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DYNAMIC RESPONSE OF SANDY AND GRAVELLY SOILS: EFFECT OF GRAIN SIZE CHARACTERISTICS ON G-γ-D CURVES

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ABSTRACT

The paper presents recent results on experimental G/G_O-logγ curves of clean sandy and gravelly soils. In addition, damping ratio values are correlated to the corresponding G/G_O values using a simple two-order equation. Twenty-four torsional resonant column tests were performed on 16 dense to very dense and eight loose to medium dense specimens. River sand of rounded particles, and a quarry gravelly sand of angular particles have been used. The effect of shearing strain amplitude, γ, mean effective confining pressure, σ_m', and grain size characteristics in terms of mean grain size, D_{50}, and coefficient of uniformity, C_u, on the non-linear response of the specimens is examined. It is underlined that not only the parameter C_u but also the mean grain size D_{50} affects significantly the non-linear behavior of the specimens. The experimental results are also compared to theoretical curves and analytical relationships proposed in the literature. It is concluded that fine sands exhibit more linear behavior at high strains in comparison to medium and coarse sands, whereas sands generally exhibit more linear behavior compared to gravels.

Keywords: resonant column, shear modulus, damping ratio, sands, gravels, grain size characteristics

INTRODUCTION

The dynamic behavior of non-plastic soils (mainly sands and gravels) at medium to high strains has been widely investigated the last five decades. In common practice and concerning the dynamic response of soils under shear waves; this behavior is expressed in terms of the normalized shear modulus and damping ratio versus shearing strain amplitude (G/G_O-logγ and DT-logγ curves).

Many researchers have indicated that not only the shearing strain amplitude but also the mean effective confining pressure, σ_m', affects significantly the response of granular soils in the non-linear region (Sherif & Ishibashi, 1976, Iwasaki et al., 1978, Kokusho, 1980, 2004, Tanaka et al., 1987, Stokoe et al., 1999, Darendeli, 2001, Menq, 2003, Zhang et al., 2005). In general, as σ_m' increases, soils exhibit more linear behavior. The predominant linear behavior of soils under higher amplitudes of σ_m' is the result of the higher reference strain, γ_{ref}, due to the increment of shear strength, τ_m, and initial shear modulus, G_O or G_{max} (Ishihara, 1996).

Seed et al. (1986) and Rollins et al. (1998) based on their experimental cyclic triaxial tests as well as on the results of other researchers proposed mean and upper-lower G/G_O-logγ and DT-logγ curves for granular soils. In addition, Darendeli (2001) and Menq (2003), based on a modification of the hyperbolic model published at first by Hardin & Drnevich (1972), proposed analytical relationships for the determination of the non-linear curves of non-plastic and plastic soils as well as sandy and gravelly soils, respectively. Menq indicated that soils of high C_u exhibit more pronounced non-linear behavior in

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comparison to uniform soils. However, it is noticeable that the available experimental results did not show any clear effect of mean grain size on the response of granular soils. This means that a fine-grained sand and a gravelly soil of the same coefficient of uniformity under the same σ_m' are expected to exhibit similar non-linear curves. Tanaka et al. (1987) noticed that granular soils of higher gravel content present more steep non-linear curves; however it should be considered that the materials tested by Tanaka et al. exhibited higher C_u values at higher gravel content. Finally, Seed et al. (1986) proposed two groups of curves concerning sandy and gravelly soils separately, but their results were based on gravels that exhibit in general high values of C_u. Thus, on writers knowledge there is not a clear experimental evidence in the literature that shows an important effect of mean grain size on the dynamic response of granular soils.

In this study, the main conclusions of an extensive resonant column testing program on clean sandy and gravelly soils performed at the laboratory of Soil Mechanics, Foundations and Geotechnical Earthquake Engineering of A.U.Th. are presented. It is underlined that uniform gravels exhibit more pronounced non-linearity in comparison to uniform sands. In addition, mean grain size affects significantly the non-linear curves of sandy soils. In particular, G/G_O-logγ and DT-logγ curves of uniform fine-grained sands are more linear compared to coarse sands. Finally, the parameter C_u is an important factor for the behavior of granular soils, but both D_{50} and C_u should be considered for an accurate prediction of the non-linear response of sands and gravels.

MATERIALS TESTED, SPECIMENS PREPARATION AND TESTING PROGRAM

Sixteen soils of different grain-size characteristics were used in this study. Three of these soils were constructed using a river sand of rounded particles as parent material, whereas thirteen soils were constructed using a quarry gravelly sand of angular particles as parent material. These materials exhibit unit weight of soil solids approximately equal to 2.67 gr/cm^3. The grain size characteristics of the materials used are summarized in Table 1. In this study, the maximum grain size of solids is assumed to range between the minimum size of the sieve that the whole amount of the material passes and the maximum size of the sieve that at least some particles are retained. Thus, in Table 1, the maximum grain size, D_{max}, is given in a range of values. Concerning the specifications and common practice, soils tested herein exhibit a D_{max} value $\leq$ 1/6 of specimen's diameter.

Eleven of the materials tested (No.1-11) are characterized as uniform to poor graded with C_u values less that 5.0. In addition, five of the materials tested (No.12-16) exhibit high values of C_u (>5), even some of them are classified as SP due to the relatively low values of C_c.

The experimental device used in this study is a resonant column apparatus that follows the fixed-free configuration (Drnevich, 1967). All specimens were tested in dry conditions, having a diameter approximately equal to 71.1 mm and a height about two times the diameter. The sixteen soils shown in Table 1 were tested at a high relative density. For this purpose, specimens were constructed into a metal mold in many layers of equal mass. To construct relatively uniform samples, the compaction tips increased at the top layers. In addition, eight of these soils were also tested in low to medium relative density. In this case, specimens were constructed by scooping the material into the mold without compaction.

Low-amplitude as well as high-amplitude torsional resonant column tests were performed at increasing steps of mean confining pressure, equal to 25, 50, 100 and 200 kPa and in some cases up to 400 kPa. In every confining pressure step, specimens were allowed about 60-80 minutes to equilibrate before the low-amplitude measurements were performed. In addition, after the high-amplitude measurements at a defined confining pressure, specimens were allowed about 30-60 minutes to recover at least 95% of their initial stiffness. The parameters of initial shear modulus, G_O, and initial damping ratio, DT_O, are defined in this study at shearing strain amplitudes in general less than $5\text{-}7\times10^{-4}\%$. Table 2 summarizes the initial values of dry unit weight, void ratio and relative density at which specimens were prepared. All tests and analysis of the results were performed according to ASTM D4015-92 specification.

Table 1. Materials tested

No.	Material code	Roundness of particles	USCS	Gravel content (%)	D_{max} (mm)	D_{50} (mm)	C_u (1)	C_c (2)
1	C2D02	Angular	SP	0	0.25-0.43	0.16	2.00	0.99
2	C2D03	Rounded	SP	0	0.25-0.43	0.27	1.58	0.93
3	C3D06	Rounded	SP	0	0.85-2.00	0.56	2.76	1.23
4	C3D09	Angular	SP	0	0.85-2.00	0.85	3.23	0.78
5	C2D1(a)	Rounded	SP	0	0.85-2.00	1.33	2.13	1.01
6	C2D1(b)	Angular	SP	0	0.85-2.00	1.33	2.13	1.01
7	C3D2	Angular	SP	0	2.00-4.75	2.00	2.50	1.07
8	C2D3	Angular	SP	15	4.75-6.35	3.00	2.45	1.10
9	C1D6	Angular	GP	100	4.75-6.35	5.50	1.17	0.96
10	C1D8	Angular	GP	100	6.35-9.53	7.80	1.22	0.94
11	C1D10	Angular	GP	100	9.53-12.70	10.10	1.03	1.00
12	C12D1	Angular	SP	20	6.35-9.53	1.33	11.80	0.68
13	C6D2	Angular	SP	21	6.35-9.53	2.00	5.40	0.50
14	C7D2	Angular	SP	25	9.53-12.70	2.00	7.30	0.65
15	C6D3	Angular	SP-SW	30	6.35-9.53	2.90	5.95	1.19
16	C13D3	Angular	SP-SW	40	6.35-9.53	3.00	12.50	0.94

(1) $C_u=D_{60}/D_{10}$ (2) $C_c=D_{30}^2/(D_{10}\times D_{60})$

Table 2. Resonant column testing program

No.	Material code	Specimen's construction into a metal mold in many compacted layers			Specimen's construction by scooping material into the metal mold		
		γ_d (kN/m^3)	e	D_r (%)	γ_d (kN/m^3)	e	D_r (%)
1	C2D02	15.6	0.683	58	13.4	0.954	<10
2	C2D03	15.8	0.661	86	-	-	-
3	C3D06	16.5	0.588	>90	15.6	0.682	64
4	C3D09	17.0	0.545	>90	15.0	0.742	39
5	C2D1(a)	16.8	0.557	85	-	-	-
6	C2D1(b)	16.4	0.594	>90	-	-	-
7	C3D2	16.9	0.553	89	14.8	0.770	28
8	C2D3	16.3	0.611	59	15.3	0.718	28
9	C1D6	15.4	0.700	>90	14.0	0.878	72
10	C1D8	15.4	0.715	>90	14.2	0.846	73
11	C1D10	16.2	0.618	>90	-	-	-
12	C12D1	19.4	0.354	>90	-	-	-
13	C6D2	18.2	0.440	67	-	-	-
14	C7D2	18.8	0.396	81	-	-	-
15	C6D3	17.7	0.479	54	-	-	-
16	C13D3	18.1	0.448	64	16.7	0.564	27

REPRESENTATIVE RESULTS

Figure 1 shows representative results of the experimental testing program on dense to very dense specimens. In the same figure the proposed non-linear curves for sandy soils by Seed et al. (1986) are also shown. It is noticed that G/G_O values decrease as well as DT values increase as shearing strain amplitude increases. In addition, G/G_O-logγ and DT-logγ curves become less steep as mean effective confining pressure increases. In Figure 2, representative results concerning the effect of relative density (or void ratio) on the non-linear behavior of the tested specimens are shown. It is noticed that dense and loose specimens of the same material exhibit similar non-linear curves. The last remark has been also reported by other researchers on similar materials (Kokusho, 1980, Menq, 2003).

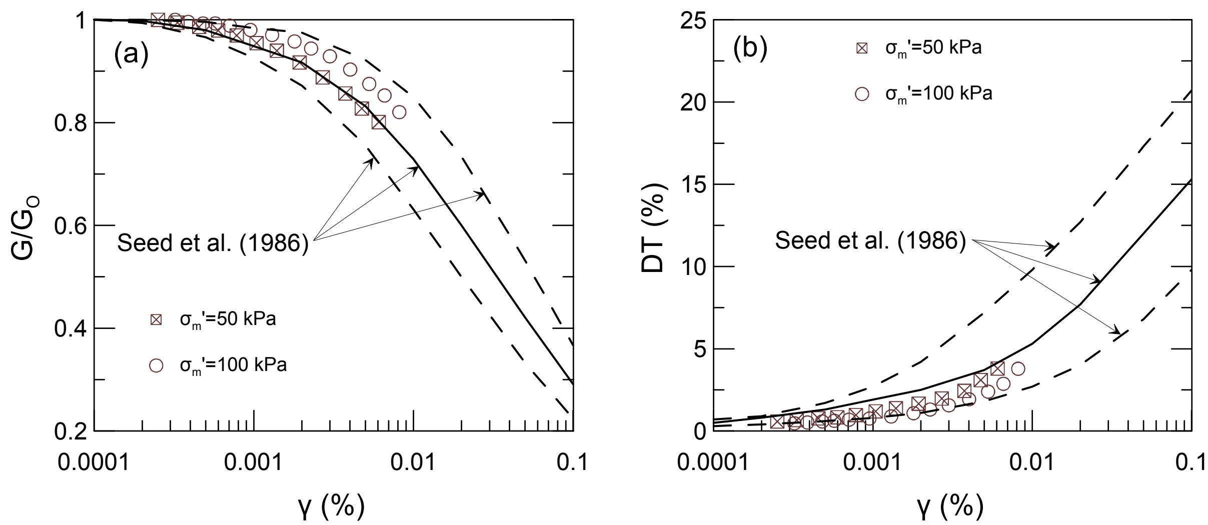

Figure 1. Effect of shearing strain amplitude and mean effective confining pressure on the non-linear G/G_O-logγ and DT-logγ curves of granular soils (dense to very dense specimen C3D2)

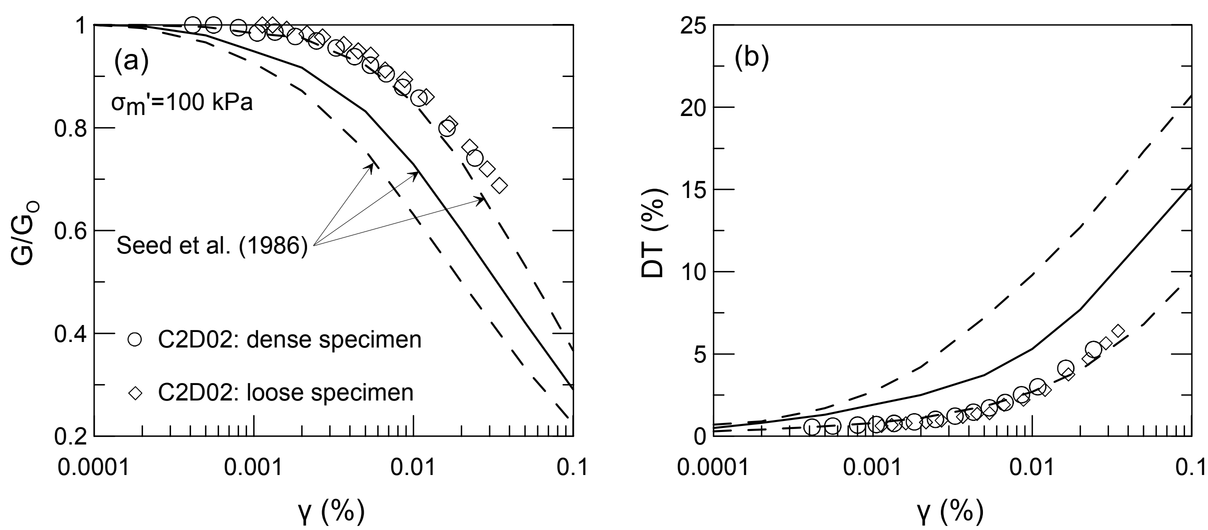

Figure 2. Effect of relative density (or void ratio) on the non-linear G/G_O-logγ and DT-logγ curves of granular soils (material C2D02 at a mean confining pressure of 100 kPa)

In Figures 3 and 4 the experimental G/G_O and DT values as a function of shearing strain, γ, are shown. Figure 3 concerns the dense to very dense specimens of fine to medium sands ($D_{50} \leq 2.00$ mm), whereas Figure 4 concerns the dense to very dense coarse sands and gravels of this study ($D_{50} > 2.00$ mm). In the same figures curves proposed by Seed et al. (1986) and Rollins et al. (1998), concerning sandy and gravelly soils respectively, are shown. It is noticed that in general the experimental results are within the upper and lower theoretical curves.

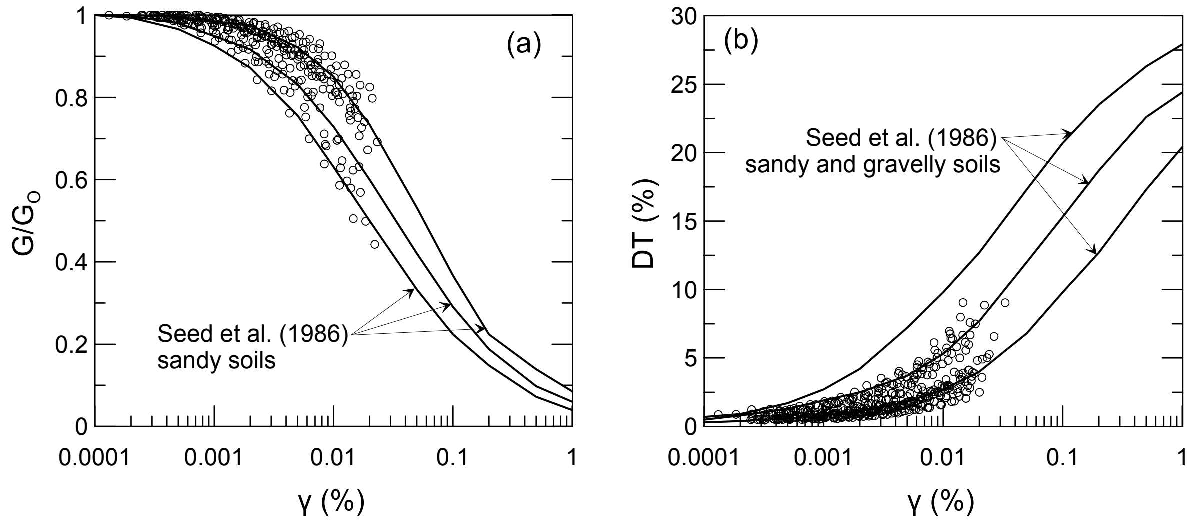

Figure 3. Measured G/G_O and DT values as a function of shearing strain amplitude of dense to very dense sands ($D_{50} \leq 2.00$ mm) in comparison to the curves proposed by Seed et al. (1986)

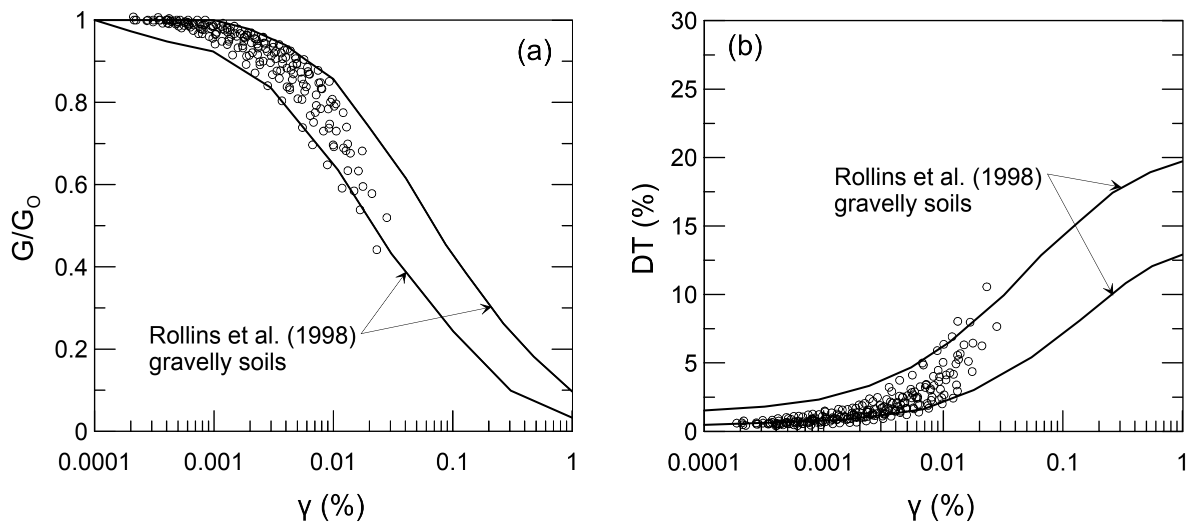

Figure 4. Measured G/G_O and DT values as a function of shearing strain amplitude of dense to very dense coarse sands and gravels ($D_{50} > 2.00$ mm) in comparison to the curves proposed by Rollins et al. (1998)

CORRELATION BETWEEN NORMALIZED SHEAR MODULUS AND DAMPING RATIO

In Figure 5 the measured damping ratio values minus the corresponding initial damping ratio ($DT\text{-}DT_O$) of the dense and loose specimens of this study are plotted against the normalized shear modulus values, G/G_O, at the same shearing strain amplitude. In the same figure the fitting curve of the experimental results as well as the curves proposed by Zhang et al. (2005) concerning physical plastic and non-plastic soils are also shown. It is noticed that $DT\text{-}DT_O$ and G/G_O may be correlated with a simple two-order relationship.

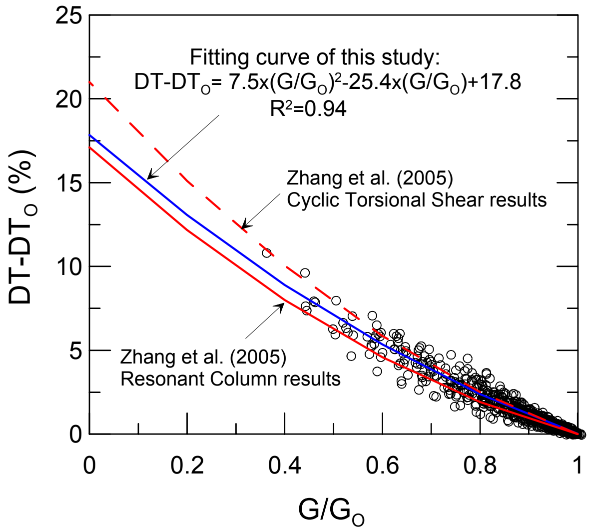

Figure 5. Correlation between G/G_O and DT values of this study (Note: DT values are normalized with respect to the corresponding initial damping ratio in terms of DT minus DT_O)

NORMALIZED SHEAR MODULUS VERSUS SHEARING STRAIN CURVES: ANALYSIS OF THE RESULTS

Analytical model used

The modified hyperbolic model proposed by Darendeli (1997, 2001) was used for the analysis of the non-linear G/G_O-logγ curves. This model applies two parameters: the reference strain γ_{ref} that corresponds to G/G_O=0.50, and the coefficient of curvature, a. Parameter (γ_{ref}) controls the linearity of the G/G_O-logγ curves, whereas the parameter (a) controls the overall slope of the G/G_O-logγ curves. Menq (2003) indicated that (γ_{ref}) of granular soils is a function of (σ_m') and (C_u), whereas (a) is a function of (σ_m'). The analytical expression of the modified hyperbolic model is shown in equation 1.

$$\frac{G}{G_O} = \frac{1}{1+\left(\dfrac{\gamma}{\gamma_{ref}}\right)^a} \qquad (1)$$

Comparison with proposed models of the literature

Figures 6 to 7 compare the reference strain values of some dense to very dense specimens tested in this study with estimated values using an analytical equation proposed by Menq (2003). In Figure 6, it is noticed that uniform to poor graded sands (C2D03, C3D06) as well as soils of high coefficient of uniformity (C13D3, C12D1) exhibit similar γ_{ref} values in comparison to proposed values by Menq (2003). In addition, the well graded materials C13D3 and C12D1 exhibit significantly lower γ_{ref} values in comparison to the uniform materials C2D03 and C3D06. However, Figure 7 shows that the uniform gravels of this study exhibit significantly more non-linear behavior expressed in terms of lower γ_{ref} values in comparison to proposed values by Menq (2003). In general, all uniform sands and well graded sands and gravels of this study exhibited a similar trend as the one shown in Figure 6, whereas uniform gravels show a general trend of more non-linearity in comparison to proposed curves. Thus, mean grain size of solids seems to be an important parameter concerning the non-linear behavior of granular soils. It is finally observed in Figures 6 and 7 that in all specimens, γ_{ref} increases with increasing σ_m'. In particular, G/G_O-logγ curves become less steep or in other words specimens exhibit higher linearity, as σ_m' increases.

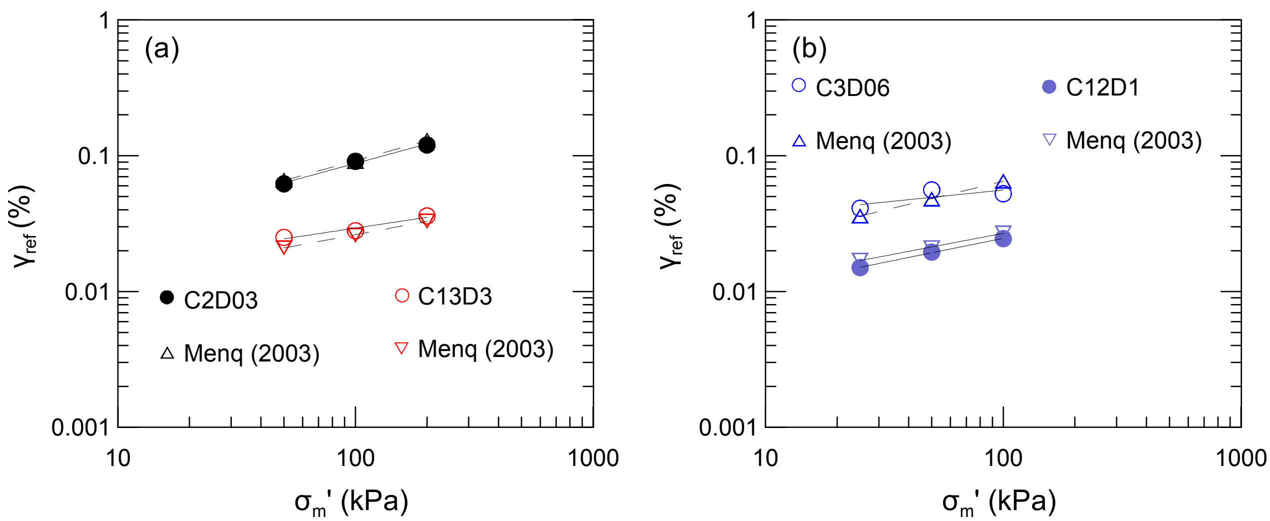

Figure 6. Reference strain values versus mean effective confining pressure of uniform to poor graded sands (C2D03, C3D06) and well graded sands and gravels (C13D3, C12D1) in comparison to estimated values from analytical relations proposed in the literature (figure refers to dense specimens)

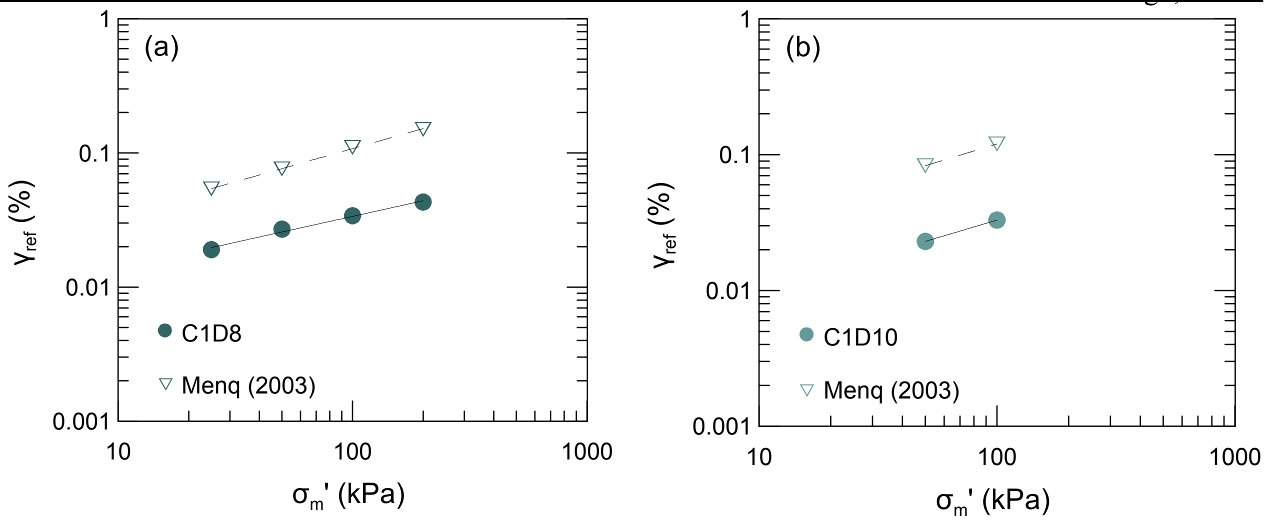

Figure 7. Reference strain values versus mean effective confining pressure of uniform to poor graded gravels in comparison to estimated values from analytical relations proposed in the literature (figure refers to dense specimens)

As mentioned before (Figure 2), dense to very dense and loose to medium dense specimens of the same material, exhibit similar non-linear curves. In fact, there was not a clear trend on the effect of relative density (or void ratio) on the parameters γ_{ref} and a. Thus, the analysis presented herein includes only the sixteen dense to very dense specimens, whereas it is assumed that loose to medium dense soils exhibit a similar behavior.

Reference strain of the tested specimens will be expressed as a function of mean effective confining pressure σ_m' and reference strain at σ_m'=100 kPa, $\gamma_{ref,100}$, using the following equations:

$$\gamma_{ref} = \gamma_{ref,100} \times A_\gamma^* \times \left(\sigma_m^{'}\right)^{n_\gamma} \tag{2}$$

$$A_\gamma = \gamma_{ref,100} \times A_\gamma^* \tag{3}$$

where, A_γ^* and n_γ are constants and the exponent n_γ expresses the effect of σ_m' effect on reference strain. Finally, equation 2 may be re-written as:

$$\gamma_{ref} = A_\gamma \times \left(\sigma_m^{'}\right)^{n_\gamma} \tag{4}$$

In Figure 8, the effect of mean grain size on the parameter $\gamma_{ref,100}$ of the dense to very dense specimens is graphically shown. It is noticed that in general $\gamma_{ref,100}$ of the fine to medium uniform sands decreases in a linear manner as D_{50} increases. More specifically, reference strain at σ_m'=100 kPa decreases from about 7-9×10^{-2}% to values less than 4×10^{-2}% as D_{50} increases from 0.16 to 2.00 mm. Considering that uniform fine-grained to medium sands of this study exhibit values of C_u in a range of 1.58-3.23, it is possible that parameter C_u affects somehow the results of Figure 8a. On the other hand, as it is shown in Figure 8b, the four uniform coarse sands and gravels of this study, exhibit similar values of $\gamma_{ref,100}$, in a range of 3.3-4.1×10^{-2}%, with an average value equal to 3.68×10^{-2}%.

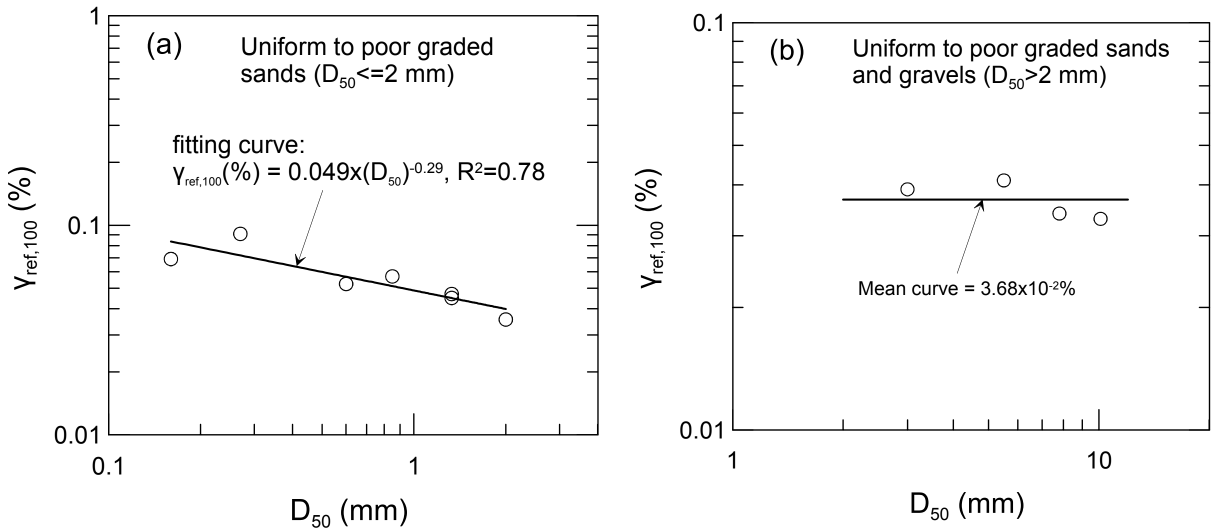

Figure 8. Effect of mean grain size on reference strain at a mean effective confining pressure of 100 kPa of uniform to poor graded sandy and gravelly soils

Tables 3 and 4 summarises the values of $\gamma_{ref,100}$ in the case of uniform and gravelly sands of this study and the corresponding values of the well-graded soils of the same D_{50}. Considering soils C2D1(b) and C12D1, as well as soils C3D2, C6D2 and C7D2, it is noticed that well graded sands exhibit a reference strain of 0.68 times on average the reference strain of the uniform soils of the same D_{50}. In addition, considering soils C2D3, C6D3 and C13D3, it is noticed that well graded gravelly sands exhibit a reference strain 0.73 times on average the corresponding value of uniform soils of the same D_{50}. It is noted that parameters $\gamma_{ref,100,uniform}$ and $\gamma_{ref,100,well\text{-}graded}$ at Tables 3 and 4 refer to specimens of C_u<5 and C_u>5, respectively.

Table 3. Effect of C_u on the parameter $\gamma_{ref,100}$ of sands

	$\gamma_{ref,100,uniform}$	$\gamma_{ref,100,well\text{-}graded}$	$\gamma_{ref,100,well\text{-}graded}/\gamma_{ref,100,uniform}$
C2D1(b)	4.50×10^{-2}%	-	-
C12D1	-	2.45×10^{-2}%	0.54
C3D2	3.55×10^{-2}%	-	-
C6D2	-	2.80×10^{-2}%	0.79
C7D2	-	2.55×10^{-2}%	0.72
		Average value:	0.68

Table 4. Effect of C_u on the parameter $\gamma_{ref,100}$ of gravelly sands

	$\gamma_{ref,100,uniform}$	$\gamma_{ref,100,well\text{-}graded}$	$\gamma_{ref,100,well\text{-}graded}/\gamma_{ref,100,uniform}$
C2D3	3.9×10^{-2}%	-	-
C6D3	-	2.9×10^{-2}%	0.74
C13D3	-	2.8×10^{-2}%	0.72
		Average value:	0.73

Figure 9 shows the parameters $A\gamma^*$ and n_γ of all specimens examined in this study. There was no a clear trend on the effect of D_{50} or C_u on these parameters. Thus, it was decided to use average values that are also shown in Figures 9a and 9b. In the same figures values of the typical standard deviation of the parameters $A\gamma^*$ and n_γ are also shown.

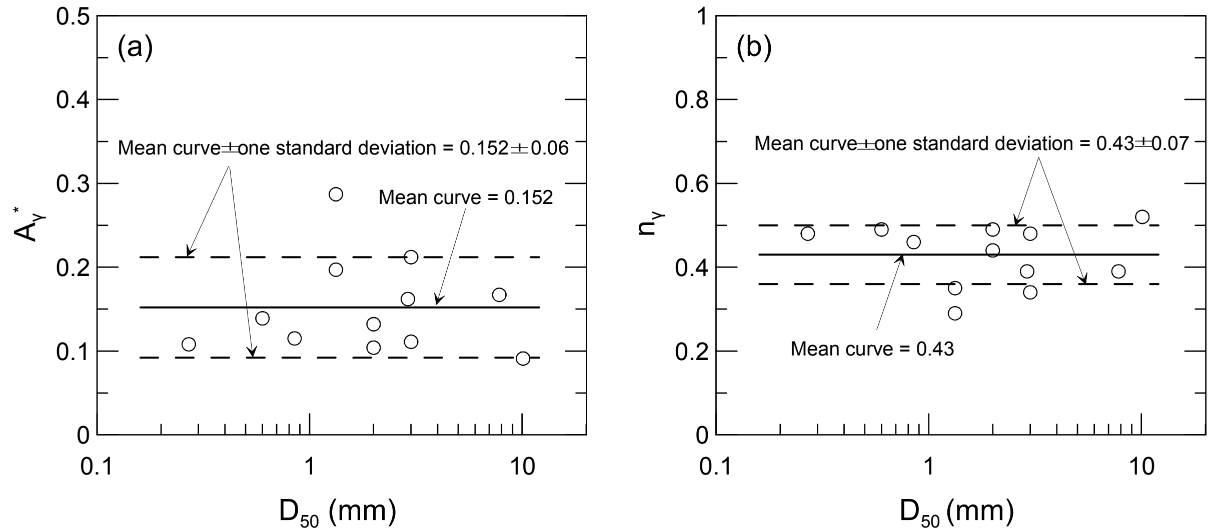

Figure 9. A_γ and n_γ parameters of tested uniform to well graded soils

Considering equations 2-4 and the experimental results of Figures 8-9 and Tables 3-4, the reference strain of the tested granular soils may be expressed with the parameters summarized in Table 5. It is noticed that D_{50} is expressed in mm, whereas parameters σ_m' and γ_{ref} of equation 4 are expressed in kPa and percent scale respectively.

Table 5. Parameters of A_γ and n_γ of tested granular soils

Type of soil	$A_\gamma=A_\gamma^*\times\gamma_{ref,100}$	n_γ
Fine to medium grained uniform sands ($D_{50}\leq2.00$ mm, $C_u<5$)	$7.45\times10^{-3}\times(D_{50})^{-0.29}$	0.43
Fine to medium grained well graded sands ($D_{50}\leq2.00$ mm, $C_u>5$)	$5.02\times10^{-3}\times(D_{50})^{-0.29}$	0.43
Uniform coarse sands and gravels ($D_{50}>2.00$ mm, $C_u<5$)	5.60×10^{-1}	0.43
Well graded coarse sands and gravels ($D_{50}>2.00$ mm, $C_u>5$)	4.10×10^{-1}	0.43

Figure 10 shows the values of curvature coefficient, (a), of the tested dense to very dense specimens, separately for fine to medium sands with $D_{50}<1.00$ and coarse sands and gravels with $D_{50}>1.00$ mm. In the same figure the mean, as well as, the typical standard deviation values of the parameter a are also shown. In fact, there was not a clear trend on the effect of grain size characteristics on the parameter a, except that fine to medium grained soils exhibit slightly lower values or in different words lower overall slope of the G/G_O-logγ curves in comparison to coarse soils.

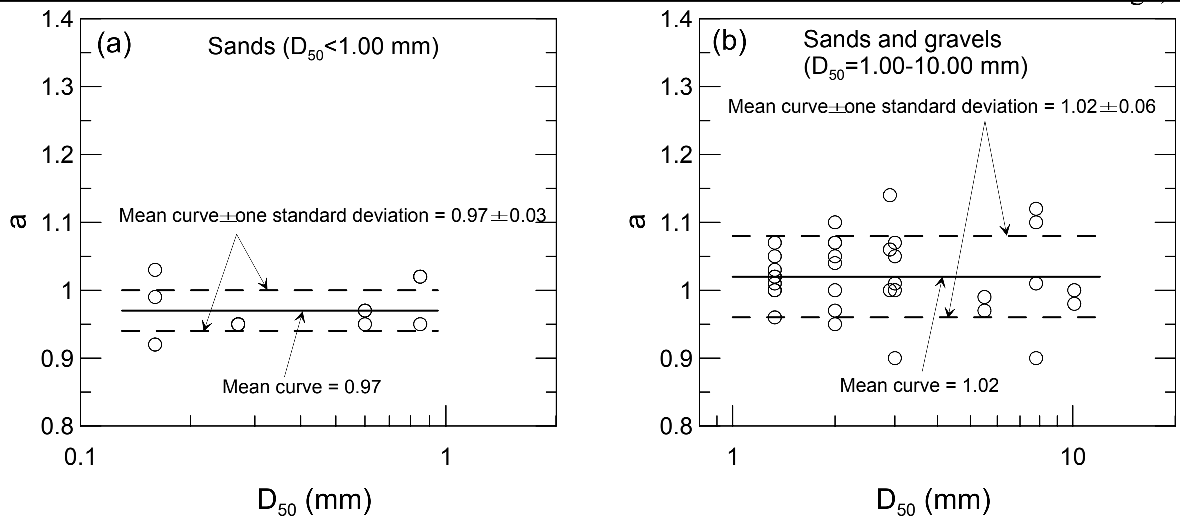

Figure 10. Parameter a of tested uniform to well graded soils

Assuming a value of curvature coefficient equal to 0.97 and 1.02 for soils of D_{50}<1.00 and D_{50}>1.00 mm, and using equations 1-4 and the parameters of Table 5, G/G_O- logγ curves of granular soils may be estimated. For the case of the dense to very dense specimens tested herein, Figure 11 compares the measured and estimated values of G/G_O. As shown in this figure, the above relationships estimate satisfactory the non-linear response of the tested specimens.

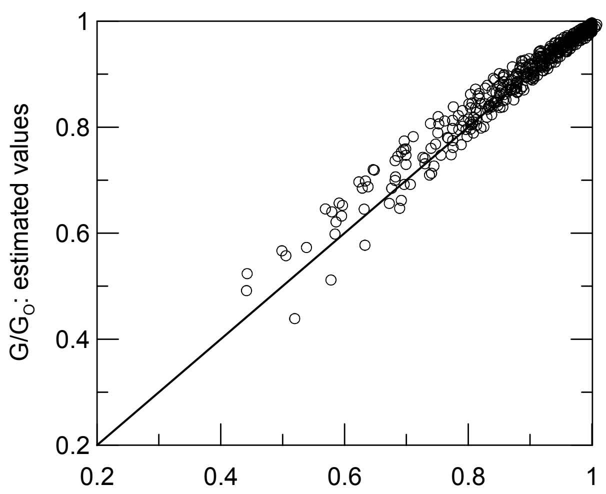

Figure 11. Measured versus estimated G/G_O values of this study

CONCLUSIONS

The present paper examined the non-linear dynamic response of granular soils using a resonant column device. G/G_O-logγ curves of the specimens were examined in terms of a modified hyperbolic model, whereas damping ratio values were correlated to the corresponding G/G_O values using a simple two-order relationship. It is concluded that not only the shearing strain amplitude, mean effective confining pressure and coefficient of uniformity are important, but in addition the mean grain size affects significantly the non-linear response of granular soils. Fine-grained sands exhibit in general more linear behavior in comparison to medium and coarse sands, whereas sands exhibit more linear behavior in comparison to gravels.

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