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DETERMINATION OF MATERIAL DAMPING IN THE SOIL BASED ON THE HALF-POWER BANDWIDTH METHOD AND SPATIAL DECAY OF THE ARIAS INTENSITY IN THE SASW TEST

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ABSTRACT

The Spectral Analysis of Surface Waves (SASW) method is widely used to determine the dynamic shear modulus and the material damping ratio of soils. It is based on an in situ experiment where surface waves are generated by means of an impact hammer, a falling weight, or a hydraulic shaker. The resulting wave field is recorded by a number of sensors at the soil's surface and used to determine dispersion and attenuation curves. An inverse problem is solved to identify the corresponding shear wave velocity and the material damping ratio profiles: the (theoretical) dispersion and attenuation curves are calculated for a given soil profile and compared to the corresponding (experimental) curves derived from the surface wave test. The soil profile is subsequently adjusted in order to minimize the distance between the experimental and the theoretical curves.

This paper focuses on the determination of the material damping ratio of the soil by means of the SASW method. Two new methods to determine the material damping ratio profile of the soil are presented, based on (1) the half-power bandwidth method and (2) the spatial decay of the Arias intensity.

The proposed methods to determine the material damping ratio are alternatives to existing techniques, where the amplitude spectrum of the wave field is used. Compared to the existing methods, both methods do not rely on the assumption that the wave field in the soil consists of a single Rayleigh mode, which may lead to incorrect results when higher modes contribute to the wave field. Both methods are successfully applied to a test site in Heverlee (Belgium).

Keywords: Material damping, Rayleigh waves, Half-power bandwidth, Arias intensity, Layered media, Shear modulus, SASW.

INTRODUCTION

The SASW method has been used in different applications over the past couple of decades: to investigate pavement systems (Nazarian and Stokoe II; 1984), to assess the quality of ground improvement (Cuellar and Valerio; 1997), to determine the thickness of waste deposits (Kavazanjian et al.; 1994), and to identify the dynamic soil properties for the prediction of ground vibrations (Madshus and Kaynia; 2000; Lombaert et al.; 2006; With and Bodare; 2007). The most important soil properties in the prediction of ground vibrations are the dynamic shear modulus and the material damping ratio.

The SASW method is a well-established technique for the determination of the soil's dynamic shear modulus (Roësset; 1998), the application to the determination of the material damping ratio has only more recently been tackled (Lai; 1998; Rix et al.; 2000). Existing techniques to determine the material damping ratio are all based on the measurement of the spatial decay of surface waves. The spatial decay is due to both

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the dissipation of energy (material damping) and the spreading of the wave fronts over an increasing area (geometrical damping). Geometrical damping is accounted for through the use of a geometric spreading factor, which is calculated based on the shear wave velocity profile of the soil.

Existing methods to determine the material damping ratio are based on the hypothesis that the response of the soil in the SASW test is due to a single surface mode. If multiple surface modes contribute to the response (e.g. due to a high stiffness contrast or the inclusion of a softer layer), this assumption does not hold and the resulting attenuation curves are incorrect (Badsar et al.; 2009). Moreover, the estimate of the attenuation curve is based on an estimate of the geometric spreading factor. The latter is computed using the shear wave velocity of the soil, which is determined by inversion of the experimental dispersion curve. Errors in the experimental dispersion curve or the inversion procedure (e.g. due to the non-uniqueness of the inverse problem) may lead to an incorrect estimate of the geometric spreading factor and, consequently, an incorrect experimental attenuation curve (Badsar et al.; 2009). In this paper, two novel methods for the determination of the material damping ratio are proposed, based on (1) the half-power bandwidth method and (2) the decay of the Arias intensity.

The half-power bandwidth method has originally been developed in the field of mechanical and structural dynamics to determine the modal damping ratio of a structure from the width of the peaks in the structure's frequency response function. In the present paper, the half-power bandwidth method is applied to the wavenumber content of the soil's response. In this approach, the experimental attenuation curve is derived directly from the experimental data, avoiding the use of a (possibly incorrect) estimate of the soil's shear wave velocity. Moreover, the occurrence of multiple Rayleigh modes does not affect the attenuation curve of either the fundamental or the dominant Rayleigh wave, as long as modes occur as separate, non-interfering peaks in the frequency-wavenumber spectrum. When modes interfere, it is not always possible to identify the mode numbers and estimate the degree of their participation, however.

The second method for the determination of the material damping ratio does not involve the determination of the attenuation curve; it is based on the decay of the Arias intensity at the soil's surface. The Arias intensity is a measure of the total energy at each receiver and is proportional to the integration over time of the acceleration squared. The Arias intensity is therefore calculated directly from the experimental data. In this method, there is no need to identify mode numbers. The occurrence and interference of higher Rayleigh modes does not affect the resulting material damping ratio, as the contribution of all modes is properly taken into account in the computation of the theoretical Arias intensity. Compared to the first method, this approach involves a slightly higher computational effort.

This paper consists of five parts, addressing (1) the in situ SASW experiment, (2) the determination of the shear wave velocity profile, (3) the determination of the material damping ratio profile using the half-power bandwidth method, (4) the determination of the material damping ratio from the spatial decay of the Arias intensity, and (5) the verification of the results, where the identified soil profiles are used to simulate the wave field registered in the SASW test.

1. EXPERIMENT

An SASW test is performed at a site in Heverlee (Belgium), where surface waves are generated by means of a hammer impact on a small aluminum foundation. The response has been measured at 90 equidistant locations from 1 m to 90 m from the source.

The wave field recorded in the SASW test is contaminated by noise. To mitigate the influence of the noise, the test is repeated a number of times (i.e. multiple hammer impacts are recorded) and the average transfer function $\hat{H}_{zz}^E(r, \omega)$ from the hammer force to the free field response at a distance r is computed. To this end,

the H_1 estimator (Ewins; 1984) is used:

$$\hat{H}_{zz}^E(r_j, \omega) = \frac{\hat{S}_{ji}(\omega)}{\hat{S}_{ii}(\omega)} \quad (1)$$

with the auto power spectrum $\hat{S}_{ii}(\omega)$ and the cross power spectrum $\hat{S}_{ij}(\omega)$ defined as:

$$\hat{S}_{ij}(\omega) = \frac{1}{N} \sum_{k=1}^N \hat{x}_i^k(\omega) \hat{x}_j^{k*}(\omega) \quad (2)$$

where $\hat{x}_i^k(\omega)$ is the frequency content of the signal recorded in channel i for impact k and $\hat{x}_j^{k*}(\omega)$ is the complex conjugate of $\hat{x}_j^k(\omega)$. The index i refers to the force channel, the index j refers to the j -th receiver, located at a distance r_j from the source.

Figure 1(a) shows the logarithm of the modulus of the transfer function $\hat{H}_{zz}^E(r_j, \omega)$ for the SASW test performed in Heverlee. These results have been obtained using $N = 85$ hammer impacts.

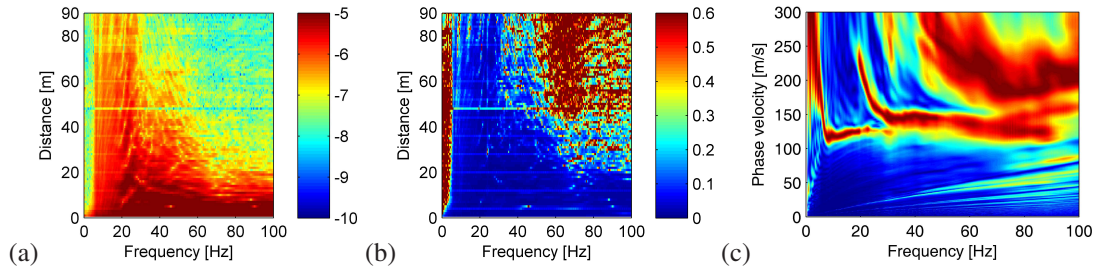


Figure 1. (a) Logarithm of the modulus of the transfer function $\hat{H}_{zz}^E(r_j, \omega)$, (b) the coefficient of variation $\hat{\sigma}_H(r_j, \omega)/|\hat{H}_{zz}^E(r_j, \omega)|$, and (c) the experimental f - k spectrum $\tilde{H}_{zz}^E(k_r, \omega)$ for the site in Heverlee.

Due to the presence of noise, the transfer function $\hat{H}_{zz}^E(r_j, \omega)$ obtained with the H_1 estimator is a random variable, in a sense that each experiment gives rise to a different estimation. Assuming that the noise is stationary, the variance $\hat{\sigma}_H^2(r_j, \omega)$ of the estimated transfer function $\hat{H}_{zz}^E(r_j, \omega)$ can be computed as:

$$\hat{\sigma}_H^2(r_j, \omega) = \frac{1 - \hat{\gamma}_{ij}^2(\omega)}{N \hat{\gamma}_{ij}^2(\omega)} |\hat{H}_{zz}^E(r_j, \omega)|^2 \quad (3)$$

with $\hat{\gamma}_{ij}(\omega)$ the coherency function defined as:

$$\hat{\gamma}_{ij}^2(\omega) = \frac{\hat{S}_{ij}(\omega) \hat{S}_{ij}^*(\omega)}{\hat{S}_{ii}(\omega) \hat{S}_{jj}^*(\omega)} \quad (4)$$

The coefficient of variation of the transfer function is obtained as the ratio $\hat{\sigma}_H(r_j, \omega)/|\hat{H}_{zz}^E(r_j, \omega)|$. A low value of the coefficient of variation corresponds to an accurate estimation of the transfer function. Equation (3) shows that the variance $\hat{\sigma}_H^2(r_j, \omega)$ is inversely proportional to the number of impacts N . As a result, the accuracy of the estimation increases proportionally to \sqrt{N} .

Figure 1(b) shows the coefficient of variation of the transfer function $\hat{H}_{zz}^E(r_j, \omega)$ for the SASW test performed in Heverlee. This coefficient is low in a frequency range with a lower bound of about 4Hz and an upper bound that decreases with the distance from the source. Outside this frequency range, the coefficient of variation increases, indicating a lower signal-to-noise ratio.

2. DETERMINATION OF THE SHEAR WAVE VELOCITY

In order to determine the shear wave velocity profile, a dispersion curve is first determined using the frequency-wavenumber content $\tilde{H}_{zz}^E(k_r, \omega)$ of the transfer function $\hat{H}_{zz}^E(r_j, \omega)$. The f - k spectrum $\tilde{H}_{zz}^E(k_r, \omega)$ is computed by means of a discrete approximation to the Hankel transformation (Forbriger; 2003). Figure 1(c) shows the f - k spectrum for the site in Heverlee. This spectrum has been normalized for each individual frequency. It exhibits peaks corresponding to the Rayleigh modes. The peak corresponding to the dominant Rayleigh wave is identified and the dispersion curve $C_R^E(\omega)$ is derived from the peak's position. The resulting dispersion curve is shown in figure 2(b). The experimental dispersion curve $C_R^T(\omega)$ exhibits a discontinuity at 20Hz, implying that multiple surface waves contribute to the experimental curve.

The shear wave velocity profile is determined from the experimental dispersion curve through the solution of an inverse problem. The ElastoDynamics Toolbox (EDT) for MATLAB (Schevenels et al.; 2009) is used to calculate the theoretical dispersion curve of a soil with a given stiffness profile. The theoretical curve is computed as a multi-mode dispersion curve, with a mode jump from the fundamental to a higher mode at 20Hz. The higher mode cannot be the second mode as this does not lead to an acceptable solution of the inverse problem. The third mode is therefore considered. The shear wave velocity profile $C_s(z)$ is iteratively adjusted in order to minimize the following misfit function that measures the distance between the theoretical and the experimental dispersion curve:

$$\min_{\mathbf{x}} \sum_{i=1}^N \left| C_R^T(\mathbf{x}, \omega_i) - C_R^E(\omega_i) \right|^2 \quad (5)$$

where \mathbf{x} is the vector of design variables. The minimization is performed using $N = 173$ frequencies ω_i from 3.9Hz to 89.9Hz with a step of 0.5Hz. An initial soil profile consisting of 7 layers on a halfspace is considered. The vector of design variables \mathbf{x} includes the thickness of the layers and the shear wave velocity in the layers and the halfspace. The other soil properties (Poisson's ratio $\nu = 1/3$, material damping ratio $D = 0.04$, and density $\rho = 1900 \text{ kg/m}^3$) are kept constant in the inversion procedure.

The minimization problem is solved with a gradient based local optimization method. The resulting shear wave velocity profile is shown in figure 2(a). The corresponding dispersion curve is shown in figure 2(b). A very good match between the experimental and the theoretical curves is observed.

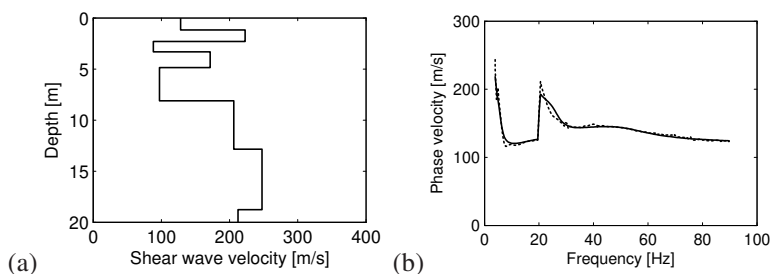


Figure 2. (a) The shear wave velocity profile as determined by the SASW method and (b) the corresponding theoretical (solid line) dispersion curve fitted to the experimental (dashed line) dispersion curve for the site in Heverlee.

3. DETERMINATION OF THE MATERIAL DAMPING RATIO WITH THE HALF-POWER BANDWIDTH METHOD

The first method to determine the material damping ratio is based on the width of the peaks in the frequency-wavenumber spectrum $\tilde{H}_{zz}^E(k_r, \omega)$. The attenuation coefficient $A_R^E(\omega)$ is derived from the peak's width: if

the attenuation is weak, the peak is high and narrow; if the attenuation is strong, the peak is low and wide. To this end, the half-power bandwidth method is used.

The half-power bandwidth method has originally been developed in the field of structural dynamics to determine the modal damping ratio ξ of a structure from the width of the peaks in its frequency response function. The half-power bandwidth $\Delta\omega$ is defined as the width of the peak where the magnitude of the frequency response function is $1/\sqrt{2}$ times the peak value (Chopra; 2007). For a weakly damped single degree of freedom system, the damping ratio ξ is then obtained as follows:

$$\xi = \frac{\Delta\omega}{2\omega_{\text{res}}} \quad (6)$$

where ω_{res} is the resonance frequency. The impulse response of the system is a harmonic function that decays exponentially with time. The temporal decay rate $A = \omega_{\text{res}}\xi$ is equal to:

$$A = \frac{\Delta\omega}{2} \quad (7)$$

The half-power bandwidth method is also applicable to the eigenmodes of a multi-degree of freedom system with widely spaced resonance frequencies. In order to avoid mixing of adjacent peaks in the frequency response function, a more general form of the method can be used where the bandwidth $\Delta\omega$ is defined as the width of the peak where the magnitude of the frequency response function is γ times the peak value, with γ smaller than but close to 1. The damping ratio ξ is then given by:

$$\xi = \frac{\Delta\omega}{2\omega_{\text{res}}\sqrt{\gamma^{-2} - 1}} \quad (8)$$

For $\gamma = 1/\sqrt{2}$, this equation reduces to equation (6). The temporal decay rate $A = \omega_{\text{res}}\xi$ of the modal impulse response function is obtained as:

$$A = \frac{\Delta\omega}{2\sqrt{\gamma^{-2} - 1}} \quad (9)$$

The half-power bandwidth method is applied to the representation of the response of a semi-infinite soil in the frequency-wavenumber domain. The frequency-wavenumber spectrum $\tilde{H}_{zz}^E(k_r, \omega)$ is computed in a similar way as in the previous subsection. However, the truncation of the integral in the discrete Hankel transformation may result in a widening of the Rayleigh peak in the frequency-wavenumber spectrum and, consequently, an overestimation of the attenuation coefficient. In order to mitigate this effect, a window $\hat{w}(r, \omega)$ that decays exponentially with the distance r is applied to the data prior to the Hankel transformation. A similar windowing technique is commonly used in structural dynamics to determine the damping ratio of weakly damped systems from a free vibration signal with a limited length in time. The application of an exponential window can be considered as the introduction of artificial damping, resulting in a stronger spatial decay of the surface waves. The window $\hat{w}(r, \omega)$ is defined as follows:

$$\hat{w}(r, \omega) = e^{-\hat{A}_{\text{art}}(\omega)r} \quad (10)$$

At every frequency ω , the peak corresponding to the fundamental Rayleigh wave is identified. The bandwidth $\Delta k_r(\omega)$ is determined as the width of the peak where the magnitude of the transfer function $\tilde{H}_{zz}^E(k_r, \omega)$ reaches γ times the peak value. The corresponding spatial decay rate or attenuation coefficient $A_R^E(\omega)$ is given by:

$$A_R^E(\omega) = \frac{\Delta k_r(\omega)}{2\sqrt{\gamma^{-2} - 1}} \quad (11)$$

The value obtained is affected by the exponential window $\hat{w}(r, \omega)$ defined in equation (10). The true value of the attenuation coefficient $A_R^E(\omega)$ is retrieved by subtracting the artificial attenuation coefficient $\hat{A}_{art}(\omega)$.

The experimental attenuation curve $A_R^E(\omega)$, shown in figure 3(b), is subsequently inverted to determine the material damping ratio D_s , defined as:

$$D_s = \frac{\Delta E_s}{4\pi E_s} \quad (12)$$

where ΔE_s is the shear energy dissipated during one cycle of shear loading applied to a soil sample and E_s is the maximum shear strain energy stored in a cycle.

In the inverse problem, the profile $D_s(z)$ is iteratively adjusted in order to minimize the following misfit function that measures the distance between the theoretical and the experimental attenuation curve:

$$\min_{\mathbf{x}} \sum_{i=1}^N \left| A_R^T(\mathbf{x}, \omega_i) - A_R^E(\omega_i) \right|^2 \quad (13)$$

where $N = 107$ frequencies ω_i from 6Hz to 59Hz with a step of 0.5Hz are used. Using two layers on a halfspace is sufficient to find a theoretical fit to the experimental attenuation curve $A_R^E(\omega)$. The vector of design variables \mathbf{x} includes the thickness of the layers and the material damping ratio in the layers and the halfspace. The other soil properties (shear wave velocity as determined in the previous section, Poisson's ratio $\nu = 1/3$, and density $\rho = 1900 \text{ kg/m}^3$) are kept constant in the inversion procedure.

The resulting material damping ratio profile is shown in figure 3(a). The corresponding attenuation curve is shown in figure 3(b). The agreement with the experimental attenuation curve is acceptable.

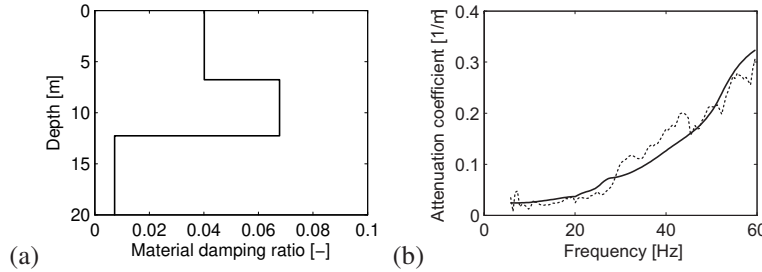


Figure 3. (a) The material damping ratio profile as determined by the half-power bandwidth method and (b) the corresponding theoretical (solid line) attenuation curve fitted to the experimental (dashed line) attenuation curve for the site in Heverlee.

4. DETERMINATION OF THE MATERIAL DAMPING RATIO FROM THE ARIAS INTENSITY

The second method for the determination of the material damping ratio does not involve the determination of the attenuation curve; it is based on the decay of the Arias intensity (Arias; 1970) at the soil's surface. In this method, the experimental Arias intensity $I_{zz}^E(r)$ at a distance r_j from the source is calculated as:

$$I_{zz}^E(r_j) = \frac{\pi}{2g} \int_{-\infty}^{\infty} a_z^2(r_j, t) dt \quad (14)$$

where g is the ground acceleration and a_z is the vertical acceleration measured at location r_j .

Figure 4(b) shows the Arias intensity $I_{zz}^E(r_j)$ at different receivers at the surface at the site Heverlee. The Arias intensity curve $I_{zz}^E(r_j)$ is inverted to give the material damping ratio profile:

$$\min_{\mathbf{x}} \sum_{j=1}^M \left| I_{zz}^T(\mathbf{x}, r_j) - I_{zz}^E(r_j) \right|^2 \quad (15)$$

where $M = 90$ distances from 1 m to 90 m from the source with a step of 1 m are used. The shear wave velocity profile, with 7 layers on a halfspace, determined from inversion of the dispersion curve is considered as the initial soil profile. The vector of design variables \mathbf{x} includes the material damping ratio in the layers and the halfspace. The other soil properties (shear wave velocity and thickness determined from inversion of the dispersion curve, Poisson's ratio $\nu = 1/3$, and density $\rho = 1900 \text{ kg/m}^3$) are kept constant in the inversion procedure.

The resulting material damping ratio profile is shown in figure 4(a). The corresponding intensity curve is shown in figure 4(b). The theoretical and experimental intensity curves correspond very well.

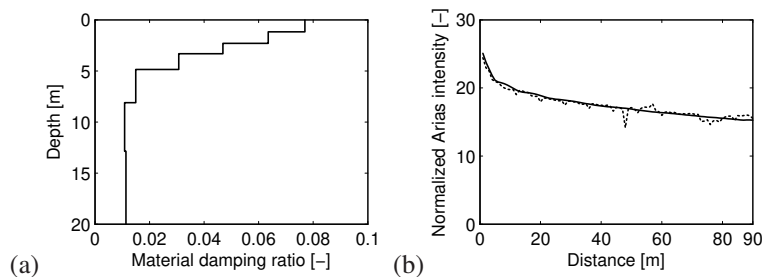


Figure 4. (a) The material damping ratio profile as determined by inversion of the intensity curve and (b) the corresponding theoretical (solid line) intensity curve fitted to the experimental (dashed line) intensity curve for the site in Heverlee.

5. VERIFICATION OF THE RESULTS

The determined material damping ratio profiles obtained from inverting the attenuation and intensity curve and shown in figures 3(a) and 4(a) do not agree well. A direct assessment of the accuracy of these soil profiles is impossible due to the lack of a reference soil profile for the test site in Heverlee. A comparison is therefore performed where the theoretical transfer function $\hat{H}_{zz}^T(r_j, \omega)$ is computed for both soil profiles and compared to the experimental transfer function $\hat{H}_{zz}^E(r_j, \omega)$. The results are shown in figure 5 for three different offsets from the source. Figure 5(a) compares the experimental and the theoretical displacement transfer functions at 2 m from the source. It is observed that both proposed methods lead to a good correspondence with the experimental results at small distances from the source, where the influence of the material damping on the response is small. At larger distances, the second method leads to a theoretical transfer function $\hat{H}_{zz}^T(r_j, \omega)$ that agrees very well with the experimental data, as shown in figures 5(b) and 5(c). The theoretical transfer function $\hat{H}_{zz}^T(r_j, \omega)$ obtained with the first method is slightly worse but still acceptable. While both material damping ratio profiles are completely different, the resulting transfer functions $\hat{H}_{zz}^T(r_j, \omega)$ are quite similar. This is explained by the non-uniqueness in the inverse problem of the SASW method: a large variety of soil profiles can be found that correspond well to the experimental data. Schevenels et al. (2008) performed a detailed study of this non-uniqueness.

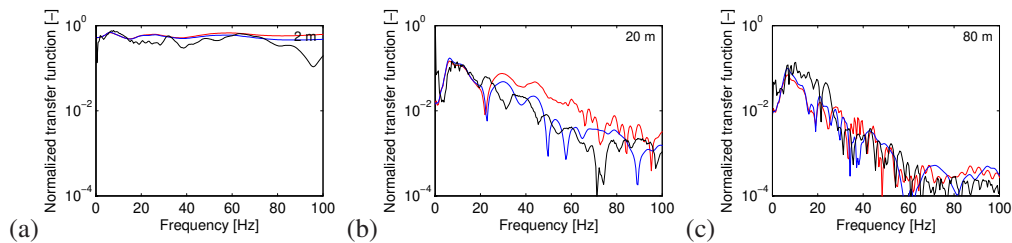


Figure 5. Comparison between experimental transfer functions (black line) with theoretical transfer functions calculated using the soil profile determined with the half-power bandwidth method (red line) and from the intensity curve (blue line) at offsets (a) 2 m, (b) 20 m, and (c) 80 m for the site in Heverlee.

6. CONCLUSION

This paper focuses on the determination of the material damping ratio in the soil by means of the SASW test. Two new methods are proposed. The first method is based on the half-power bandwidth method applied to the width of the peaks in the $f-k$ spectrum to determine the attenuation of Rayleigh waves. The second method uses the spatial decay of the Arias intensity at the surface to determine the material damping ratio. Both methods are used to determine the material damping ratio in the soil at a site in Belgium. The identified soil profiles are used to simulate the wave field registered in the SASW test, which is confronted with the experimental data. Good results are obtained with both methods, the second method performing slightly better than the first.

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