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DISAGGREGATION BASED REGIONAL MAPS FOR LIQUEFACTION ANALYSIS

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ABSTRACT

Probabilistic Seismic Hazard Analysis (PSHA) is one of the main tools used in seismic risk analysis. Throughout the years it has been used for the determination of Peak Ground Accelerations (PGA), uniform hazard spectra, and even liquefaction potential. Disaggregation is a powerful tool that allows understanding of the different contributors to the risk, in terms of magnitude, M_w and distance, R. It also allows attributing statistical weights to the various events (M_w,R) which lead to the attained exceedance value (for example PGA). Using disaggregation, one can choose suitable events for simulation (say by Monte-Carlo analysis) which will lead to the representative PSHA response. When calculating the factor of safety against liquefaction one must consider the earthquake magnitude (which allows scaling of the Cyclic Resistance Ratio, CRR, to magnitudes other than 7.5). This paper suggests the use of regional maps, which are based on PSHA disaggregation, for the calculation of the Magnitude Scaling Factor (MSF). As an example, the paper presents maps for the area of Israel. It is demonstrated that different sites may have significantly different MSF values and coefficients of variation, both are required to assess the probability of having a factor of safety smaller than one. The suggested maps may be seen as a rigorous alternative that eliminates the need to decide, arbitrarily, on a magnitude to be used in the scaling factor.

Keywords: Liquefaction, PSHA, Earthquakes, Magnitude scaling factor, Seismic hazard, Regional maps

INTRODUCTION

Liquefaction is one of the most hazardous phenomena involved with geotechnical earthquake engineering. Even though it has been significantly researched throughout the last 30 years, the most common engineering approach for its evaluation is still based on the “simplified procedure” of Seed and Idriss (1971) and its various modifications (e.g. Seed, 1979; Seed and Idriss, 1982; Youd et al., 2002). In this approach the factor of safety, FS, against liquefaction is calculated as the ratio between a cyclic resistance ratio (CRR) and a cyclic stress ratio (CSR):

$$\mathrm{FS} = \frac{\mathrm{CRR}_{7.5}}{\mathrm{CSR}}\mathrm{MSF} = \frac{\mathrm{CRR}_{7.5}}{0.65(\mathrm{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}\mathrm{MSF} \qquad (1)$$

where PGA is the peak ground acceleration, g is the acceleration of gravity, σ_{v0} and σ'_{v0} are the total and effective overburden stresses, respectively, r_d is the stress reduction coefficient, $\mathrm{CRR}_{7.5}$ is the cyclic resistance ratio for earthquakes of magnitude of 7.5, and MSF is the magnitude scaling factor that allows

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evaluation of the factor of safety at magnitudes other than 7.5. The PGA can be evaluated by either attenuation models (with PSHA approach) or by site amplification calculation following a PSHA for the bedrock. The $CRR_{7.5}$ is commonly obtained through a correlation to the standard penetration test value $(N_1)_{60}$.

This approach is suggested in various seismic provisions and standards (e.g. FEMA 356 or in more details in FEMA 274, SI-940), but without a clear indication of the value that should be assigned to the MSF or the magnitude that should be used to calculate it based on different relations (e.g. Seed and Idriss, 1982; Ambraseys, 1988; Youd and Noble, 1997; Idriss, 1999; Andrus and Stokoe, 2000; Youd et al. 2002).
This paper suggests a method to determine what representative value of MSF should be assigned to a specific site based on disaggregation of the probabilistic seismic hazard analysis (PSHA) of the PGA.

DISAGGRAGATION AND CHOICE OF REPRESENTATIVE EVENTS

Disaggregation (or deaggregation) is a powerful tool that can assist engineers in selecting events that control the specific site. Essentially, it allows insight to the different contributors of the PSHA value. In all probabilistic seismic hazard analyses the following integral is involved (Thenhaus and Campbell, 2003):

$$\lambda[z \geq z^*] = \sum_{i=1}^{N} \alpha_i \int_{M_{\min,i}}^{M_{\max,i}} \int_{R_{\min,i}}^{R_{\max,i}} P[z \geq z^* | M, R] f_{M,i}(m) f_{R,i}(r) dr dm \qquad (2)$$

where $\lambda[z \geq z^*]$ is the annual frequency that the ground motion, at the site, will exceeds the value z^*, α_i is the annual rate of earthquake occurrence at the seismic zone i, $M_{min,i}$ is the minimal magnitude of engineering importance and $M_{max,i}$ is the maximum magnitude of seismic zone i, $f_{M,i}(m)$ is the probability density function of earthquake magnitude (usually corresponds to the Gutenberg-Richter law with upper and lower bounds on the magnitude) of seismic zone i, $R_{min,i}$ and $R_{max,i}$ are the minimal and maximal distances between the site and seismic zone i, and $f_{R,i}(r)$ is the probability density function of the distance from the site to the source at seismic zone i. In the PSHA the inverse problem is answered, in which the value z^* that leads to a prescribe value of λ is found. For example, a λ value of 1/475y (i.e. return period of 475 years) is associated with an exceedance probability of 10% in 50 years. Fig. 1 shows the solution of this inverse problem for the state of Israel for λ value of 1/475y. Fig. 1a shows the different seismogenic zones involved in the analysis, and Fig. 1b shows the obtained z^* values. The parameters for the Gutenberg-Richter law of each seismogenic zone are given in the Appendix. In this example the PGA attenuation model of Boore et al. (1997,2005) is used with a V_s value of 310 m/s, corresponding to the shear wave velocity of soils as suggested by Boore et al. The attenuation model of Boore et al. was used in the creation of the hazards maps of Israel (SI-413), and is used here for consistency.

In conventional liquefaction potential evaluation the MSF is usually computed using the mean or modal magnitude from the disaggregation of the PGA (Mayfield et al., 2010). In the USA one can attain these values from the USGS web site (www.usgs.gov). Such as a service does not exist in all countries and can be replaced by regional maps. Fig. 2 shows a PSHA disaggregation for two sites on the map; one along the cost of Haifa's bay and the other along the cost of Tel-Aviv. As can be seen the two sites are significantly different. Haifa's bay is dominated by a single scenario, while Tel-Aviv is dominated by two (one close with a small magnitude and the other far with a larger magnitude). Commonly, the dominant scenario is defined by either the mean or the mode (the most likely group) of the disaggregation (e.g. Abrahamson, 2006; Katsanos et al., 2010). Selecting the mean value of M_w for determination of MSF may correspond to a scenario that is not realistic in the case of multimodal disaggregation, as in the example of Tel-Aviv, although the mean value is considered a robust parameter due its insensitivity to the bin size of the disaggregation. On the other hand, selecting a representative M_w based on the highest mode is not a robust process, as the mode depends on the bin size (Abrahamson, 2006). An alternative selection of a representative M_w value can result from the following procedure.

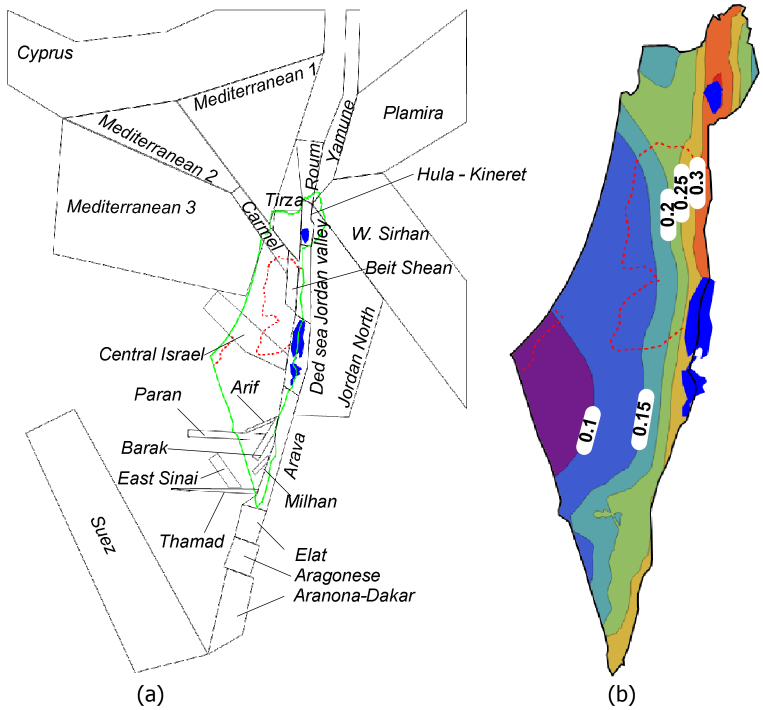


Figure 1. (a) Seismogenic zones and (b) the resultant PGA (g) maps for V_s=310m/s for the state of Israel, return period of 475 years

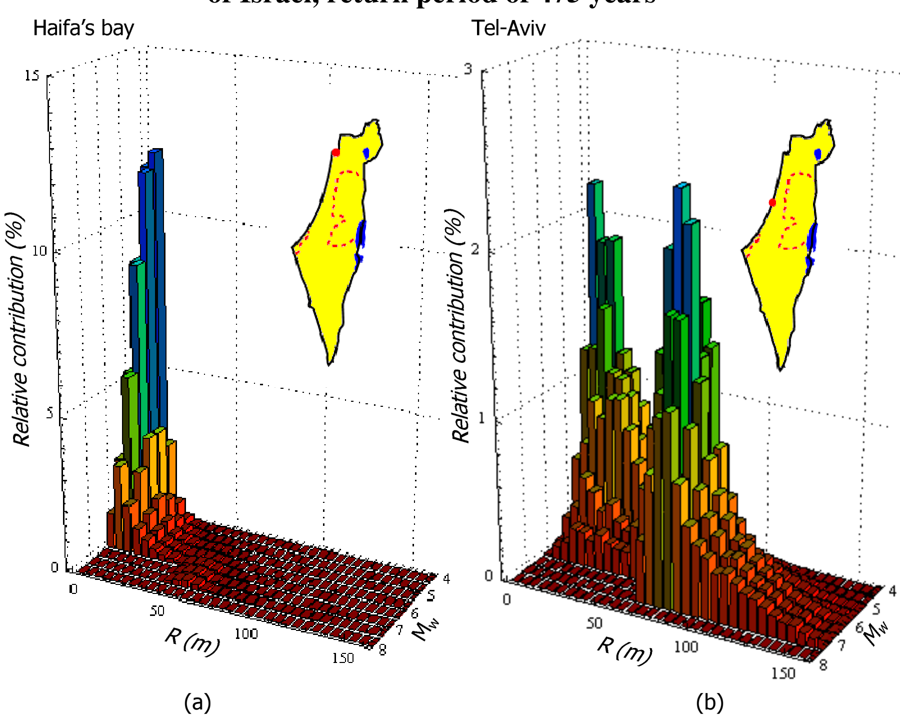


Figure 2. Disaggregation of PGA value at (a) Haifa (PGA=0.23g) and (b) Tel-Aviv (PGA=0.11)

WEIGHTED AVERAGE OF MSF BASED ON THE DISAGGREGATION

A mean value of the FS, which considers all possible events, can be defined as follows:

$$\begin{aligned}\overline{\mathrm{FS}} &= \sum_{i=1}^{N_M}\sum_{j=1}^{N_R} p(i,j)\mathrm{FS}(M_i) \\ &= \sum_{i=1}^{N_M}\sum_{j=1}^{N_R} p(i,j)\frac{\mathrm{CRR}_{7.5}}{0.65\cdot(\mathrm{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}\mathrm{MSF}(M_i) \\ &= \frac{\mathrm{CRR}_{7.5}}{0.65(\mathrm{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}\sum_{i=1}^{N_M}\sum_{j=1}^{N_R} p(i,j)\mathrm{MSF}(M_i)\end{aligned} \tag{3}$$

where N_M and N_R are the number of bins of magnitude and distance, respectively, M_i is the mid magnitude value of bin i, $p(i,j)$ is the relative contribution of bin i,j to the PGA value according to the disaggregation, and MSF(M_i) is the value of the magnitude scaling factor for each M_i. If Idriss's revised MSF values are selected (i.e. MSF=$10^{2.24}/M_w^{2.56}$), as recommended by the NCEER workshop (Youd et al. 2001), Eq. (3) becomes:

$$\overline{\mathrm{FS}} = \frac{\mathrm{CRR}_{7.5}}{0.65(\mathrm{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}\overbrace{10^{2.24}\sum_{i=1}^{N_M}\sum_{j=1}^{N_R} p(i,j)M_i^{-2.56}}^{\overline{\mathrm{MSF}}} \tag{4}$$

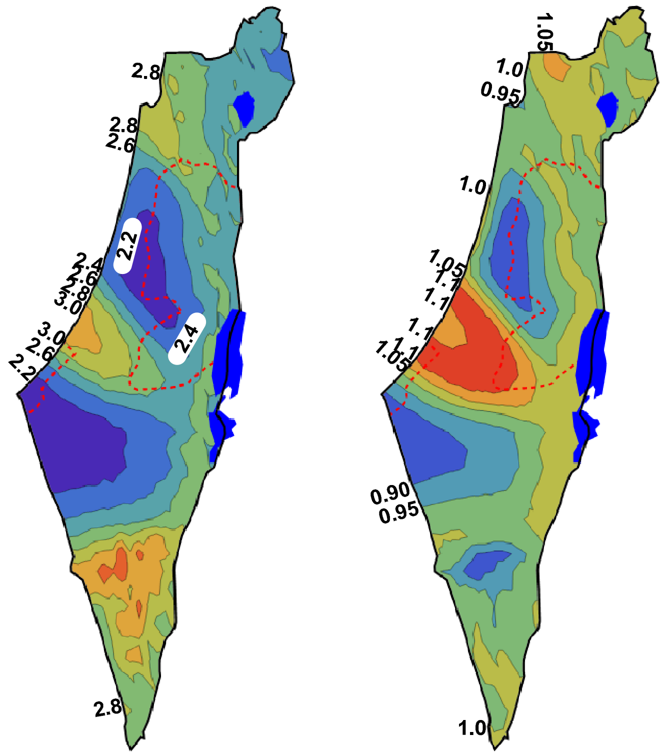
where $\overline{\mathrm{MSF}}$ is the weighted average of MSF that leads Eq. (1) to results with the mean value of factor of safety according to the disaggregation:

$$\overline{\mathrm{FS}} = \frac{\mathrm{CRR}_{7.5}}{\mathrm{CSR}}\overline{\mathrm{MSF}} \tag{5}$$

A regional map of $\overline{\mathrm{MSF}}$ can be constructed based on the disaggregation. Fig. 3(a) shows the $\overline{\mathrm{MSF}}$ map associated with the PGA map of Fig. 1. The $\overline{\mathrm{MSF}}$ is not too sensitive to the specific attenuation model and hence robust. Consequently, it can also be used in conjunction with PGA values obtained from nonlinear site response analysis. Note that this approach of weighted average of MSF is not limited to a specific function, and can be conducted for other MSF functions. Furthermore, this concept, of weighted averages, can be extended to other problems and result in a sequence of maps of values to be multiplied one by another to obtained engineering values of interest.

It should be noted that the derived $\overline{\mathrm{FS}}$ is not necessarily the one associated with exceedance probability of 10%@50year, but with a PGA with exceedance probability of 10%@50years. In a sense, this procedure slightly deviates from the conventional PSHA concept, with respect to the value FS, but allows consistency with the calculation approach in which the PGA value is determined statistically from PSHA. One may view the above procedure as equivalent to Monte-Carlo analysis in which an infinite number of scenarios is considered from all possible sources, all of which with the PGA associated with a return period of 475 years.

Fig. 3(b) shows the standard deviation of $\overline{\mathrm{MSF}}$, $\sigma_{\overline{\mathrm{MSF}}}$. One can use the disaggregation to generate a cumulative density function for the MSF for a PGA event with a return period of 475. Fig. 4 shows the cumulative distribution of the MSF for Haifa and Tel-Aviv with the addition of cumulative density function based on normal and lognormal distribution with $\overline{\text{MSF}}$ and $\sigma_{\overline{\text{MSF}}}$ values from the maps. Since the calculation involves events of magnitude equal or larger than 4 the MSF value cannot be larger than 5 (i.e. $10^{2.24}/4^{2.56}$).



(a) (b)

Figure 3. (a) MSF map, and (b) standard deviation value

This cumulative density function can be used to define the probability, $P[\text{FS}<1]$, for an event with a PGA of return period of 475 year, simply by entering the horizontal axis of Fig. 4 with the value of $0.65\cdot(\text{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d/\text{CRR}_{7.5}$:

$$P[\text{FS}<1]=P[\frac{\text{CRR}_{7.5}}{0.65\cdot(\text{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}\text{MSF}<1]=P[\text{MSF}<\frac{0.65\cdot(\text{PGA}/g)(\sigma_{v0}/\sigma'_{v0})r_d}{\text{CRR}_{7.5}}] \quad (5)$$

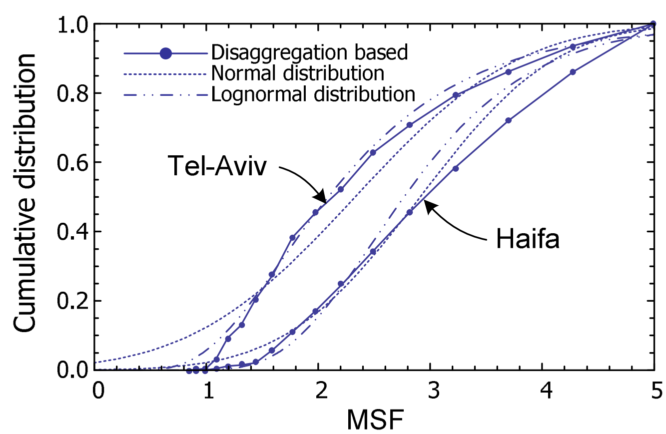


Figure 4. Cumulative distribution for MSF at Haifa and Tel-Aviv

EXAMPLE PROBLEM

Let us consider the two sites mentioned earlier (i.e. Haifa's bay and Tel Aviv). Let us view a certain point in a sand profile, at depth of 5m and with $(N_1)_{60}$ equal to 10. For calculation purposes the water table is assumed to be located at the surface and the unit weight equal to 20 kN/m^3. Both sites have a $CRR_{7.5}$ value of 0.11 and r_d of 0.96. Table 1 shows the calculation values for these two sites.

Table 1. Calculated values for liquefaction in Haifa and Tel-Aviv, $(N_1)_{60}$=10

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Site	PGA	$\overline{\mathrm{MSF}}$	$\sigma_{\overline{\mathrm{MSF}}}$	$\overline{\mathrm{FS}}$	$\overline{\mathrm{FS}}/\mathrm{FS}_{\overline{M_w}}$	$P[\mathrm{FS}<1]$	$P_{ND}[\mathrm{FS}<1]$	$P_{LN}[\mathrm{FS}<1]$
Haifa's bay	0.23	2.91	0.95	1.12	0.97	0.38	0.37	0.42
Tel-Aviv	0.11	2.33	1.15	1.87	1.09	0.11	0.17	0.10

If the factor of safety against liquefaction is calculated with MSF=1, the obtained values are FS=0.38 for Haifa, and 0.80 for Tel-Aviv. Using the average MSF value one obtains a FS of 1.12 for Haifa, and 1.87 for Tel-Aviv, for the given PGAs. These values are in good agreement with those calculated using a magnitude scaling factor that is based on the average magnitude according to the disaggregation (shown in column (f) in the table). The probability that the factor of safety will be smaller than 1.0 can be obtained using Fig. 4, with horizontal axis values of 2.63 (i.e. 1/0.38) for Haifa and 1.25 (i.e. 1/0.80) for Tel-Aviv. These lead to probability of 0.38 for Haifa and 0.11 for Tel-Aviv. The cumulative distribution of FS can also be plotted by scaling the horizontal axis of Fig. 4 according to those factors. Fig. 5 shows this representation. Note that this calculation assumes that $(N_1)_{60}$ is deterministic and there is no uncertainty in the value $CRR_{7.5}$.

The probability of FS being smaller than 1.0 can also be calculated using $\overline{\mathrm{MSF}}$ and $\sigma_{\overline{\mathrm{MSF}}}$. In this case an assumption regarding the nature of FS distribution must be made. If a normal distribution is assumed, the obtained values are 0.37 for Haifa and 0.17 for Tel-Aviv (column (*h*) in the table). If a log-normal distribution is assumed, the obtained values are 0.42 for Haifa and 0.10 for Tel-Aviv (column (*i*) in the table). When comparing the accurate probability and the one based on the assumed distributions, a better agreement exists between the values of Haifa than those of Tel-Aviv. This is probably due to the fact that Haifa is controlled by a single mode while Tel-Aviv by two.

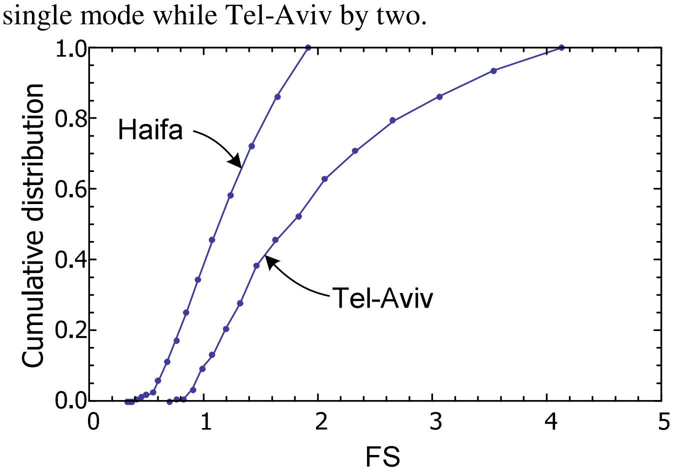


Figure 5. Cumulative distribution for the factor of safety, $(N_1)_{60}$=10

Table 2 shows results of the same calculation, only for the case of $(N_1)_{60}$ equal to 20 (i.e. $CRR_{7.5}$=0.215). As can be seen the agreement between the log-normal distribution value of FS and that calculated based on the disaggregation cumulative distribution are in better agreement for Haifa (same order of magnitude) than for Tel-Aviv. Yet, overall, it appears that relatively to the amount of effort (i.e. the use of only two maps) the calculation based on these assumed distributions for FS gives valuable approximations.

Table 2. Calculated values for liquefaction in Haifa and Tel-Aviv, $(N_1)_{60}$=20

Site	PGA	$\overline{\mathrm{MSF}}$	$\sigma_{\overline{\mathrm{MSF}}}$	$\overline{\mathrm{FS}}$	$\overline{\mathrm{FS}}/\mathrm{FS}_{\overline{M_w}}$	$P[\mathrm{FS}<1]$	$P_{ND}[\mathrm{FS}<1]$	$P_{LN}[\mathrm{FS}<1]$
Haifa's bay	0.23	2.91	0.95	2.17	0.97	0.016	0.049	0.011
Tel-Aviv	0.11	2.33	1.15	3.65	1.09	0.000	0.071	0.005

It should be noted that *P*[FS<1] is not equal to the probability of liquefaction, since the $CRR_{7.5}$ values (or the CRR curve) are associated with the conservative scenario of almost zero liquefaction chance. In other words, the probability for liquefaction is smaller than *P*[FS<1].

The aforementioned probability of liquefaction is not the one the site may experience in an exposure period of 50 years, but the one the site may experience when it is exposed to an earthquake event with a PGA with exceedance probability of 10% in 50 years (i.e. return period of 475 years). In this sense, the approach is more consistent with the design process involved in structural engineering, where the structure is designed to withstand a certain acceleration level with a well defined return period. That is, the FS here is calculated for a well defined acceleration event.

SUMMARY AND CONCLUSION

The magnitude scaling factor is an important component of the "simplified procedure" for liquefaction analysis. However, some of the existing standards and previsions do not give sufficient guidelines for its determination. The use of a magnitude value of 7.5 for all cases (i.e. MSF=1) may lead to over-conservatism in many cases, while the mean or modal values from a disaggregation process may either not be representative or robust. This paper suggests determining the averaged MSF using weighting of the magnitudes obtained from the disaggregation of the PGA. This calculation may results in two maps, one for the averaged MSF and the other for its standard deviation. The paper presents such maps for the State of Israel.

The averaged MSF embodies all possible earthquake events with known exceedance value for the PGA (10% in 50 years in the presented maps), and eliminates the need to choose either the disaggregation mode or the weighted arithmetic mean for determination of the magnitude scaling factor. It is believed that the averaged MSF is relatively insensitive to the site condition and attenuation model, and hence can be used in conjunction with PGA values from nonlinear site response analysis for determination of the factor of safety.

To establish the factor of safety for a site, one may use the suggested MSF maps with general PGA and $CRR_{7.5}$ values specific for the site (and not related to the maps). One can use the standard deviation of the MSF to evaluate the probability that the factor of safety will be smaller than 1.0 in the event of an earthquake with PGA of return period of 475 years. The suggested procedure is demonstrated for two sites in Israel, one at Haifa's bay and the other at Tel-Aviv.

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APPENDIX A

Eq. (2) involves the distribution function $f_M(m)$, taken in the current work as the Guterberg-Richter recurrence law with upper and lower bounds on magnitude:

$$f_M(m) = \frac{\beta \exp[-\beta(m - M_{\min})]}{1 - \exp[-\beta(M_{\max} - M_{\min})]} \tag{A.1}$$

Table A.1 shows the values of α and β associated with each seismogenic zone shown in Fig. 1. The values were taken from the seismological institute of Israel (Shapira, A., 2009; personal communication).

Table A.1. Parameters of seismogenic zones

Zone name	α	β	M_{min}	M_{max}
W. Shiarnb	0.1546	2.21	4	6.0
Med1	0.3956	2.21	4	6.5
Beit Shean	0.0599	2.21	4	6.5
Jordan North	0.1044	2.21	4	5.5
Galil	0.0348	2.21	4	5.5
Roum	0.2887	2.21	4	7.5
Yamune	0.9144	2.21	4	7.75
Jordan V.	0.3729	2.21	4	7.5
Hula-Kinert	0.2526	2.21	4	7.5
Carmel	0.1199	2.21	4	6.5
Dead sea	0.2887	2.21	4	7.5
Central Israel	0.0232	2.21	4	5.5
Med2	0.2277	2.21	4	6.5
Med3	0.2158	2.21	4	6.5
Arava	0.3007	2.21	4	7.5
Suez	2.0425	2.46	4	7.0
Elat	0.1925	2.21	4	7.5
Aragonese	0.1925	2.21	4	7.5
Arnona	0.1925	2.21	4	7.5
Paran	0.0238	2.21	4	6.0
Barak	0.0371	2.21	4	5.5
Thamad	0.0642	2.21	4	6.0
Milhan	0.0162	2.21	4	5.5
Arif	0.0302	2.21	4	5.5
East Sinai	0.0333	2.21	4	6.0
Cyprus	2.7769	2.25	4	8.0
Palmira	0.1189	2.21	4	6.0

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