5ICEGE
5th International Conference on Earthquake Geotechnical Engineering
January 2011, 10-13 | Santiago, Chile
Paper No. AFFBR

VALIDATION OF SEISMIC RESPONSE ANALYSES USING SEISMOMETER ARRAYS

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ABSTRACT

A framework for the validation of computational models used to predict seismic response based on observations from seismometer arrays is presented. The framework explicitly accounts for the epistemic uncertainty related to the unknown characteristics of the 'site' (i.e. the problem under consideration) and constitutive model parameters. A mathematical framework which makes use of multiple prediction-observation pairs is used to improve the statistical significance of inferences regarding the accuracy and precision of the computational methodology and constitutive model. The benefits of such a formal validation framework include: (i) development of consistent methods for determination of constitutive model parameters; (ii) rigorous, objective and unbiased assessment of the validity of various constitutive models and computational methodologies for various problem types and ground motion intensities; and (iii) an improved understanding of the uncertainties in computational model assumptions, constitutive models and their parameters, relative to other seismic response uncertainties such as ground motion variability.

Keywords: Validation, seismometer array, seismic response analysis, epistemic uncertainty.

INTRODUCTION

The continuing evolution toward the seismic design of engineered facilities based on their expected seismic performance places increasing emphasis on the use of computational models to predict the seismic response of such facilities. Despite our best efforts in the design and assessment of facilities to reduce their vulnerability to earthquake-induced hazards, the occurrence of every large earthquake seems to provide new evidence of the complex phenomenon producing strong ground motions at the earth's surface, and weaknesses in these contemporary seismic design and/or assessment methods.

Quantitative data from seismometers represent one of the primary interactions between observations and computational simulation in earthquake engineering, with other interactions including: element testing, testing of subsystems, or testing of entire systems at full or reduced scales. Seismometer data offers several advantages over these other forms of quantitative data in that the instrumented facilities automatically have the correct in situ and boundary conditions which can be difficult, if not impossible, to reproduce in laboratory experiments.

This paper provides an overview of a framework in which seismic response models can be validated with seismic array recordings. Firstly, the conventional use of seismometer arrays in validation of seismic response modelling and its limitations are discussed. The details of the proposed framework, which

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addresses conventional limitations, are developed and its benefits for use in seismic response prediction are examined. Further details of the proposed framework can be found in Bradley (2010).

CONVENTIONAL COMPARISON OF SEISMIC RESPONSE MODELS WITH ARRAY OBSERVATIONS

Examples of the use of seismometer arrays to examine the capabilities of geotechnical site response analysis include Cubrinovski *et al*. (2000); Bernardie *et al.* (2006); Elgamal *et al.* (1996); Finn *et al.* (1993), among others. Figure 1 illustrates a schematic example of a seismic instrumentation array which can be used to validate a one-dimensional seismic response analysis. A computational seismic response model could be constructed, and the input excitation applied to the model can be obtained from one (or possibly more) of the seismometer recordings. Thus, in this case, the 'input' motion (i.e. that at the base of the one-dimensional computational model in Figure 1) is known explicitly, such that any difference between the computational prediction and seismometer observations is due to the computational model (including how the input motion is applied as a boundary condition). This feature is a prerequisite to enable seismic arrays to be used for seismic response validation.

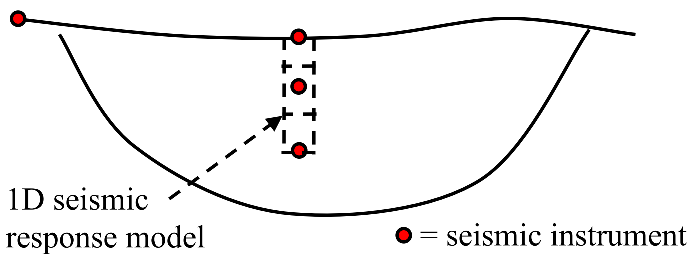

Figure 1: Illustration of a site response example in which seismometer arrays can be used to provide validation of seismic site response analyses.

The conventional use of seismometer arrays for validation of seismic response computational models, as exemplified by the aforementioned references, can be regarded as deterministic in the sense that no uncertainties in the seismic response model are considered. One of the consequences of constructing a computational model of a site which exists in reality (rather than a model which is created under laboratory-type conditions) is that it is not possible to fully characterise the physical and mechanical properties of the site. Hence there exists significant uncertainty in the characterisation of the problem under consideration. This consequently results in uncertainty in the parameters of the constitutive models required for computational analyses. A consequence of the failure to account for these uncertainties in the computational model is that it cannot be determined if a good agreement between a single prediction and observation is due to a capable computational model or is in fact due to 'cancellation' of errors that result from the unknown site characterisation and inconsistencies in the computational model.

A FORMAL FRAMEWORK FOR SEISMIC RESPONSE VALIDATION

Uncertainty classification

There are numerous significant uncertainties in any seismic response problem, and these must be considered in a robust validation framework. Here, such seismic response uncertainties are differentiated into four classes (Figure 2): (i) site characterisation uncertainties; (ii) constitutive model parameter uncertainties; (iii) constitutive model uncertainties; and (iv) model methodology uncertainties.

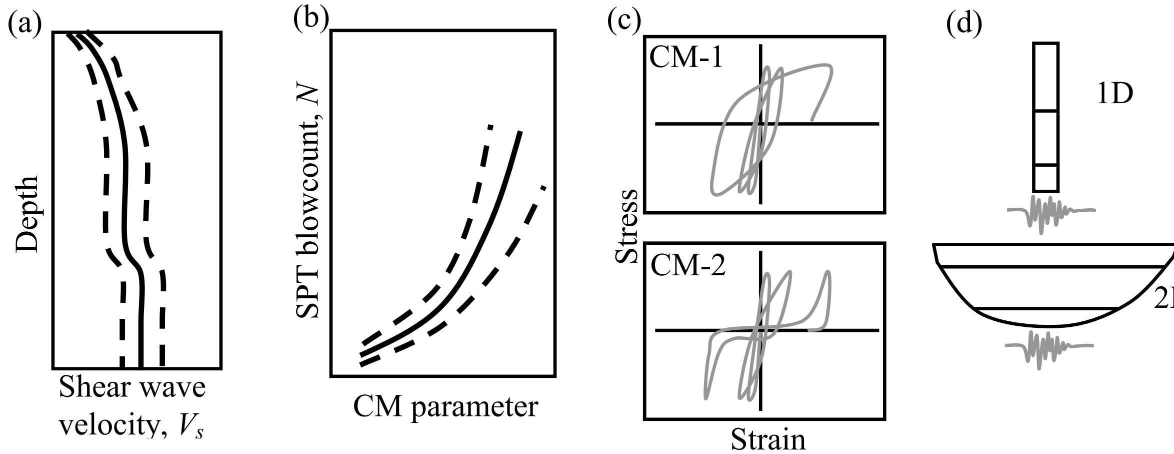

Figure 2: Examples of the four different types of uncertainties in the case of a geotechnical seismic site response analysis: (a) site characterisation uncertainty; (b) constitutive model (CM) parameter uncertainty; (c) constitutive model uncertainty; and (d) computational model methodology uncertainty.

Consideration of site characterisation and constitutive model parameter uncertainties

Consider initially that the computational model methodology and constitutive relations are an exact representation of the physical problem of interest. Therefore the only uncertainties in the seismic response predicted by the computational model are related to the measured values of mechanical and physical properties and the uncertain relationships between measured properties and the parameters of constitutive relationships (i.e. type (i) and (ii) uncertainties in Figure 2). When type (i) and (ii) uncertainties are considered in the computational model then the resulting seismic response, measured by one of more engineering demand parameters (EDPs), will have a distribution (with each EDP having a different value for each possible realisation of the uncertain parameters). Figure 3a illustrates this uncertainty in the form of a probability density function of a predicted EDP from the computational model. Figure 3a also illustrates the unique value of the seismic response quantity, $edp_{i,j,k}$, as measured from the seismometer array.

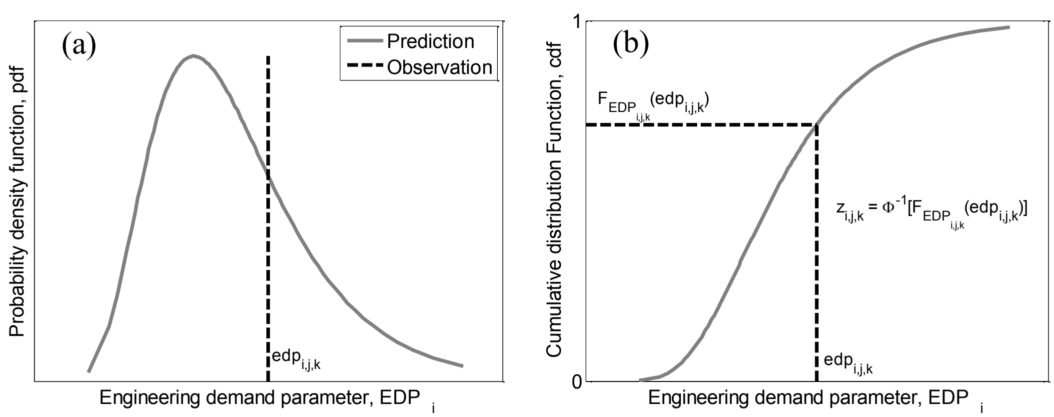

Figure 3: (a) Schematic comparison between prediction probability density function and observation; (b) computation of normalised residual based on cumulative prediction distribution and observation.

The probability density function (pdf) of the prediction of a particular demand measure, EDP_i, (e.g. peak displacement at the surface) for a single observation *k,* at a single site *j,* $f_{EDP_{i,j,k}}$, shown in Figure 3a gives the likelihood that a particular value of EDP_i is observed based on the computational model. Because of the aforementioned uncertainties it is not possible to make robust inferences on the predictive capability of a particular computational model based on a single observation, and therefore multiple-prediction observation pairs are needed.

Consideration of multiple observations and sites

Consider the uncertain prediction from the computational model in terms of the cumulative density function (CDF) shown in Figure 3b (rather than the pdf in Figure 3a). Using this CDF the actual seismometer observation (the k^{th} observation at site *j* of EDP_i), $edp_{i,j,k}$, corresponds to a value $F_{EDP_{i,j,k}}\left(edp_{i,j,k}\right)$. The normalised residual of the seismometer observation for $edp_{i,j,k}$ relative to the computational model prediction can then be computed from:

$$z_{i,j,k} = \Phi^{-1}\left[F_{EDP_{i,j,k}}\left(edp_{i,j,k}\right)\right] \tag{1}$$

where $\Phi^{-1}[\;]$ is the inverse normal cumulative density function. Based on its definition, $z_{i,j,k}$ represents a random observation from a standard normal distribution. In order to account for the dependence between multiple observations at a single site this normalised (total) residual is expressed as:

$$z_{i,j,k} = a + \eta_{i,j} + \varepsilon_{i,j,k} \tag{2}$$

where *a* is a constant; $\eta_{i,j}$ is the inter-site residual for EDP_i and site *j*, and $\varepsilon_{i,j,k}$ is the intra-site residual for the k^{th} observation of EDP_i at site *j*. It is assumed that $\eta_{i,j}$ and $\varepsilon_{i,j,k}$ are independent and are characterised by a normal distribution with zero means and variances σ_S^2 and σ_O^2, respectively. Upon conducting regression to determine the unknown parameters in Equation (2) (i.e. *a,* σ_S^2 and σ_O^2), the mean and variance of the regression model of the normalised residuals, are given by:

$$\hat{\mu}_Z = a \tag{3}$$

$$\hat{\sigma}_Z^2 = \sqrt{\sigma_S^2 + \sigma_O^2} \tag{4}$$

where $\hat{\mu}_Z$ is the point-estimate of the mean of *Z*; and $\hat{\sigma}_Z^2$ is the point-estimate of the variance of *Z*. Based on the aforementioned assumption that the computational methodology and constitutive model are exact, each $z_{i,j,k}$ represents a random variable from a standard normal distribution. Hence, comparison of the mean and variance of the regression model for Z with that of a standard normal distribution can be used to examine the bias and precision of the computational methodology and constitutive model.

APPLICATION OF THE PROPOSED FRAMEWORK

Hypothetical observations

Figure 4 illustrates possible situations which may arise when comparing the predicted distribution of Z (i.e. using Equations (2)-(4)), with the theoretical standard normal distribution for a particular seismic response problem. Figure 4a illustrates the case in which the mean and variance of Z are very similar to the standard normal distribution. It can be seen that the 90% confidence interval of μ_Z easily encompasses the theoretical value of zero, and hence the bias of the computational model methodology and constitutive model for the sites considered is relatively small. Figure 4b illustrates a situation where the computational model systematically over predicts the response for some EDP_i, resulting in residuals which are predominantly negative. This over-prediction bias is significant as can be seen from the 90% confidence interval for μ_Z not including the theoretical value of zero. Figure 4c illustrates a situation in which there is little bias in the computational model (similar to Figure 4a), but that the variance of Z, σ_Z^2, is significantly larger than that of the theoretical value of 1, indicating that the computational model is imprecise.

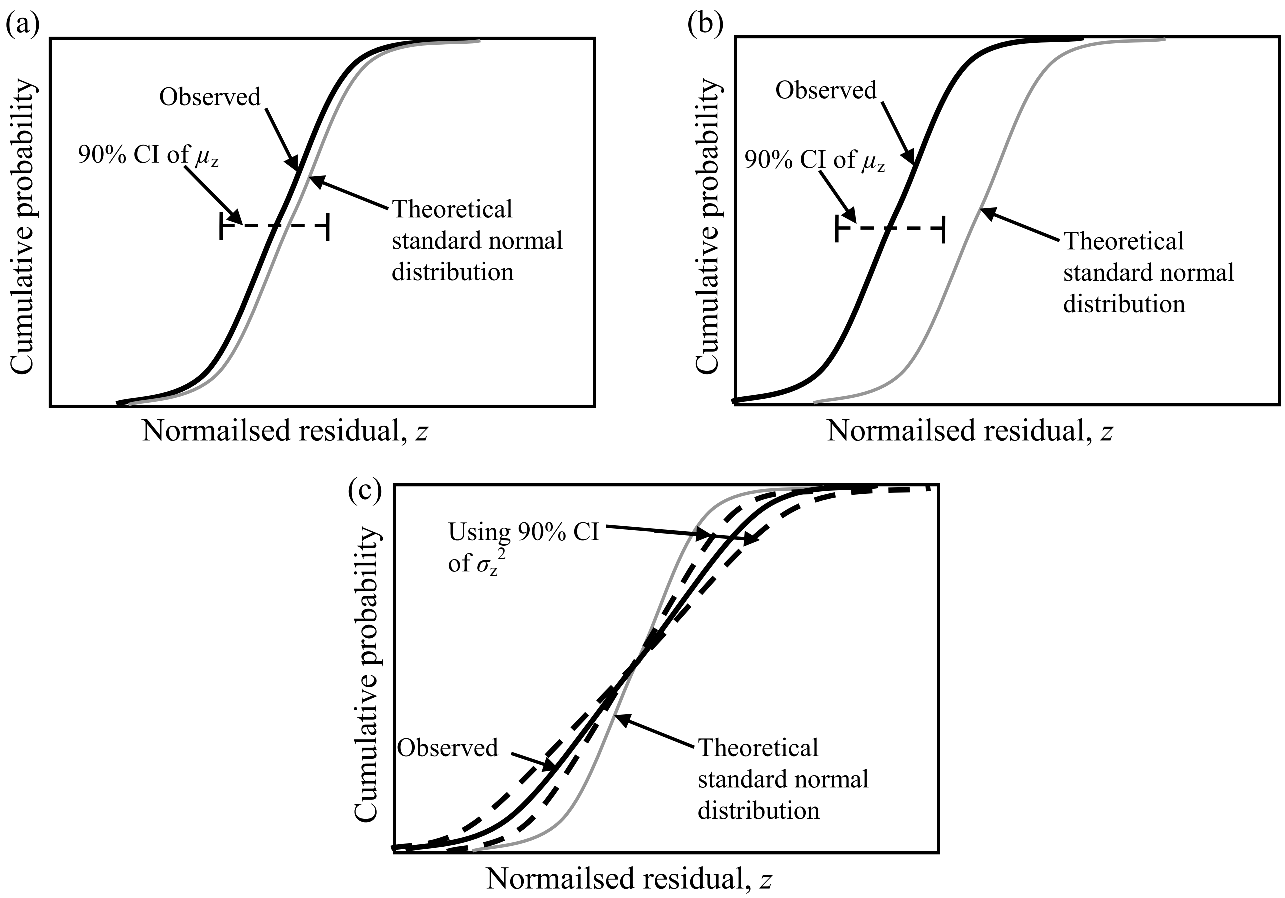

Figure 4: Illustration of the resulting distribution of the normalised residuals compared to the theoretical standard normal distribution (and statistical error bounds) in the cases of: (a) insignificant bias; (b) significant bias; and (c) imprecision.

Consideration of alternative constitutive models

Because constitutive models used in seismic response analyses are typically empirically constructed based on direct observations, or theoretically derived based on various assumptions, then it is unlikely that a single constitutive model is perfectly representative of an engineering material. Consequently, this imperfection leads to uncertainty in the prediction of the seismic response of such a material (i.e. type (iii) uncertainty in Figure 2). It is well recognised that significant differences in computational model predictions can be obtained using various commonly adopted constitutive models for certain problems (Arulanandan &Scott, 1993).

The proposed seismic response analysis validation framework offers the opportunity to quantify a hierarchy of constitutive model validity based on the observed bias and precision of the alternative models, and therefore avoid problems associated with a significant reliance on expert opinion. Figure 5a schematically illustrates the distribution of the normalised residuals for a given computational model methodology, but using three different constitutive models. It can be seen that the use of constitutive models 1 and 2 leads to a small over-prediction and under-prediction bias, respectively, as indicated by the small negative and positive mean values of the normalised residuals, respectively. It is also noted that the use of constitutive models 1 and 2 leads to an appropriate level of prediction precision (as indicated by the similarity in the variance of the normalised residuals relative to the theoretical standard normal distribution). On the other hand, the use of constitutive model 3 leads to a large over-prediction bias, as indicated by the mean value of the normalised residuals being significantly different than zero. It addition, the variance of the normalised residuals obtained using constitutive model 3 is significantly larger than one, indicating that the use of constitutive model 3 also leads to significant prediction imprecision. Hence on the basis of Figure 5a, an analyst could comfortably reject the use of constitutive model 3 (for a seismic response problem which is 'within' those encompassed by the array recordings providing the observed normalised residuals), and consider only constitutive models 1 and 2 when accounting for constitutive model (type (iii)) uncertainty.

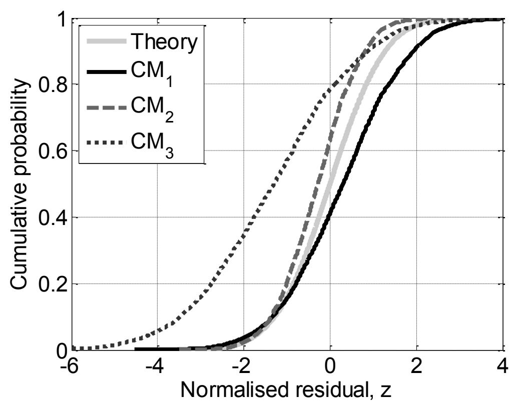

Figure 5: Use of the validation framework to assess the capability of various constitutive models (CM) based on the distribution of the normalised residuals.

PREDICTION CAPABILITY FOR DIFFERENT PROBLEM TYPES

Consider for example, a set of similar sites being all of the down-array soil sites in Japan which are instrumented as part of the KiK-Net project (Kinoshita, 1998). The computational model methodology adopted for such sites could be one-dimensional wave propagation models to predict the seismic response of surficial soil deposits. In this case, the results which would be observed in the form of Figure 4 and Figure 5 would provide validation for the use of one-dimensional models of site response.

Clearly, the number of prediction-observation pairs which could be used to obtain normalised residuals in this example would be large (Kinoshita, 1998), and based on the observed distribution of the normalised residuals (i.e.Figure 4) it may be desirable to understand the predictive capability of the computational model within these such sites as a function of one or more 'problem characterisation parameters'. Such problem characterisation parameters could be the intensity of the input ground motion observed at the base of the down-hole array, or the aspect ratio (depth divided by width) of the sedimentary basin, among others. In order to examine such trends, the distributional properties of $z_{i,j,k}$ can be considered as dependent on the various problem characterisation parameters, $\boldsymbol{X}$, of interest. When this dependence on $\boldsymbol{X}$ is considered, the mixed-effects regression of the normalised residuals can be expressed as:

$$z_{i,j,k} = f(\boldsymbol{X}) + \eta_{i,j} + \varepsilon_{i,j,k} \tag{5}$$

where $f(\boldsymbol{X})$ represents some pre-determined function (with unknown parameters obtained from regression) of the site characterisation parameters; and $\eta_{i,j}$ and $\varepsilon_{i,j,k}$, as before, are the inter-site and intra-site residuals with zero means, but now with variances $\sigma_S^2(\boldsymbol{X})$ and $\sigma_O^2(\boldsymbol{X})$, respectively. The point-estimates of the mean and variance of the model of the residuals in Equation (5) is hence given by:

$$\hat{\mu}_z(\boldsymbol{X}) = f(\boldsymbol{X}) \tag{6}$$

$$\hat{\sigma}_z^2(\boldsymbol{X}) = \sqrt{\sigma_S^2(\boldsymbol{X}) + \sigma_O^2(\boldsymbol{X})} \tag{7}$$

Figure 6 illustrates two possible cases which may be observed. With reference to the aforementioned site response example, Figure 6a may represent that as the intensity of the input ground motion (the problem characterisation parameter) increases the computational model systematically over-predicts (leading to negative normalised residuals) the observed peak accelerations at the ground surface. On the other hand, Figure 6b may illustrate that an increasing basin aspect ratio (the problem characterisation parameter), which leads to an increasing effect of two-dimensional wave propagation, causes a significant increase in the imprecision of the computational model.

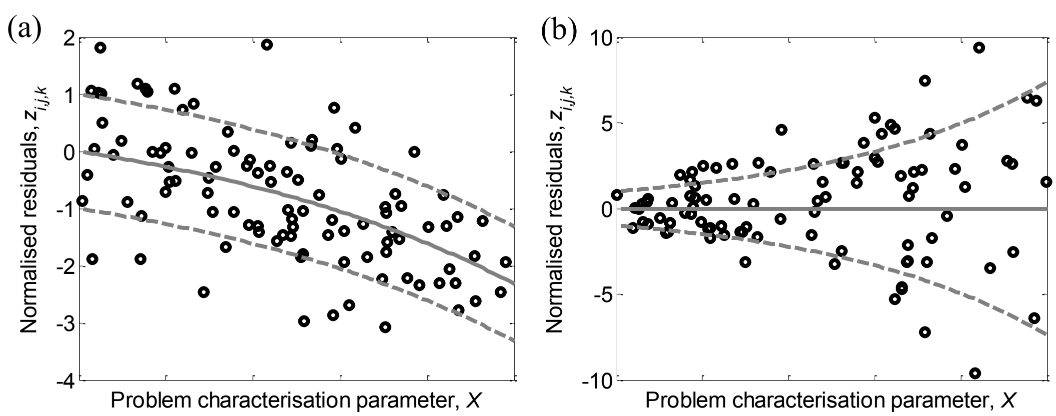

Figure 6: Possible observed trends in the normalised residuals with a particular parameter characterising the problem: (a) observed model bias with increasing *X*; and (b) increasing model imprecision with increasing *X*.

The proposed validation framework has so far being presented in a general sense, but its application for specific problems requires potentially additional considerations. As the complexity of the seismic response analysis increases the importance of an 'expert analyst' becomes pivotal in the development of a computational model based on interpretation of the physical problem. Therefore the 'expertise' of an analyst represents a further uncertainty in the seismic response analysis. Although no attempt is made here to consider this in the proposed framework, it follows that those computational models which require the least input from analysts (e.g. total stress equivalent-linear one-dimensional site response) are more directly amenable to utilization of the proposed framework. The proposed framework is still applicable however to complex computational models, but care is needed to ensure that the obtained results are not devoid of the correct use and interpretation of the model which an expert user offers.

CONCLUSIONS

This manuscript has presented a framework for the validation of computational models used to predict seismic response based on observations from seismometer arrays. The framework explicitly accounts for the epistemic uncertainties related to the unknown characteristics of the 'site' (i.e. the problem under consideration) and constitutive model parameters. The statistical significance of inferences regarding the accuracy and precision of the computational modelling methodologies and constitutive models is enhanced via the use of a mixed-effects model in which multiple prediction-observation pairs are considered. The benefits of the formal validation framework include: (i) development of consistent methods for determination of constitutive model parameters; (ii) rigorous, objective and unbiased assessment of the validity of various constitutive models and computational methodologies for various problem types and ground motion intensities; and (iii) an improved understanding of the uncertainties in computational model assumptions, constitutive models and their parameters, relative to other seismic response uncertainties such as ground motion variability.

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