5ICEGE
5th International Conference on Earthquake Geotechnical Engineering
January 2011, 10-13 | Santiago, Chile
Paper No. IOSEV

INTERPRETATION OF SURFACE WAVE METHODS IN 2D STRATIGRAPHIC CONDITION

Lorenza EVANGELISTA [1], **Filippo SANTUCCI de MAGISTRIS**[2]

ABSTRACT

Some considerations on the use of the non-invasive Multichannel Analysis of Surface Wave technique and its uncertainty in the interpretation of test results are described here. The common assumption of one-dimensional model, which does not take into account the morphology of the superficial and deep layers on the dispersion curve and the influence of bi-dimensional stratigraphic condition on it are discussed. Several simulations of the MASW tests using a finite element code have been carried out, showing significant variations in the dispersion curve in the presence of complex site conditions.

.

Keywords: Surface waves, MASW tests, Finite-element method, Bi-dimensional stratigraphy condition

INTRODUCTION

Over the past few decades, a considerable number of studies have been made on the development of seismic methods based on measurements of surface-waves; in particular, the Spectral Analysis of Surface Wave (SASW method and its evolution to the multi-channels configuration, MASW), aimed at understanding the evaluation of shear wave velocity profiles. The non-invasive nature of surface waves methods allows performing tests efficiently, based on the dispersion behaviour of Rayleigh waves in a vertical heterogeneous soil deposit, but, the flexibility of the methods, compared to the relative simplicity of experimental procedures, require a complex interpretation of the measurements taken at a site.

Despite the recent developments in the technique, several issues remain uncertain or unresolved. These issues include how to properly account for near-field effects, non-uniqueness of the inversion problem, and difficulties in detecting punctual soil stiffness at a given depth, especially when it is necessary to analyze in detail the influence of surface and buried morphology on the particular problem in hand. The interpretation of 'indirect' measures, such as those relating to the MASW technique is typically performed by introducing simplifying assumption (Constable et al., 1987). The larger-one is related to subsoil model, that is assumed being as one-dimensional with horizontal stratification, while the behaviour of the soil is approximate as elastic, homogeneous and isotropic (Lai, 1998).

In the following, after describing the main features of the MASW experimental procedure, the results of numerical analyses carried out with the finite element code Plaxis (Brinkgrave, 2002), in order to simulate numerical dispersion curves in stratigraphic bi-dimensional conditions, are presented.

---

[1] Researcher, Department of Hydraulic, Geotechnical and Environmental Engineering, University of Naples Federico II, e-mail: lorenza.evangelista@unina.it
[2] Lecturer, Structural and Geotechnical Dynamic Lab., StreGa, S.A.V.A. Department, University of Molise.

SURFACE WAVE METHOD

Rayleigh waves are surface waves that travel in a confined zone along the free surface of the propagation medium involving only a limited superficial portion of it, whose thickness is nearly equal to a wavelength. The basic principle of the application of the experimental technique is founded on the dispersivity of surface waves in vertically heterogeneous media: the velocity of each harmonic component of the surface wave depends on the properties of the medium affected by the wave propagation. Rayleigh waves of longer wavelengths (high frequency) tend to propagate faster than Rayleigh waves of shorter wavelength (low frequency). The dispersive nature of Rayleigh waves is due to the direct relationship between their wavelength and their zone of influence: waves with different wavelength (or frequencies) sample different parts of the medium (Stokoe et al., 1994), allowing them to be used in determining the variation of material properties with depth (Figure 1a). Therefore, the Surface Waved Method consists of conducting field tests, determining experimental dispersion curves, and constructing the stiffness profile (Jones 1962; Stokoe et al, 1994). The key point in a MASW field test is to generate primary Rayleigh waves and measure the vertical particle motions of Rayleigh waves.

Here, the attention is focused on surface wave tests for shallow survey in soil, considering the typical depth of interest of geotechnical engineering problems (depth < 50m). Then, the basic testing equipment employed in this research is composed of a source of surface waves and some receivers connected to instrumentation to record and process the data (Figure 1b). The source of mechanical waves is an electro-mechanical shaker controlled by a function generator, which allows the generation of a harmonic force in the frequency range of 5-120 Hz. The receivers are 14 piezo-electric 1-D accelerometers with a wide dynamic range (0.1 - 300 Hz), placed following a linear array with a total length of 29 m (Figure 1b).

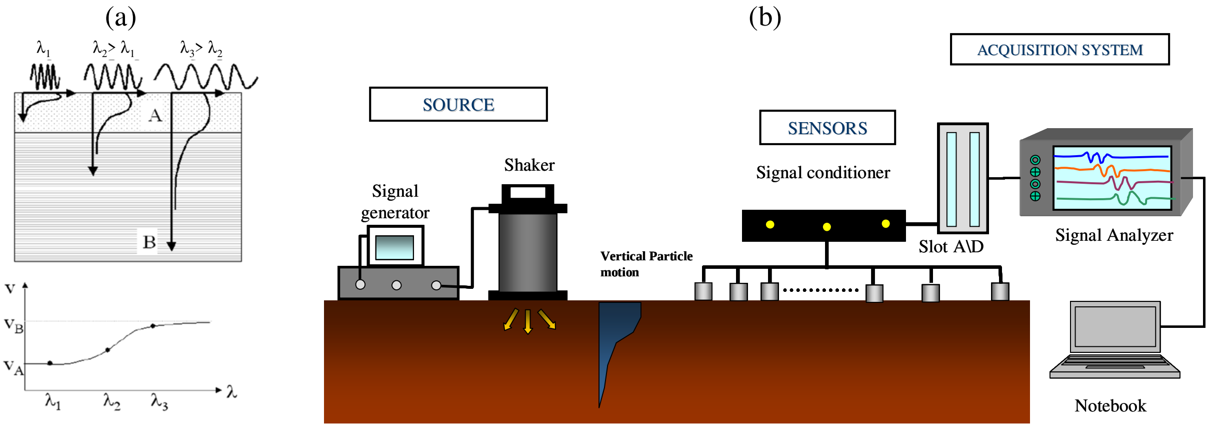


Figure 1: (a) Surface waves in a layered subsoil; (b) experimental configuration for an active MASW test (Evangelista, 2009)

The dispersion calculations are performed in the frequency domain to let both the magnitude and the phase to be elegantly and efficiently represented using complex notation.

Current dispersion calculations are conducted using a frequency wavenumber *(f-k)* procedure (Zywicki & Rix, 1999), obtained as the amplitude peaks in the f-k (frequency-wavenumber) domain, where different curve branches identify the multi-modal response of the layered subsoil.

The evaluation of the shear wave velocity profile is obtained with the resolution of the inversion problem with global algorithms, such as Monte Carlo (Evangelista, 2009). The inversion methods are based on a simple idea: a large number of shear wave velocity profiles are randomly generated, with some given constraints. For each profile, the correspondence with the experimental data is assessed in terms of dispersion curves. Only the profiles for which the misfit of the dispersion curve with the experimental data is below a fixed threshold are accepted.

In this approach, the inversion of surface wave dispersion is done with horizontally homogeneous model. In real observations, the subsurface structure can be bi- and three-dimensional with layer thicknesses expected to vary along the path of propagation, so that the ray path of the waves often crosses different geological provinces. The measured curves are most sensitive to the vertical variation of V_S, while lateral variations are largely averaged off during data processing. On the consequence, 1-D model is taken to represent an average of the structure along the path. This means that the Surface Wave Method gives the 1-D V_S structure most representative of the surface materials below the receiver spread by approximating them as a layered soil profile model despite the fact that, in reality, a certain degree of lateral variation often exists.

Because of the nature of multi-channels processing, it is usually assumed the 1-D V_S profile at location of the source. As previously explained, the Surface Wave Method asses the average V_S profile of the subsoil volume detected during the test, and do not provide punctual soil stiffness. Clearly, this aspect influences the correct evaluation of impedance contrast depth and the possibility to keep the correct subsoil geometry in 2-D conditions.

The use of numerical simulations is a straightforward means for studying this kind of problem, especially when laterally-varying velocity structure or complex topographic relief is encountered.

NUMERICAL SIMULATION

The close-form solution of surface wave propagation through a finite two-dimensional medium is relatively complex. The finite-element commercial code Plaxis (Brinkgreve, 2002) is used in this work to study the surface wave propagation characters in bi-dimensional models.

It is worth mentioning that the equation of the motion is solved with the Newmark time integration scheme and it can simulate direct and reflected body wave and material attenuation. The analyses are aimed at simulating by numerical tools the real execution of the MASW test, therefore it is assumed that the wave propagation occurs in a vertical plane and no lateral reflected waves are present. The Surface Waves Method is modelled as an axisymmetric problem with an impulsive or harmonic source at the origin of the system.

In Figure 2 the definition of the geometrical dimension of the model for two-dimensional analysis (H=50 m, D=100 m) satisfies the following conditions:
-the time record should be long enough to let the Rayleigh wave reaching the last receivers, whose position (its coordinate) is here indicated as R_N:

$$t_{\max} > \frac{R_N}{V_R} \tag{1}$$

-the model should be large enough that compression waves are not reflected from the boundaries:

$$t_{\max} < \min\left\{\frac{2D - R_N}{V_P};\frac{2D + R_1}{V_P};\sqrt{\frac{H^2 + (R_1/2)^2}{V_P}}\right\} \tag{2}$$

In eq. (1) V_R is the Rayleigh wave velocity; in eq. (2) V_P is the P wave velocity and R_1 is the coordinate of the first receiver.

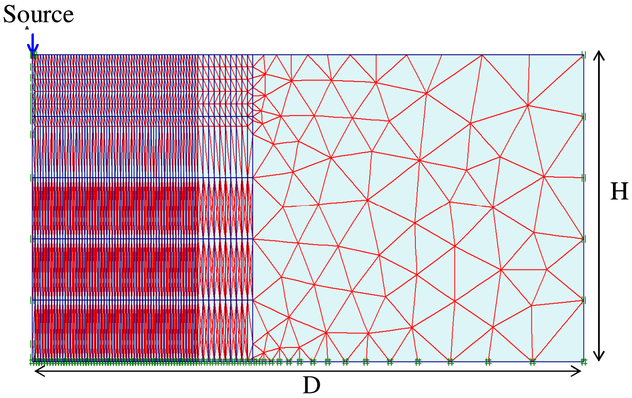


Figure 2. Axisymmetric finite element mesh

The properties of the medium (Young's modulus E, Poisson ratio ν, mass density ρ and material damping D) and frequency content of the propagating pulse govern the wave propagation in MASW tests. Since the induced shear strains are smaller than 0.001%, an elastic medium is assumed. Material damping is not considered in this study.

Regarding the mesh, the size of each element is determined by minimizing numerical errors in the phase velocity for the wavelength of interest. An element size not more than λ/6 (Lysmer & Kuhlemeyer, 1969) is chosen for the area between the source and the last receivers (29 m from the origin) consisting in a uniform grid, after that the element size increases gradually toward the other boundary. The time step is small enough to capture all of the desired frequencies accurately, and to provide a numerically stable integration over time for transient dynamic analysis. The stability limit is approximately equal to the time needed for the wave to cross the smallest element in the model.

The model calibration has been performed by comparing the results in terms of geometrical spreading and dispersion curve with the theoretical solution proposed for the field of motion generated by harmonic load acting on the surface of a horizontal homogenous half-space. The seismic source is modelled by a series of harmonic loads with a duration of 0.2 s at frequencies of 6.3; 9.45; 25.5; 50.4; 97 Hz. Measures of vertical displacements and vertical accelerations involving the first 25 meters from the source are analysed.

From Figure 3a it is also possible to observe also the first arrive of body waves that can be seen as an impulse, followed by wave packets and dispersed surface waves. It is clear the time delay in the arrival of waves at each point receivers. As required by the theoretical solution, in the numerical analyses the motion of surface waves generated has a polarization of backward type and fall exponentially with depth (Figure 3b). Analyzing the shape of the particle motion, the positive values of u_x/u_z indicate that the particle orbit is prograde or direct. Because of the horizontal particle motion is not either behind or ahead the vertical on 90 degree in phase, like in the ideal case, it is confirmed the condition for which the wavefield consists not only of plane Rayleigh waves.

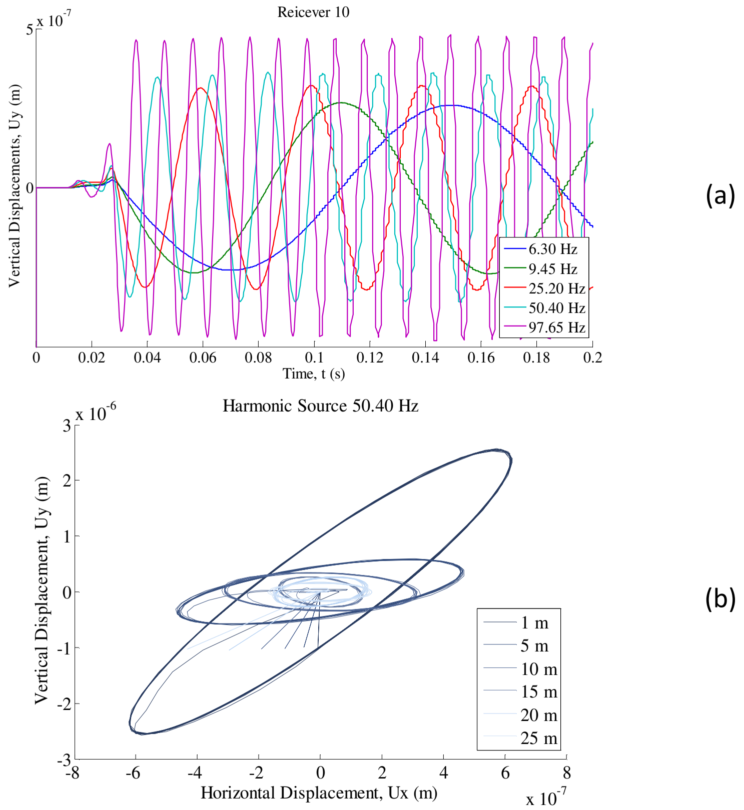


Figure 3: (a) Example of vertical displacements distribution;
(b) Example of particle motion for a harmonic source of 50.40 Hz

Two types of body waves appear in the wavefield: one propagating at the P-wave velocity and the other propagating at the S-wave velocity. When attempting to monitor particle motions associated with the P-waves, the motions associated with the S-waves represent a near-field effect, which decreases with increasing a distance from the source. When exciting pure S-wave motions, the P-wave motions result in a near-field effect. This aspect is clearly visible comparing the theoretical attenuation law of body and surface waves with the geometrical attenuation evaluated for the simulated displacement distribution at difference frequencies (Figure 4).

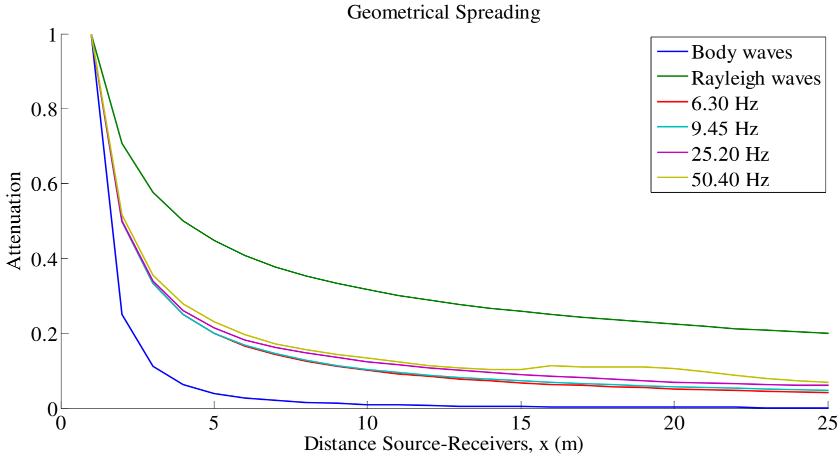


Figure 4. Geometrical Spreading

Comparing the theoretical attenuation law of body and surface waves with the geometrical attenuation at different frequencies (Figure 4) it is observed the influence of near field effects on the propagation wave simulation. The presence of body and surface waves increases the attenuation compared with the theoretical attenuation law of Rayleigh waves; this is related to near field effects, which are more observable in the deeper layers. In fact, these effects decrease with increasing frequency.

One-dimensional soil model

As previously underlined, the inversion procedure for the MASW test is made in the hypothesis of one-dimensional subsoil model for the experimental dispersion curve.

Here, numerical simulations of a transient in situ test in presence of different two-layer normal dispersive soil profiles to obtained three different impedance ratios are performed. In Table 1, the geometrical parameters and material properties of the soil models, used in the numerical simulation, are presented.

Table 1. Parameters of soil models

		Impedance 1.5	*Impedance 1.75*	*Impedance 2*
Layer 1	V_S (m/s)	400	400	400
Layer 2	V_S (m/s)	600	700	800
Depth of interface	z (m)	5 – 7 – 9 - 11	5	5

A transient in situ test is simulated; the seismic source is an impulse with duration of 1 s and the sampling frequency is assumed equal to 640 Hz. Measures of vertical acceleration, sampled with an inhomogeneous 14 channels array [“virtual” measurement points are placed at 2.4; 3.0; 3.7; 4.6; 5.5; 6.7; 8.5; 10.4; 12.8; 15.2; 18.3; 21.3; 24.4; 29.0 metres from the source] are elaborated with the f-k analysis to obtain the dispersion curve.

Figure 5 shows an example of tree-dimensional plots of array signal processing analysis, conducted with the vertical accelerations provided by the numerical simulations. Both the absence of errors caused by noise from data measurement and the analysis of the data to create the dispersion curve attempting to represent an assumed real model and a real wave propagation path can be appreciated from the figure.

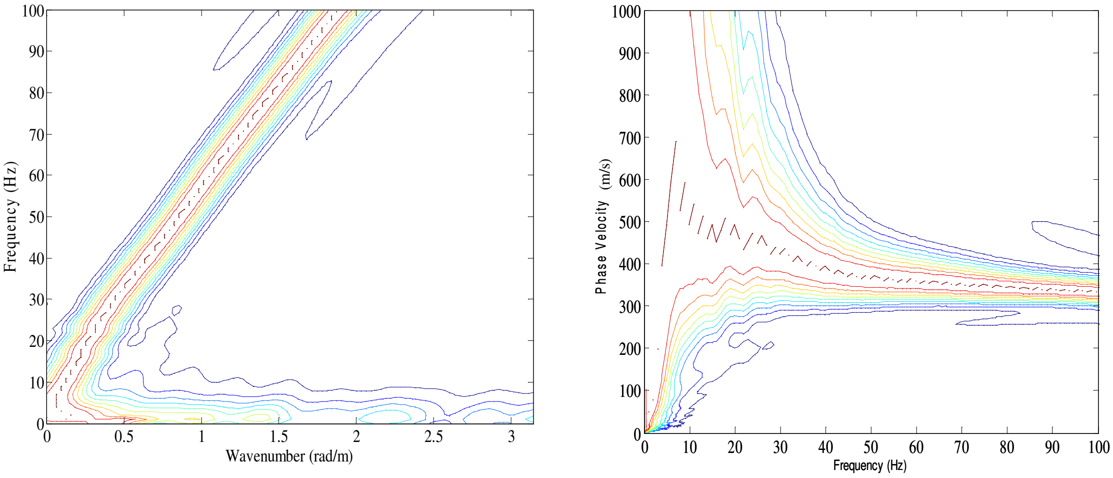


Figure 5: Frequency-Wavenumber Beamforming Analysis

The latter aspect is confirmed by solving the forward problem to evaluate the theoretical dispersion curve for the same soil profile that could be assumed as a dispersion curve free from near-field effects and array effects. Note that the dispersion curve of full wavefield, on the contrary, includes certainly near-field and array effects.

The comparison between the numerical and the dispersion curves in Figure 6a shows a good agreement. For the assumed array configuration, for which the near-field effects are negligible, the gap at shallow layer may be explained due to stability problems in the numerical analyses.

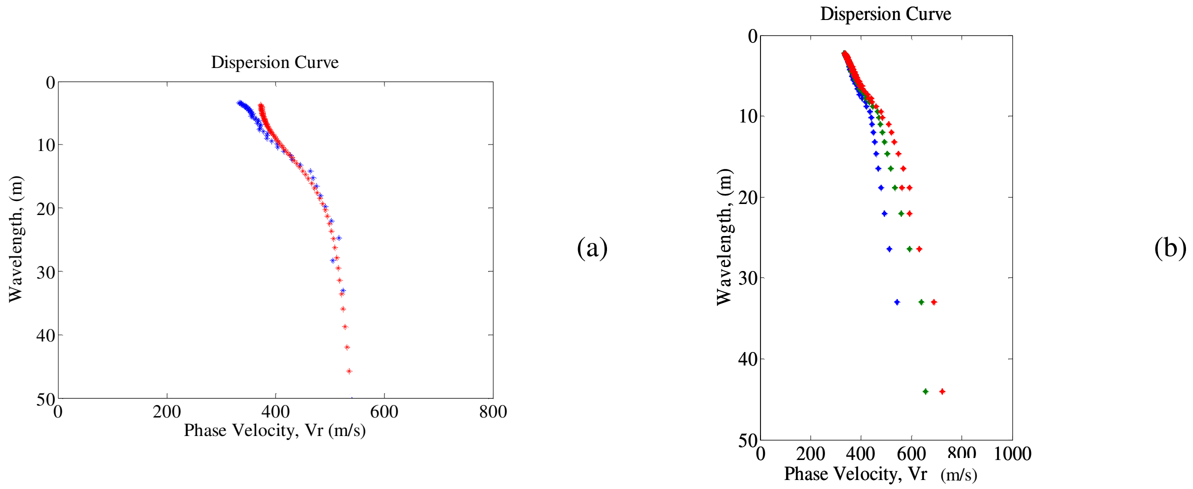


Figure 6: (a) Example of comparison between numerical (blue) and theoretical (red) dispersion curve for a two-layer normal dispersive soil profile (400m/s-600m/s, 5m-45m); (b) Comparison between numerical (blue) and theoretical (red) dispersion curve for the 3 soil profiles with layer interface at 5 m

To give more consistency to the validation of the model, in Figure 6b the dispersion curves for all the adopted V_S profile are compared. It is evident the correct interpretation of the increment of the impedance ratio between the layers, as well as the detection of the depth of the horizontal layer interface, fixed at 5 m.

Transient in situ tests are also simulated for the same two layer normal dispersive soil profile at impedance ratio 1.5 (Table 1), but assuming four different subsoil configurations, with depth interface moved from 5 to 11 m, with a step of 2 m. Among others, these analyses are carried out to verify the correct interpretation of the dispersive behaviour of surface wave by the numerical analyses in layered profile. The temporal and spatial sampling and the signal processing are the same as previously illustrated, as well as the array signal processing. Numerical analyses capture correctly the layered interface at the different depth (see Figure 7). By the distributions of phase velocity it is possible to observe that when increasing of the thickness of the first layer, the dispersion curves tend to the ones relevant for the homogenous half-space.

It is possible to observe, also, that the phase velocity is lower than the reference value. The underestimation of the dispersion value, especially for the shallow layer, could be primary caused by near-field effects.

Bi-dimensional simplified soil model

The effects of stratigraphic irregularities on the dispersion curve are analyzed through the simulation of a transient test, whose experimental setup was already described.

The 2-D stratigraphic configuration is modelled assuming the two- layer soil profile, previously described, introducing a variable inclination of the interface between two layers.

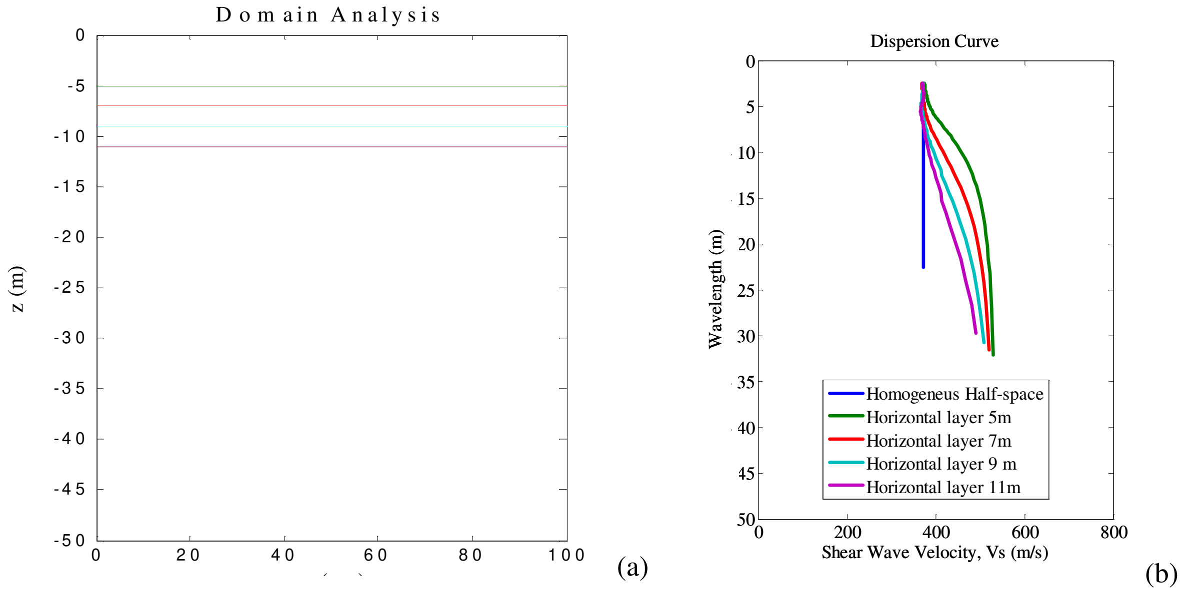


Figure 7: (a) One-dimensional soil model; (b) Dispersion Curve varying the depth of the soil interface

Different values of slope interface are assumed from 5% to 25% from the condition of horizontal stratification at 5 metres from the ground level (Figure 8). Different values of the impedance ratio are also considered. For each value of the slope inclination interface, both upslope and downslope test are performed to evaluate the influence on the dispersion curve of the source position, compared to the immersion of the layered interface. All the values for the analysis are presented schematically in Table 2

Table 2. Parameters of soil models

		Impedance 1.5	*Impedance 1.75*	*Impedance 2*
Layer 1	V_S (m/s)	400	400	400
Layer 2	V_S (m/s)	600	700	800
Slope interface	(%)	5 – 10 – 15 – 20 -25	5 – 15 - 25	5 – 15 - 25

The analyses are performed simulating a transient in situ test: the seismic source is an impulse with duration 1 s and the sampling frequency equal to 640 Hz.

Measures of vertical acceleration, sampled with a homogeneous 29 channels array (“virtual” measurement points are placed from the source at 2 m for a total length of 30 m with an equally spacing of 1 m). The choice of using larger points of observation is in order to obtain a better wavenumber resolution in the valuation of the dispersion curve, to avoid errors due to the array configuration.

To distinguish the dispersion curve trend, the numerical curves for each slope interface are compared with the numerical one relating to the one-dimensional subsoil condition.

In long wavelength range of the downslope tests, phase velocity of the Rayleigh waves is visible smaller compared with the one-dimensional condition, showing visible different dispersion behaviour. In particular, at the increasing of slope interface, phase velocity tends to the values of the phase velocity expected in the upper layer, which has a predominant effect on the light of the characteristics of the geometrical model. This behaviour could be explained by the body waves that reflect in the interface between the upper and lower layer, disturbing phase information of the captured signals.

It is clear that the surface wave method provide an average value of the elastic properties of the volume investigated, in contrast with the traditional assumption of punctual information referred to the subsoil profile near the source.

For strong lateral variations, the average dispersion curve can produce very different 1-D inverted models depending on the parameterization. Also, the 1-D inverted models might differ significantly from the average properties of the inhomogeneous path, and wrong depths to interfaces might be inferred.

In this framework, the interpretation of dispersion curves in complex stratigraphic condition, using a one-dimensional model, would lead to an overestimation of the thickness of the surface actually affected by the phenomenon of the propagation.

The downslope and upslope dispersion curve are not the same, but the same observation can be reached analyzing the dispersion behaviour. The influence of the source position on the immersion of the slope interface is evident: as the inclination are getting larger, dispersion curve in low frequency range have the bigger difference.

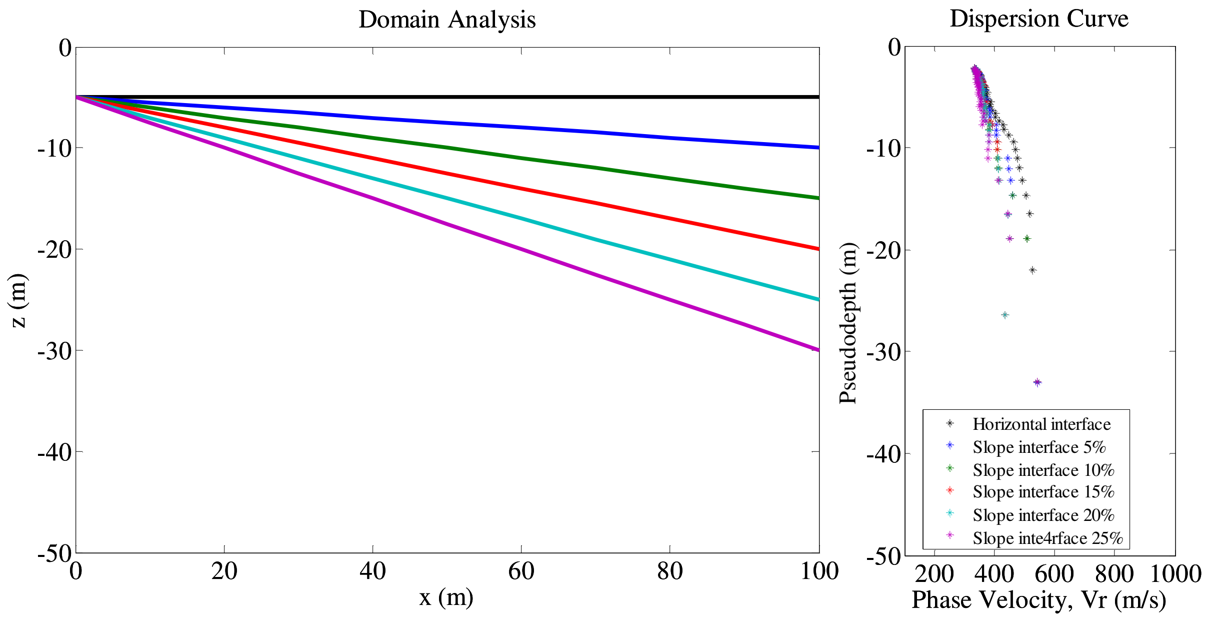


Figure 8. Example of dispersion curves for the analyses of non sub-horizontal sites

ANALYSIS OF THE RESULTS

To quantify the uncertainty, related to the improper application of dispersion theory from 1-D model into real 2-D subsoil profile, the following root mean square (*rms*) error is applied:

$$e_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} \left(V_{i1D} - V_{i2D}\right)^2}{N}} \qquad (3)$$

where N is the number of frequency and V_{1D} and V_{2D} are the shear wave velocities obtained, respectively, by 1-D and 2-D models.

To avoid large rms error (i.e., difference between 1-D and 2-D models) at low frequencies that going to dominate over those occurred at high frequencies (thus, causing a loss of resolution for the more superficial layers) the i^{th} e_{RMS} (which refers to the frequency f_i) is multiplied by a factor w_i calculated as:

$$w_i = \sqrt{\frac{f_i}{f_{\max}}} \qquad (4)$$

where f_{max} is the maximum frequency in the dispersion curve analyzed.

The results are summarized in Figure 9, plotting the rms error against the slope interface.

The same errors are evaluated for the three impedance ratio assumed. It is confirmed the increasing of error assumption with the increasing of the slope interface, but also a linear relation between the parameters can be observed. Also, the relation could be considered approximately the same in the both situation. However the error is higher when the slope interface is in opposition with the direction of the wavefield propagation.

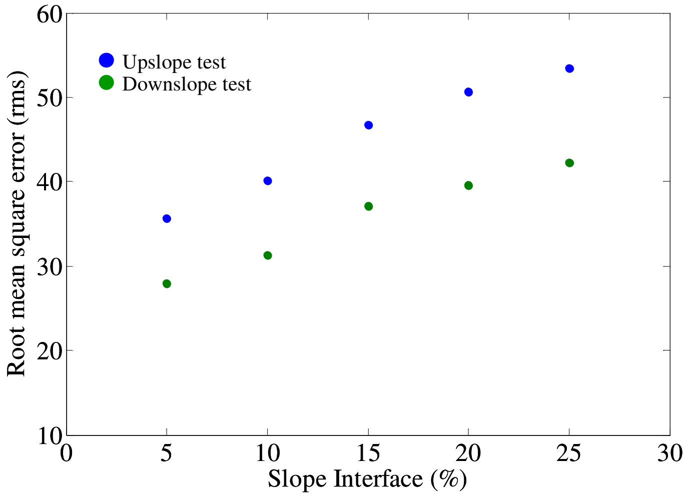
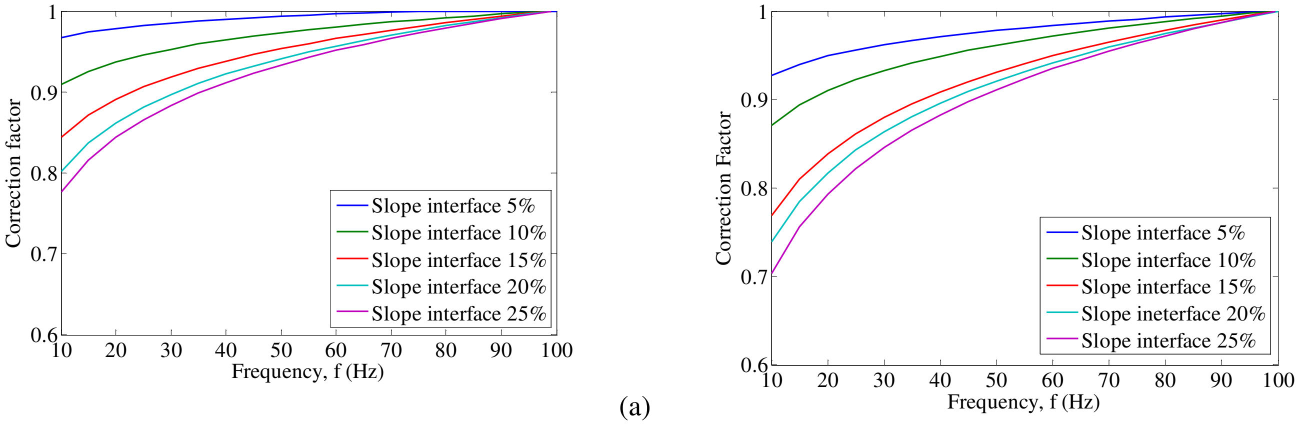


Figure 9: Root mean square error vs. slope interface: upslope test (blue dots); downslope test (green dots)

These results could be explained by the multiple reflections that occurred in presence of a stiff formation in the propagation path. The result indicates that the velocity model is more important than the frequency contents of the source in order to image clear dispersion curves. It seems reasonable supposing that the resolution of dispersion curves is determined by velocity models, together with that the number of traces and the inclusion of further offset. The frequency contents of the source could have small effects on the dispersion curves, as long as the data acquisition is precise.



(a) (b)

Figure 10. Correction Factor: (a) downslope test; (b) upslope test

It is observable, that the error is dependent also by the array configuration, especially by its length. It is clear that the greater the extent of the array, the greater is the instigated volume and consequently the uncertainty on the elastic properties of the deposit.

Dimensionless correction factors (Figure 10), as a function of the frequency, are evaluated for all the stratigraphic condition analyzed, being considered an independent variable of the impedance ratio between the layers.

CONCLUSION

This article analyzes some aspects of the interpretation procedures for non-invasive MASW method. In particular, the assumption of the one-dimensional subsoil model, introduced in the inversion process of the experimental dispersion curve is assessed through a series of numerical analysis by finite element method.

A simplified bi-dimensional stratigraphic condition is analyzed, putting into account also the influence of the position of the source in the experimental set up configuration and the impedance of the soil deposit.

The numerical analyses remark the influence of the morphology of the superficial and deep layers on the dispersion curve and the errors that are introduced during the inversion process. When simple 2-D stratigraphic condition occurred, correction factors to the shear wave velocity values are proposed.

The proposed procedure seems to be a promising pathway in the interpretation of the results when the experimental technique has uncertainties due to the non–regularity of the morphological and stratigraphic conditions, with the introduction of correction factors in the inversion problem.

NOTE

The work presented in this paper is a part of the Ph.D. thesis of Lorenza Evangelista that was supervised by late Prof. Filippo Vinale. The Authors would like to remember here his passion for the University, his precious suggestions and brightness in developing research ideas in and his great capability in organizing his research team.
The support from the Soil Dynamic group at the University of Naples and from the StreGa lab of University of Molise in developing the MASW technique is also acknowledged.

REFERENCES

Brinkgreve, R.B.J. (2002). Plaxis 2-D version 8. A.A. Balkema Publisher, Lisse.

Constable, S.C., Parker, R.L., and Constable, C.G. (1987). Occam's Inversion: A Practical Algorithm for Generating Smooth Models from Electromagnetic Sounding Data. Geophysics, 52, pp.289-300.

Evangelista, L. (2009). A Critical Review of the MASW Technique for Site Investigation in Geotechnical Engineering. PhD. Thesis. University of Naples Federico II, http://www.fedoatd.unina.it/2034/

Jones, R. (1962). Surface Wave Technique for Measuring the Elastic Properties and Thicknesses of Roads: Theoretical Development. British Journal of Applied Physics, 13.

Lai, C.G. (1998). Simultaneous Inversion of Rayleigh Phase Velocity and Attenuation for Near-Surface Site Characterization. Ph.D. Dissertation, Georgia Institute of Technology.

Lysmer, J. and Kuhlemeyer, R.L. (1969). Finite dynamic model for infinite media. ASCE Journal of the. Engineering Mechanics Division, ASCE, pp. 859-877.

Stokoe, K.H.II, Wright, S.G., Bay, J.A. and Röesset, J.M. (1994). Characterization of Geotechnical Sites by SASW Method. Geophysical Characterization of Sites. XIII ICSMFE. Oxford & IBH Publishing Co.PVT.Ltd. New Delhi.

Zywicki, D.J. and Rix G.J. (1999). Frequency-Wavenumber Analysis of Passive Surface Waves. Symposium on the Application of Geophysics to Engineering and Environmental Problems, 75-84, Oakland, CA.