PROBABILISTIC SEISMIC SITE RESPONSE ANALYSIS

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ABSTRACT

Determination of seismic site response is the important first step in any earthquake – related engineering studies. The most popular approach in current practice, adopted first in the SHAKE computer program in 1972, assumes that the ground profile consisting of an assembly of horizontal soil/rock layers with different material properties. This approach requires selection of an acceleration time history as the representative motion for the design earthquake, usually obtained with a spectral matching process from a recorded "seed" motion. Due to the uncertain nature at many steps of this procedure, the results of a site response analysis from a single time history record may scatter significantly. For a practical nuclear facility site, it may be necessary to perform the same site response analysis many times using 30-60 different time histories to get statistically stable results. Thus, the procedure becomes cumbersome and very time consuming.

An alternative procedure is proposed in this paper to overcome the shortcomings. In this new procedure, the original SHAKE framework of site response analysis is preserved. However, instead of using an acceleration time history as the seismic input, a design spectrum is used as the input motion directly. Power spectrum densities in each step of the procedure are calculated. Extreme values of stress, strain, acceleration and response spectra are derived directly from the power spectrum densities based on relationships obtained from random-vibration-theory. The results represent statistical means of the interested quantities from all possible input time histories fitting the same design spectra. This procedure is coded in a new program P-SHAKE.

Numerical examples included in the paper demonstrate the compatibility of P-SHAKE results with the results of "conventional" SHAKE runs, and the efficiency and easiness of this new procedure in generating statistically meaningful and stable results. This new approach has been used successfully in the site response analysis work of several large scale projects.

Keywords: Seismic Site Response Analysis. Random Vibration Theory, P-SHAKE

INTRODUCTION

In the current engineering practice, seismic design motions at the sites of most critical structures are developed by first generating the rock motions using probabilistic seismic hazard analysis (PSHA) and then by conducting site response analysis through the soil column to include local soil effects. The most popular approach used in a site response analysis, adopted first in the computer program SHAKE (Schnabel et al 1972, Idriss and Sun 1992) and its linear and nonlinear variations, assumes that the ground profile consisting of an assembly of horizontal soil/rock layers with different material properties. One acceleration time history, which usually starts with a recorded motion as the "seed" time history and is

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modified to fit a given design response spectra, is specified at a certain elevation, and the responses of the soil profiles, including stresses, strains, maximum accelerations, response spectra, etc., are computed. Soil non-linearity can be considered through either equivalent linear or non-linear numerical iterative procedures. Figure 1 shows schematically the approach.

Uncertainties arise in using the above procedure on practical engineering projects. One major uncertainty is the appropriate selection of the input time history. It is well known that two acceleration time histories may fit the same design response spectrum, but are different in other important characteristics, i.e., velocity, displacement, Arias intensity, power spectrum density, etc. This is mainly due to the fact that the phasing and energy characteristics of the time history play a significant role on soil column responses, particularly for site conditions where soil nonlinearity becomes important (e.g., Ostadan et al, 1996). The most commonly used approach to reduce this uncertainty is to (1) generate multiple time histories, all are fitted to the same design spectrum and orthogonal to each other but originated from different earthquake recordings, (2) run the site response computation many times, each time using a different time history, and (3) take statistic measures and bonding values from the multiple runs for the quantities interested in the engineering project. This approach can be very cumbersome and time consuming, especially for site response studies of critical structures, e.g., a nuclear power plant, in which a group of 30 – 60 time histories are usually required to obtain statistically stable results. Selection of such a large suite of time histories at sites where only limited recorded motions are available (e.g. Eastern U.S.) is very challenging and often involves modifying the motions from other regions to the project site.

This paper presents an alternative approach for conducting seismic site response analysis which eliminates the need of time-history generation. This approach follows the SHAKE theoretical framework but using random vibration theory (RVT) based formulation for input motion and soil column analysis. This new approach follows three basic steps:

- The input target rock response spectrum is first converted to a power spectrum density (PSD) function.
- The PSD of responses in the soil column are computed based on the input PSD and the transfer functions of the site soil column. The statistical means of the maximum shear strains and effective strains are obtained based on the PSD, and the process is repeated until the strain-compatibility is reached over the entire soil column.
- The PSDs and the statistical means of the maximum responses of other required quantities, such as the acceleration response spectra and maximum accelerations, are computed once convergence on soil properties has been reached.

Figure 2 shows schematically the new approach.

THEORY

Converting an Acceleration Response Spectrum to a Power Spectrum Density Function

It is well known from basic RVT theory (e.g., Der Kiureghian, 1983) that the following relation exists

$$S_d(\omega) = \left|H^2(\omega)\right| S_a(\omega) \tag{1}$$

where $S_d(\omega)$ is the relative displacement PSD, $S_a(\omega)$ is the acceleration PSD, and $H(\omega)$ is the transfer function between displacement response and absolute acceleration input of a single degree of freedom oscillator with frequency ω_o and damping ξ

$$\left|H^2(\omega)\right| = \frac{1}{(\omega_o^2 - \omega^2)^2 + 4\xi^2\omega_o^2\omega^2} \tag{2}$$

The mean of the maximum relative displacement response of the oscillator (definition of a mean relative displacement response spectrum) is given by:

$$D = p\sqrt{\lambda_0} \tag{3}$$

Where p is a peak factor, and λ_0 is the zero moment of the response defined in Equation (6). Following Davenport (1964) and Der Kiureghian (1980)

$$p = \sqrt{2\ln\nu(0)\tau} + \frac{0.5772}{\sqrt{2\ln\nu(0)\tau}} \tag{4}$$

$\nu(0)$ is the mean zero crossing of the response between 0 and τ and equal to:

$$\nu(0) = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{5}$$

τ is taken as the strong motion duration of the earthquake.

The moments of the response are defined as the following

$$\lambda_n = \int_0^\infty \omega^n S_d(\omega)d\omega \tag{6}$$

$n = 0, 1, 2$ for the zero (λ_0), first (λ_1), and second (λ_2) moments of the response.

Following Igusa and Der Kiureghian (1983) and Venmarcke (1975), $\nu(0)$ necessarily is adjusted with the parameter δ, where

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}} \tag{7}$$

The steps to calculate the acceleration power spectral density function from a given acceleration response spectrum are as follows.

1. Convert the acceleration response spectrum $RS_a(\omega)$ to a relative displacement response spectrum $RS_d(\omega)$,
2. Assume an initial acceleration power spectral density function $S_{a,0}(\omega)$, usually a constant value of unity is assumed as the initial value over the frequency range.
3. With the assumed $S_{a,0}(\omega)$ and the relations given above, calculate the mean of the maximum relative displacement response for all the frequencies defining the response spectrum. This will be a new relative displacement response spectrum $RS_{d,1}(\omega)$.

4. Calculate the ratio $R(\omega) = RS_d(\omega)/RS_{d,1}(\omega)$.
5. Correct the assumed acceleration power spectral density function $S_{a,0}(\omega)$ by $R^2(\omega)$ to calculate a new acceleration power spectral density function $S_{a,1}(\omega)$
6. Iterate from step 3 to step 5 until the desired accuracy is reached in the calculation of the displacement response spectrum.

Determine the Mean of Maximum Responses

Having the acceleration PSD $S_a(\omega)$ of the input motion and the transfer function between the input and any desired response $H_r(\omega)$, which is calculated following the normal SHAKE procedure, the steps to calculate the mean of the maximum response are the following:

1. Calculate the PSD of the desired response

$$SR(\omega) = \left|H_r^2(\omega)\right| S_a(\omega) \tag{8}$$

2. Calculate the moments λ_0, λ_1, λ_2 of the response

$$\lambda_n = \int_0^\infty \omega^n SR(\omega) d\omega \tag{9}$$

3. Calculate the peak factor p with these moments as described in Step 1

4. Calculate the mean of the maximum response

$$M_R = p\sqrt{\lambda_0} \tag{10}$$

Where $\boldsymbol{p}$ is the peak factor for the desire response, following the same procedure outlined in Equations (4) through (7) but with the response PSD in Equation (8)

NUMBERICAL EXAMPLE

The above procedure is coded in a computer program P-SHAKE (Bechtel, 2009). We have demonstrated in an earlier study (Deng and Ostadan, 2008) that P-SHAKE results generally are in very good agreement with SHAKE results for an individual earthquake time history. The following numerical example illustrates compatibility of the P-SHAKE results with the SHAKE analysis results and the efficiency of the new approach.

A 1630-ft deep soil profile consisting of various sand, clay and soft rock layers overlaying a rock half-space is being analyzed. The site shear wave velocity profile is shown in Figure 3. A group of strain-degradation curves for shear moduli and damping ratios was assigned to various soil layers, but are not presented here due to space limitations.

The uniform hazard spectrum (UHS) at the rock surface outcrop with 10^{-5} recurrence period was developed through probabilistic seismic hazard analysis (Figure 4). Thirty (30) acceleration time histories were selected as the "seeds" from historical recordings around the site and other earthquakes with similar geological and seismological conditions. Figure 5 shows the overall match of the 30 time histories with the rock UHS, and Figure 6 shows a few of these matched rock time histories.

Two parallel analyses are performed. The first one utilizes the SHAKE program and repeated 30 times using the matched time histories as input motions, one time history at a time. The second one utilizes the P-SHAKE program and uses the 5%-damped UHS the input motion. In both cases, the input motion is specified at top of the rock half-space as outcrop motion.

Figure 7 shows the maximum shear strains developed in the soil profile after convergence has been reached on soil properties. Figure 8 shows the maximum acceleration profile of the results of analysis. And Figure 9 shows the acceleration response spectra at the ground surface. In all the figures, the thin gray lines are from the 30 individual SHAKE analyses which show, as expected, large variations from results of different time histories. The thick red line is the average of all SHAKE analyses. And the thick black line is the P-SHAKE results. It can be observed quite clearly that the two sets of results are in very good to excellent agreement. However, SHAKE requires 30 analyses for the same profile while P-SHAKE needs only one to achieve essentially the same results.

It worth to mention that we have tested many cases in numerous different soil profiles with multiple time histories to demonstrate the compatibility and close agreement between SHAKE and P-SHAKE results. These results are not shown here due to space limitations. The P-SHAKE program is now widely used for major Bechtel projects.

CONCLUSIONS

An alternative approach for seismic site response analysis is presented in this paper. This approach is based on the random vibration theory and works within the theoretical framework of the computer program SHAKE. In this approach, the design input motion is characterized by the design response spectrum directly, all intermediate computations are calculated through PSD and transfer functions, and all responses of interest are calculated as the statistical averages. This approach avoids the difficulties associated with generating multiple spectrum-matching input time histories and is most suitable with the current approach of using a suite of randomized soil profiles for soil amplification.

Numerical examples show that the results computed by the new approach are in good agreement in statistical average with the results computed by the SHAKE program. Thus, all practical engineering experiences and empirical relationships built upon SHAKE are still applicable. This approach has been adopted successfully in site response analysis work of several major nuclear power plant sites and has been accepted by the U.S. Nuclear Regulatory Commission.

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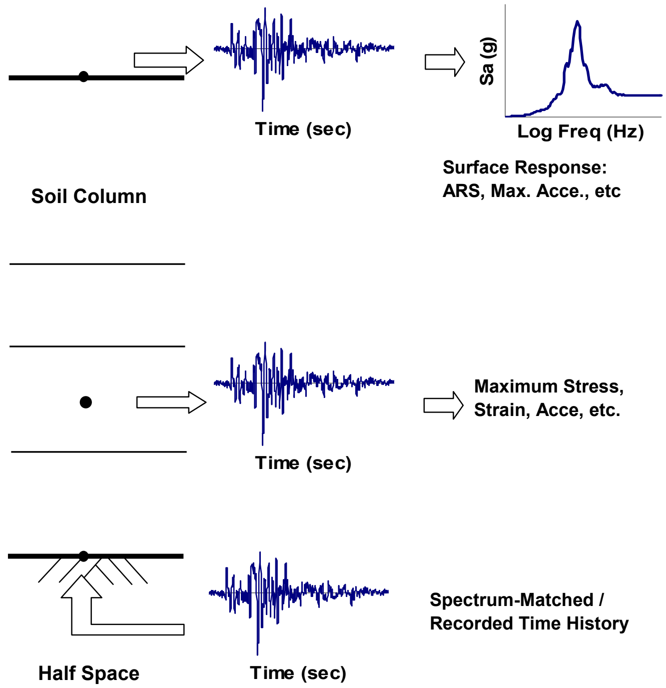

Figure 1. Site response analysis – Time history approach

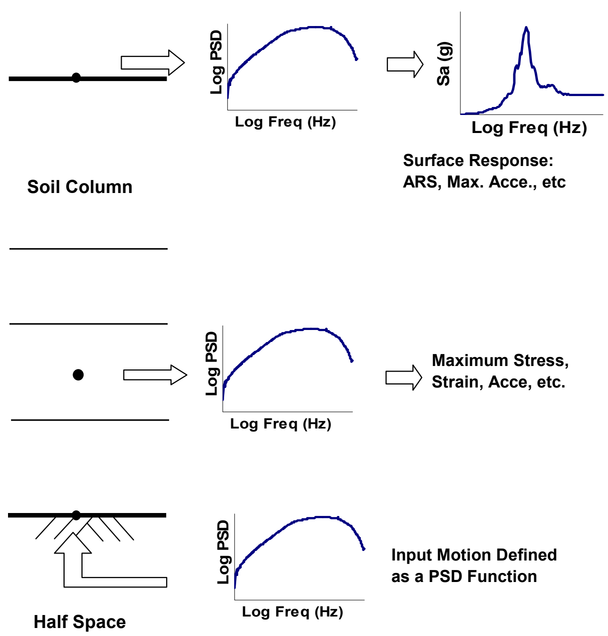

Figure 2. Site response analysis – Alternative approach

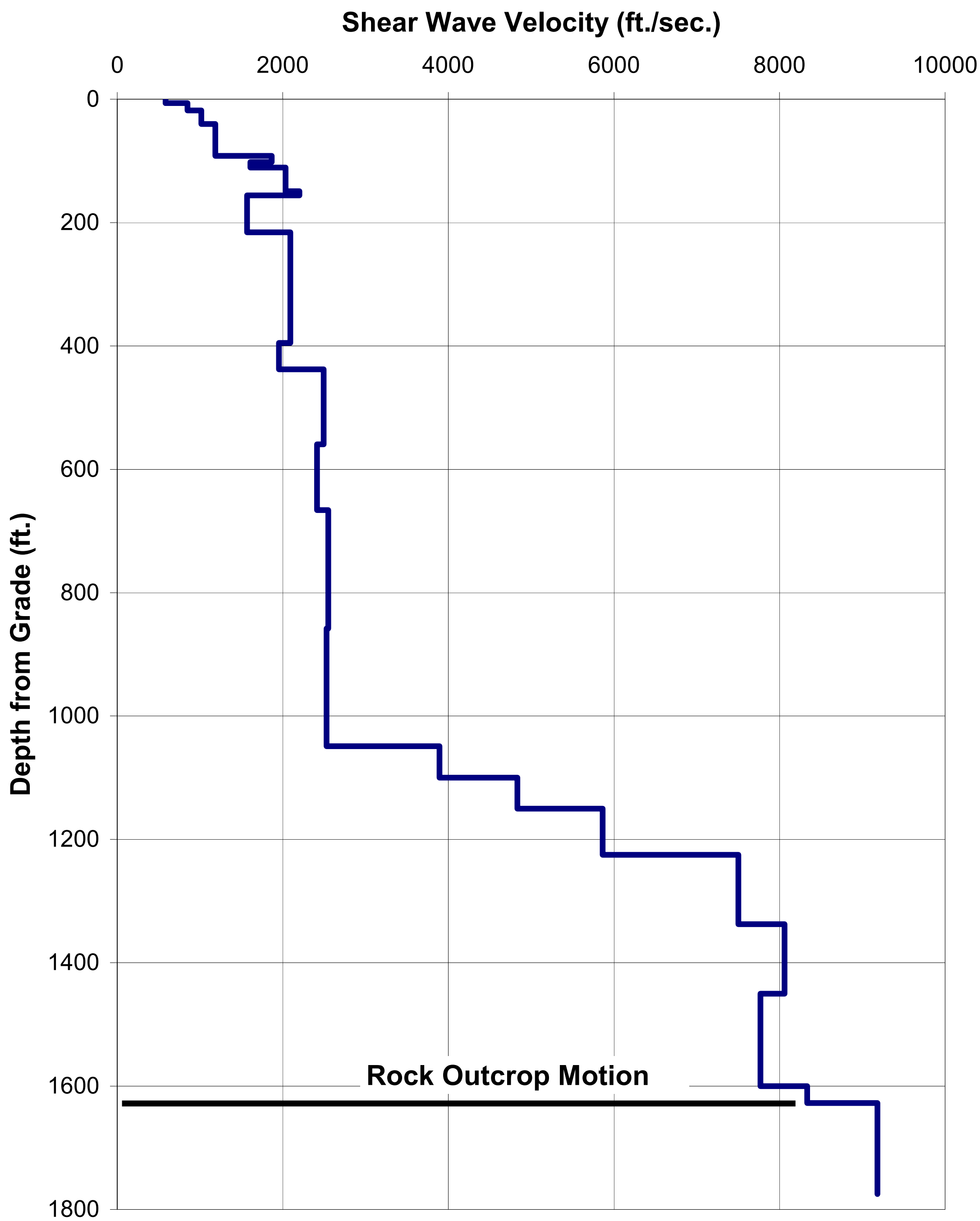

Figure 3. Shear wave velocity profile of the example problem

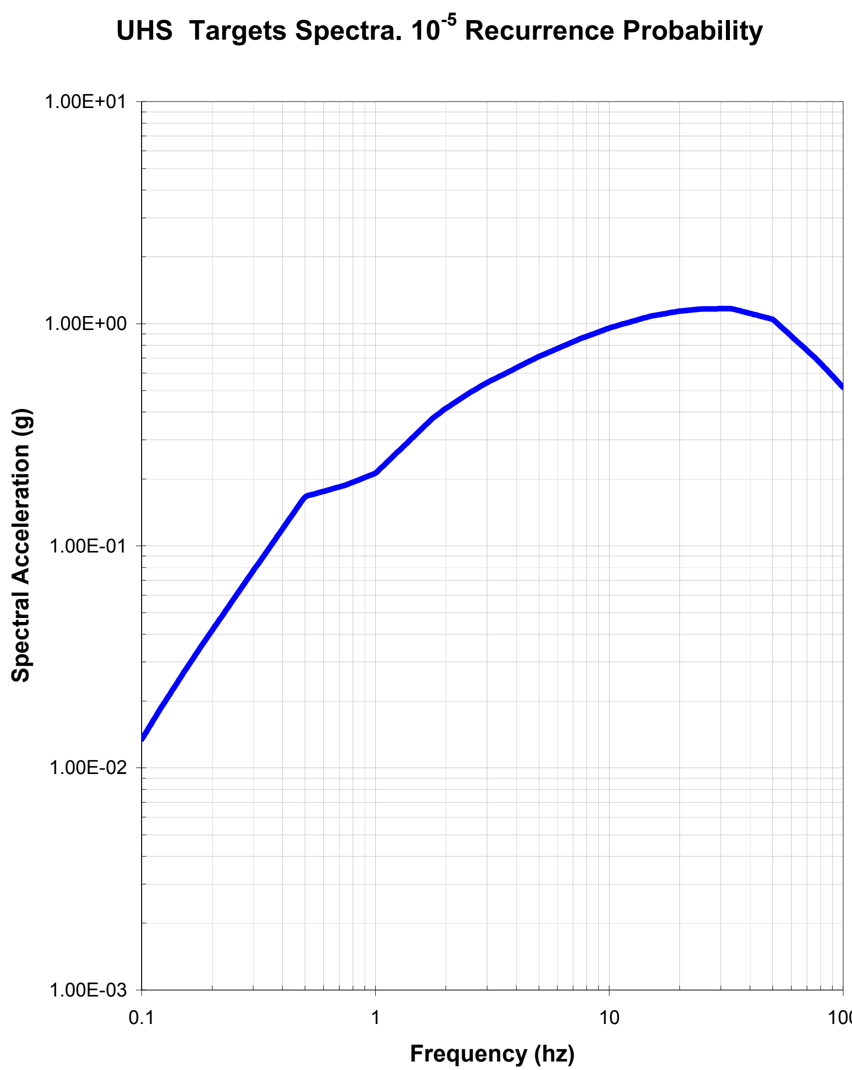

Figure 4. Uniform hazard spectrum of rock motion

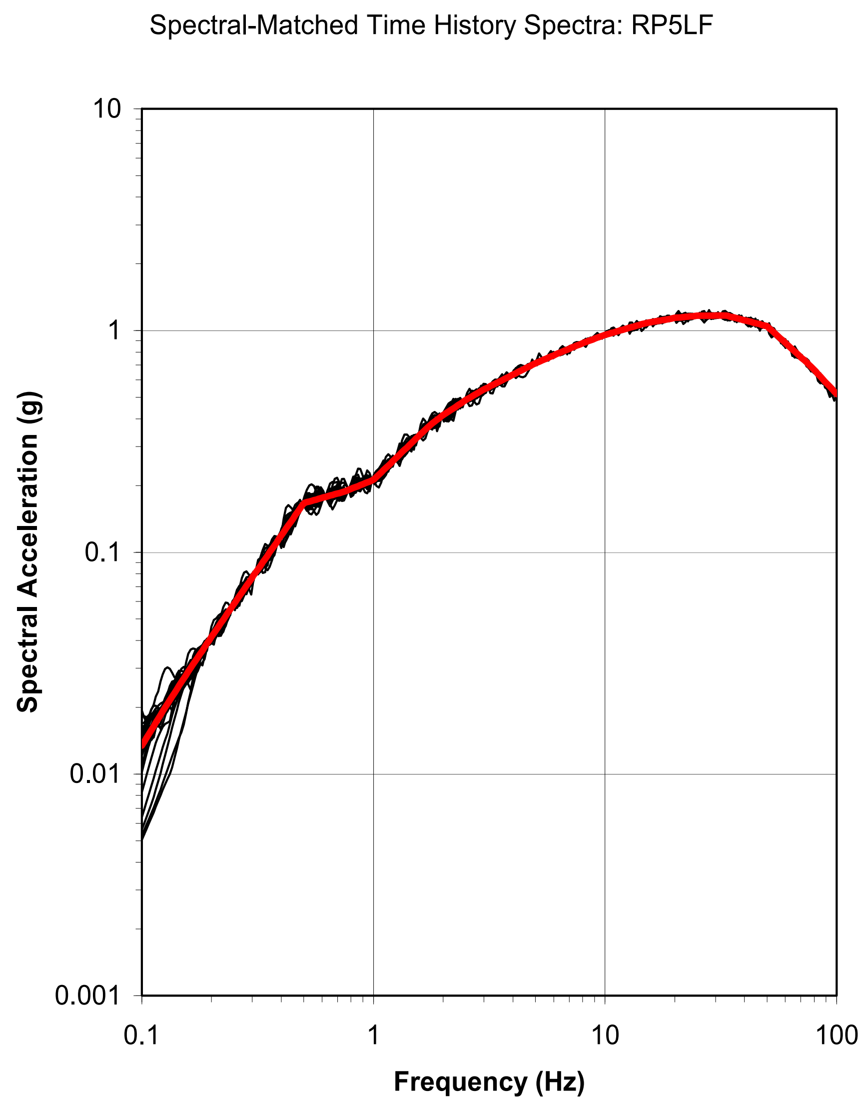

Figure 5. Target spectra-matching of 30 time histories

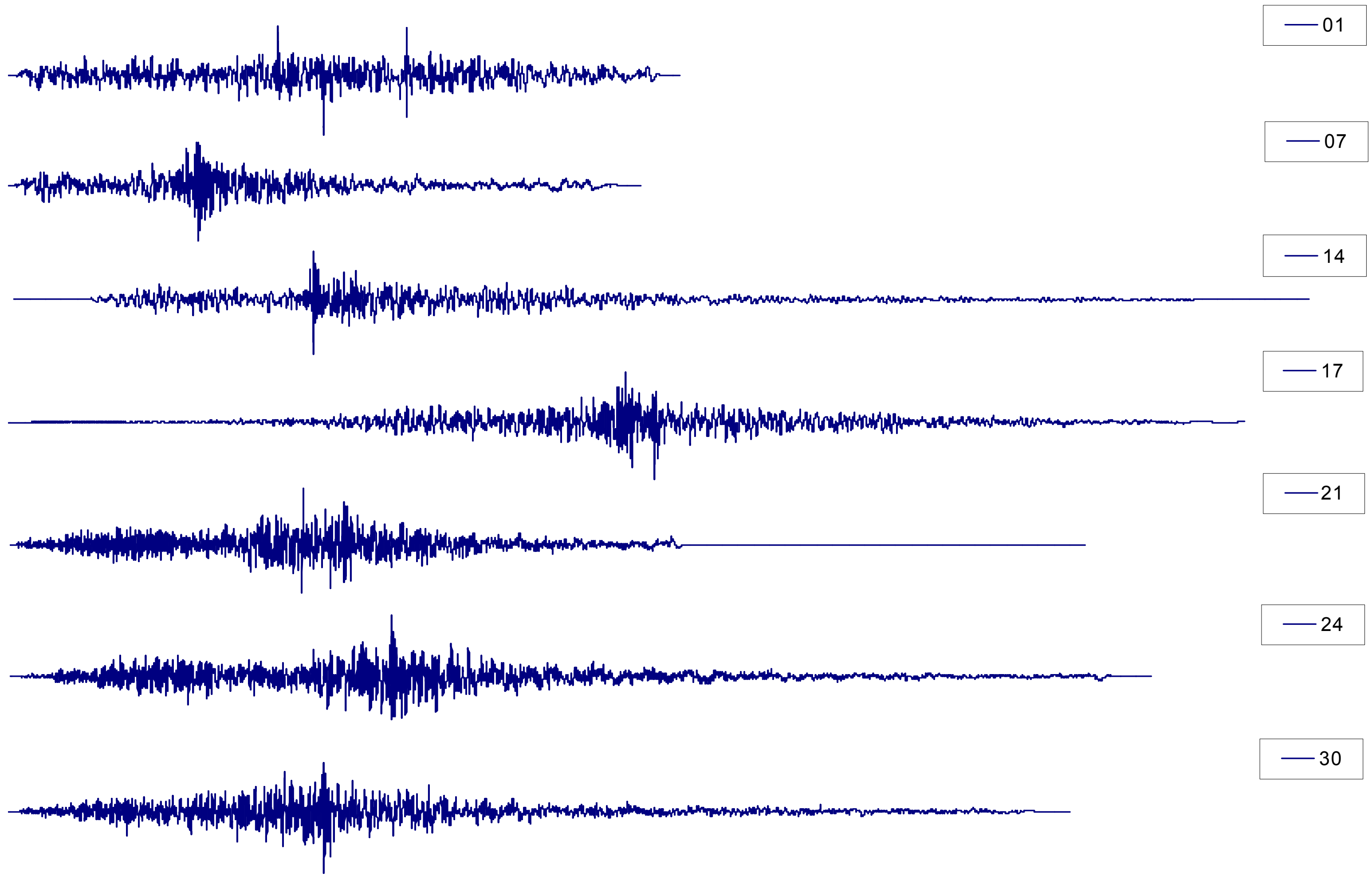

Figure 6. Selected plots of spectra-matched time histories (# in the box is the sequential number)

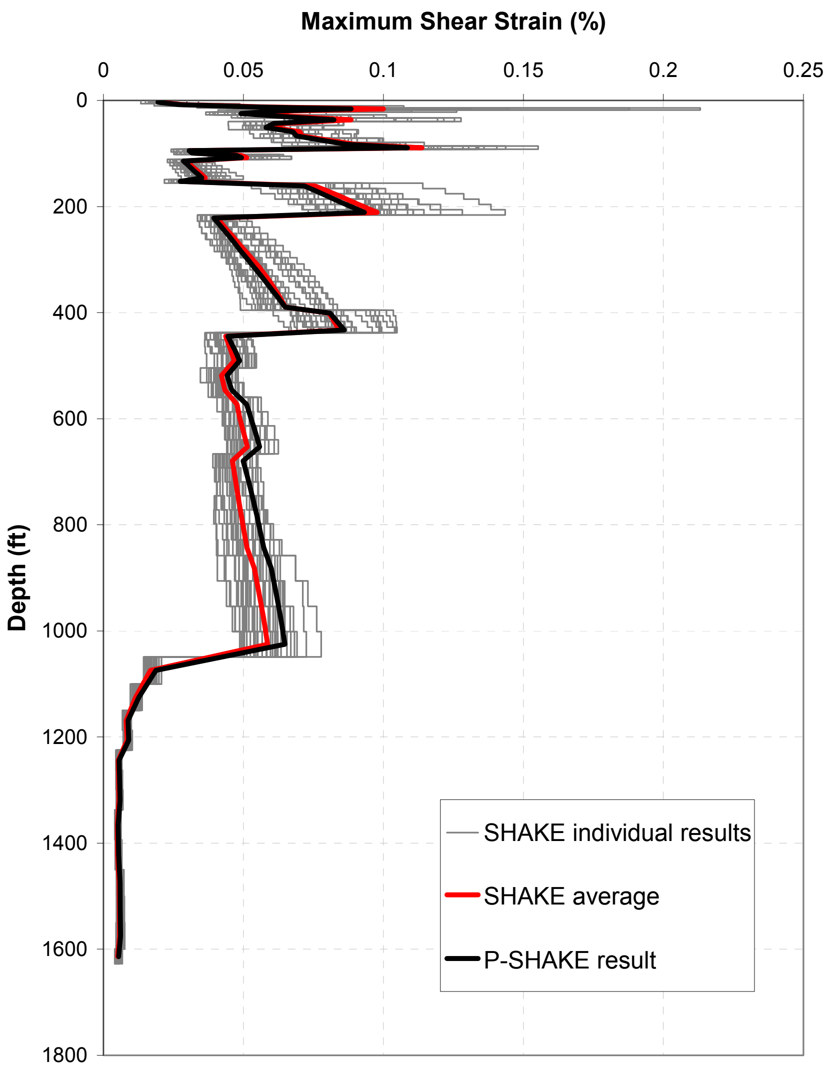

Figure 7. Computed maximum shear strain profile

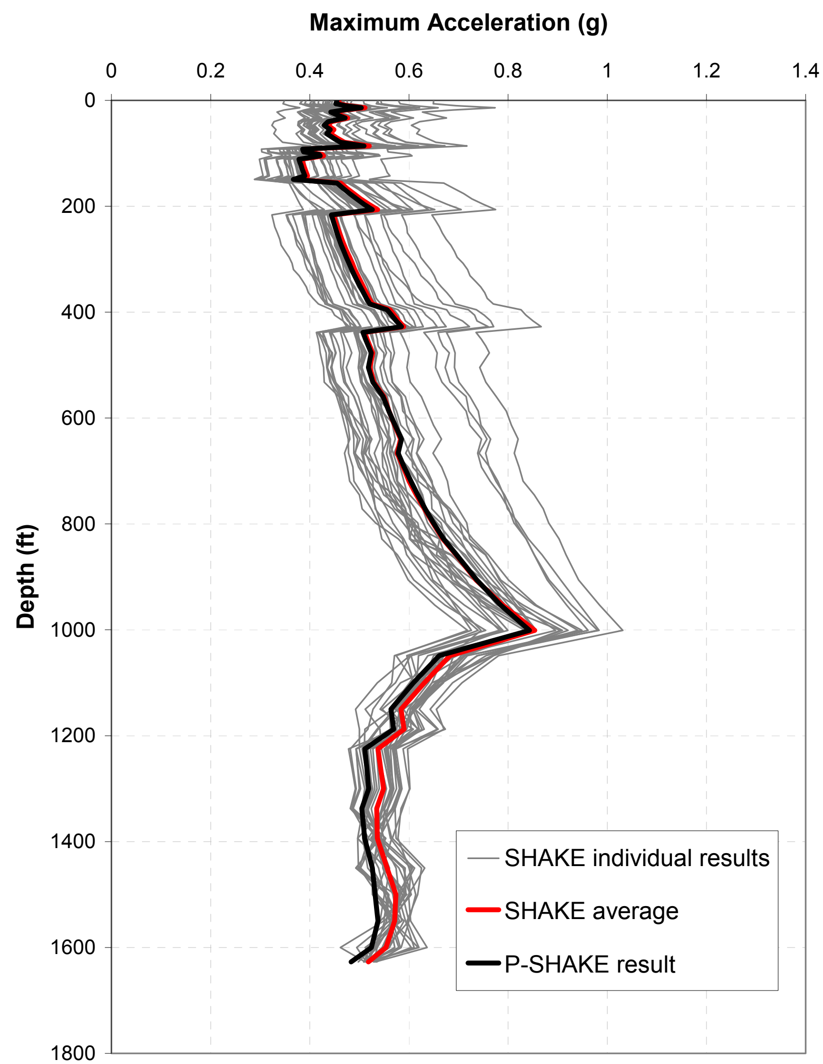

Figure 8. Computed maximum acceleration profile

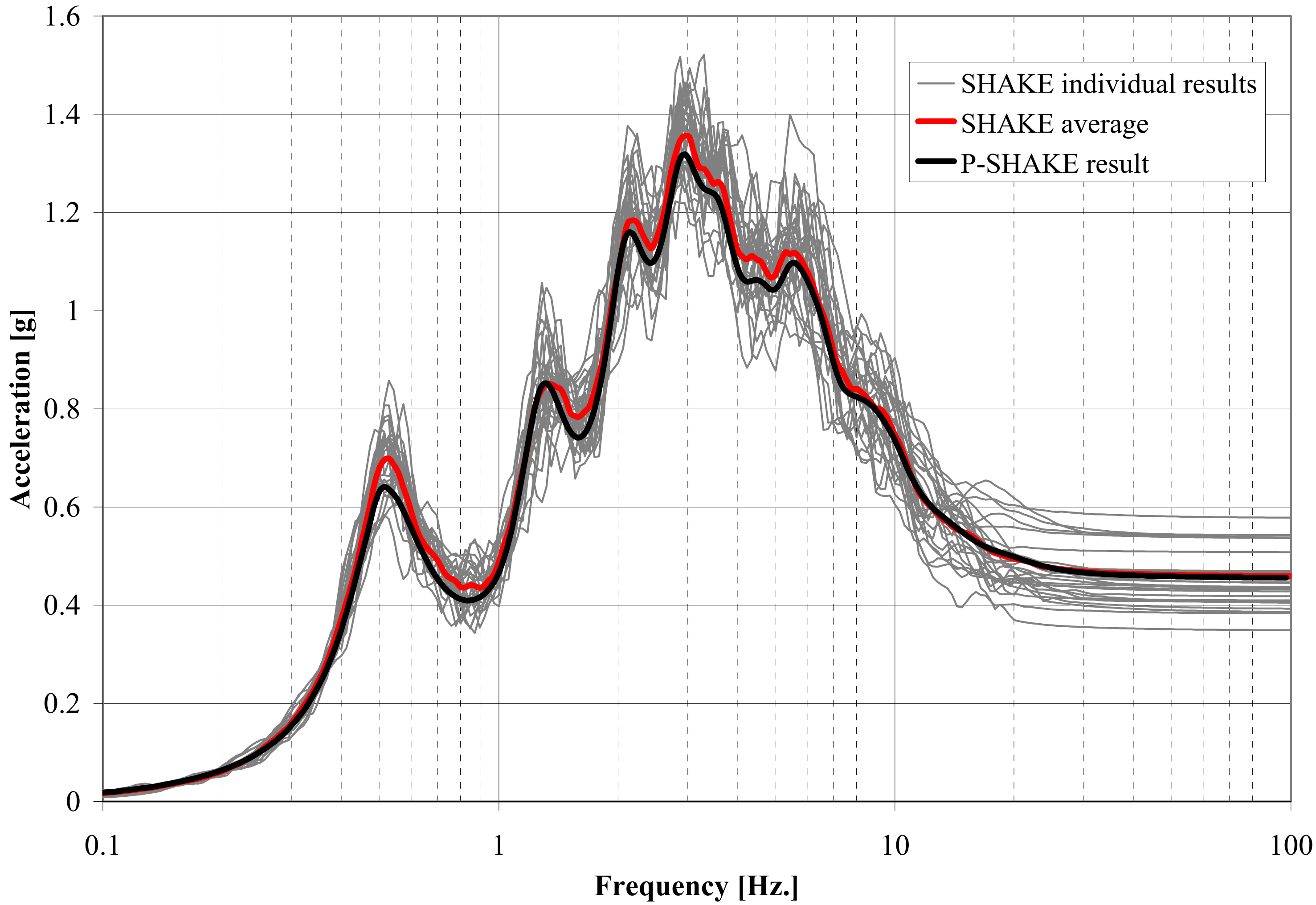

Figure 9. Acceleration response spectra at ground surface