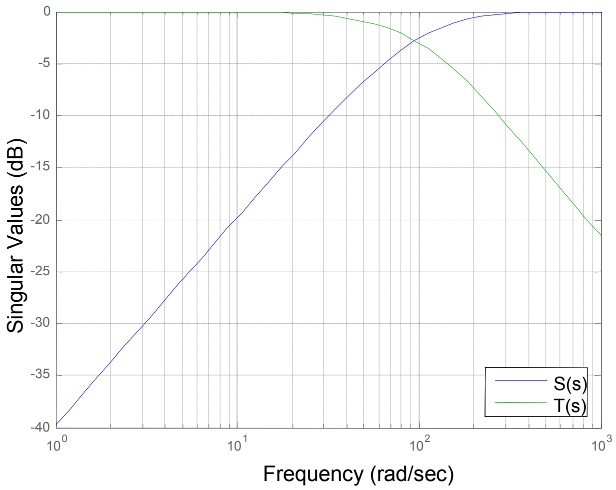


Figure 3. Bode of the sensivity function(S(s)) and complementary sensivity of the Kalman's feedback Filter

Using the loop gain $L(s) = G(s)K_f$ this can be analyzed in a simpler way, showing that while the frequencies are low, error tends to cero and while frequencies are high, a high rejection to this kind of signal appears. In state variables $L(s)$ is represented by the matrices $A_{l(s)} = A_a \quad B_{l(s)} = K_f \quad C_{l(s)} = C_a$

3. *Design of the feedback state matrix (LQ).*
   As it was mentioned before, the calculation of the feedback state matrix $(\mathbb{u} = -K_c\mathbb{x})$ tries to minimize an objective function that is the following:

$$J = \int_0^\infty (\mathbb{x}^T Q \mathbb{x} + \mathbb{u}^T R \mathbb{u})dt \qquad (14)$$

Where, Q and R are weight matrices that are useful to differentiate the significance of the outputs and inputs of the system. In this case it is selected:

$$Q = C_a^T C_a \qquad R \rightarrow \rho$$

In order to achieve that the full system (LQG+ shaking table) has a behavior similar to that of a Kalman filter, R goes to zero, therefore, with $\rho = 10^{-10}$ both responses are similar in the bandwidth (Figure 4), However, R is given the value of $\rho = 10^{-8}$,because the poles of K(s) tend to infinite which would be difficult to discretize it. The representation in state variables of K(s) has the following matrices$A_{k(s)} = A_a - B_a K_c - K_f C_a$, $B_{k(s)} = K_f$ *and* $C_{k(s)} = -K_c$.

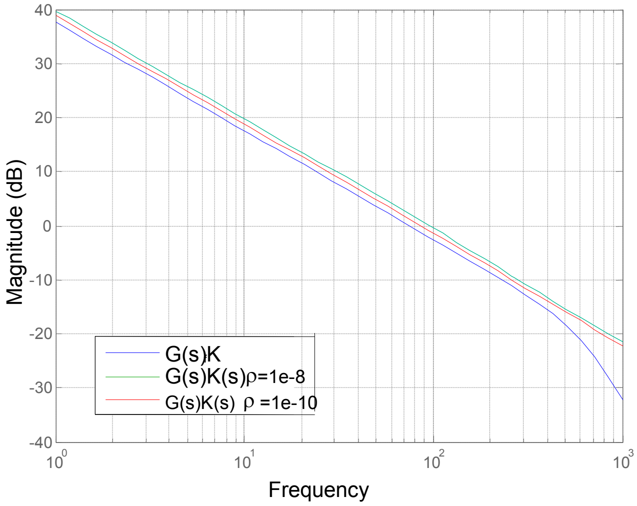


Figure 4. Magnitude of the Loop Gain $G(j\omega)K_f$ vs $G(j\omega)K(s)$ with different values of ρ

Model Predictive Control

The predictive control is a technique that allows estimating future control signals calculating the future value of the output signal from a model [16] . Although, the model can be generated by different ways like diffuse logic, no linear models, neural networks and transfers function [4] which presents a model predictive control based in fuzzy logic model, this work used a classic approach of this techniques using a discrete transfer function of the plant. After the model is defined it is necessary to define the function to minimize, which can have restrictions in the input signals or in the output signals [17], [18] or the Model Predictive Control Toolbox of Matlab.

1. *The Model.*
   The Model selected was the linear state representation of (9) which is discretized and the input u_2 taken as 0 to get a discrete difference equation.

2. *Controller design.*
   The controller tries to minimize (15)

$$J = \sum_{k=N_1}^{N_2} e^2(k) + \sum_{0}^{N_2-1} \lambda \Delta u^2(k) \tag{15}$$

Where, e is the error between the reference and the output of the system is a scope of future time (N_k); Δu is the change in the input, which represent the energy consumed; λ is a weight factor between the error and the energy consumed, N_1 is the lower limit time of the horizon of prediction, which depends on the delay of the system; and N_2 is the higher limit time of the horizon of prediction, which depends on the response time of system.

Solving (15) is a optimization problem, which in this case is linear and is solved by using a filter structure known as RST (16), this algorithm was developed by one of the authors in Matlab.

$$u = \frac{1}{S(z^{-1})}\left(T(z^{-1})w - R(z^{-1})y\right) \tag{16}$$

Where, w is the reference signal and y is the output of the system. After some iterations the values selected were $N_u = 1$,$N_1 = 1$,$N_2 = 5$ and $\lambda = 1x10^{-6}$.

SIMULATIONS

This section presents the results in simulation of the controllers made in Simulink using ODE45 over the no linear model of the shaking table with the equations *(2), (5)* and *(7)*. The performance in this case is defined by the quadratic error between the magnitude of the spectrum of the reference and that is obtained in the output of the closed loop system (13).

$$I = 2\sum_{n=0}^{N}\left((|w(f)| - |y(f)|)\right)^2 T_c \;\therefore\; f = \frac{n}{2T_c} \qquad (17)$$

Where, $w(f) = \mathfrak{F}\{w(nT_c)\}$ is the spectrum of the reference, $y(f) = \mathfrak{F}\{y(nT_c)\}$ is the spectrum of the output and N is the horizon of the spectrum which depends on the bandwidth of the reference (Earthquake).

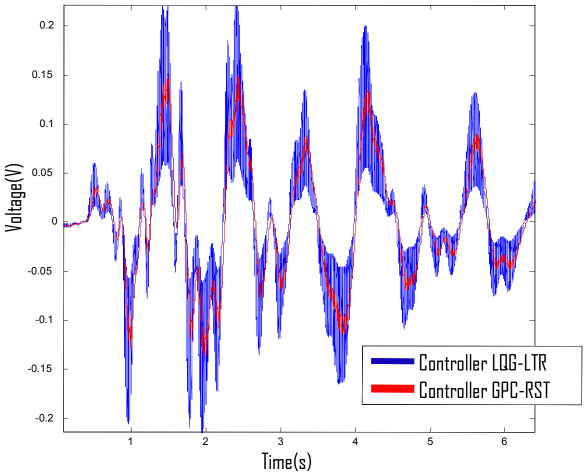
There were analyzed three different cases. The first without centrifugal acceleration under the system with three different reference signals: The earthquakes of Umbria –Italia of 19-04-1980, Mammoth Lakes - Aftershock U.SA of 26-05-1980 and Mexico City of 19-09-1985; results shown in Table 2. Although the results with the controller LQG-LTR are better than the other one, if the control signal is analyzed (Figure 5) the variation is higher in the LQG-LTR which can be dangerous to the life time of the machine.

Table 2. Results of the performance of the controller under different reference without centrifugal acceleration.

Controller	*Earthquake Umbria-Italia 19-04-1980*	*Earthquake Long Valley-USA 26-05-1980*	*Earthquake de México 19-09-1985*
LQGLTR	*3.34e^{-13}*	*2.58e^{-13}*	*6.42e-14*
GPC-RST	*3.07e-11*	*4.38e-11*	*5.1e-12*

Table 3. Results of the performance of the controller under different centrifugal accelerations without friction in the servo valve

Controller	*w(t) ap=5g*	*w(t/5) ap=5g*	*w(t/20) ap=20g*
LQGLTR	*1.64e-14*	*8.8e-12*	*7.923e-10*
GPC-RST	*1.06e-12*	*3.18e-10*	*15.66e-10*



FIGURE

Figure 5. Signal control of the two controllers reproducing a part of the earthquake of Umbria-Italia 19-04-1980 without acceleration centrifugal

The second at different centrifugal acceleration without the no linear friction in the servo valve with reference the earthquake of Mexico City of 19-09-1985 at the same magnitude in all case but multiplying the spectrum by the gravity; results are shown in

Table **3**.When the spectrum is bigger some attenuation in the performance is found as can be seen in the Figure 6

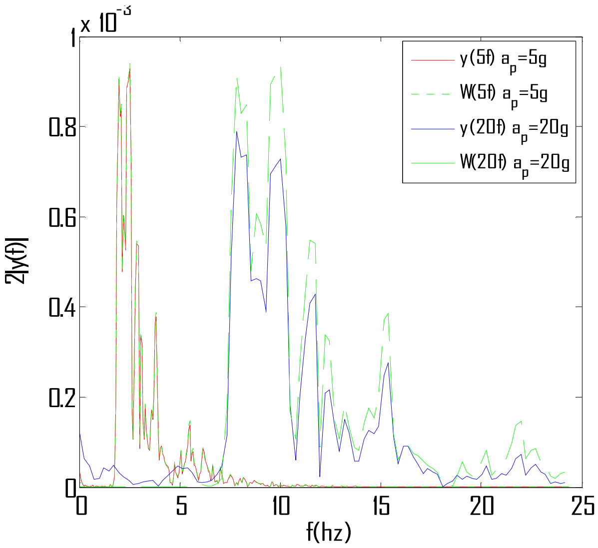


Figure 6. Spectrum magnitude of the reference and the output with different scales for the earthquake of Mexico city and the controller LQG-RST

The last one at different centrifugal acceleration with the no linear friction in the servo valve and reference the earthquake of Mexico City of 19-09-1985 at the same magnitude in all case but multiplying the spectrum by the gravity; results are shown in

Table **4**. The last column shows the result adding a signal to u known as Dither [19], which is a typical solution to compensate the no linear friction in the servo valve in electro-hydraulics control systems.

Table 4. Results of the performance of the controller under different centrifugal accelerations with friction in the servo valve

Controller	*w(t/10) ap=0g*	*w(t/10) ap=10g*	*w(t/10) ap=10g +Dither*
LQGLTR	*3.661e-12*	*2.696e-10*	*1.6963e-10*
GPC-RST *	*3.324e-14*	*1.798e-10*	*1.315e-10*
**λ is divided by10*			

CONCLUSIONS

- When the centrifugal acceleration is high the signal control is “stronger” because the system needs to respond quickly.
- The two techniques used let to obtain similar bandwidth value which is very high considering the open loop characteristics of the model.
- The controller GPC-RST allows tuning the compromise between performance and energy consumed with only one parameter (Q).
- The design method LQG-LTR can be a good tool because when the centrifugal acceleration increases the bandwidth required by the system increase too.

The preliminary experimental results are presented in [20] which has been developed in a embedded system CompactRio of National Instrument[21].

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